Analytical investigation on tsunamis generated by submarine slides

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Abstract
Tsunamis induced by landslides are a topic on which growing attention is being paid especially under the pressure of recent events in which movement of underwater masses have been recognised to be the certain or likely cause of the observed tsunami. Here analytical methods and idealised cases are used to investigate tsunami generation by submarine slides that undergo negligible deformation during their motion, such as slumps. The general solution of the 1D Cauchy linear problem for long water waves is specialised to deal with rigid bodies and is used systematically to explore the main characteristics of the generated waves. Relationships between body motion, that is prescribed in terms of the slide Froude number, and wave pattern, wave amplitude and wave energy are studied in dimensionless space. Wave generation in various flow conditions (from subcritical to supercritical) is handled, though most attention is given to analysing tsunamis induced by submarine slides at subcritical speed which are by far the most common cases. From numerical experiments it is found that good estimates of the tsunami wave amplitude can be calculated by means of simple expressions based on the maximum value and on the average value of the Froude number during the main generation phase.

Key words *analytical methods – Froude number – shallow-water approximation – tsunami – underwater landslides*

1. Introduction

Tsunamis generated by slumps and landslides can be very severe and even disastrous, as demonstrated by tsunami catalogues (Lander *et al.*, 1993; Tinti and Maramai, 1996; Soloviev *et al.*, 1997) and by recent examples. The last devastating tsunami that killed more than 2200 people in Papua New Guinea in July 1998 was possibly caused by a submarine slump triggered by an offshore earthquake (Davies, 1999; Kawata *et al.*, 1999; Tappin *et al.*, 1999). Nevertheless, most of the research on tsunamis in recent decades has been devoted to tsunamis of seismic origin rather than tsunamis produced by landslides. This preferential orientation can be explained by two main reasons: 1) earthquake-induced tsunamis are more frequent, though not always more catastrophic, and 2) tsunamis engendered by landslides are more difficult to identify and more complex to study. Examples of investigations based on numerical models can be found especially in recent papers (Johnsgard and Pedersen, 1996; Assier Rzadkiewicz, 1997; Watts, 1998; Heinrich *et al.*, 1999; Fine, 1999; Tinti *et al.*, 1999a,b) dealing with simulations of real natural events, or of artificial events produced in hydraulic tanks, or with envisaged scenarios of possible future events. Analytical studies normally address only idealised cases since they cannot handle complications associated with the geometrical complexity of the ocean basin and of

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the landslide, or with complicated details of interaction between sliding body and water. However, these investigations are widely recognised to be essential because they help clarify the fundamental characteristics of the phenomenon and improve our insight and knowledge of basic physical processes (Kajiura, 1970; Noda, 1970, 1971; Iwasaki, 1982; Sabatier, 1983; Pelinovsky, 1996; Pelinovsky and Poplavsky, 1996; Harbitz and Elverhøi, 1999; Tinti and Bortolucci, 2000).

This paper uses analytical methods to investigate tsunami generation by underwater slides. Supposedly the sliding body does not deform during motion and produces waves that can be treated by the linear shallow-water approximation. Submarine slumps detaching from steep slopes at the continental margins are often characterised by little deformation, or at least their deformation is rather ineffectual on tsunami production. Different may be the case of tsunamis generated in a volcanic environment, since debris avalanches from volcanic flanks are formed by cohesionless material that interacting with water experiences substantial changes of shape, and can even evolve partially to turbidity currents. Assuming that the slide behaves like a rigid body is therefore adequate for studying waves induced by underwater slumps. In the case of tsunami generation, the linear long wave theory has been shown (Tinti *et al.*, 2000) to be appropriate when slide height is small with respect to, and slide horizontal scale is large compared to, the water mean depth. These assumptions are not unrealistic, since failures of large sectors of oceanic continental slopes, with lateral dimensions of tens of kilometres and thickness in the range of tens to hundreds of meters moving in 1-4 km deep oceans, have been identified by marine geologists in several places in the world (Hampton *et al.*, 1996). Noticeable, for instance, is the sequence of large submarine landslides which occurred in the Norwegian Sea and known as Storegga slides, the last of which caused a tsunami circa 6-7 ka ago that was proven to hit Norway and Scotland (Dawson *et al.*, 1988; Harbitz, 1992; Bondevik *et al.*, 1997). A rigid body is characterised by its shape and its velocity. The ratio of the instant body speed to the local phase velocity of the free water waves, depending on the local ocean depth, is known as Froude number and plays a fundamental role in determining the generation and evolution of the induced tsunami. In a previous paper (Tinti *et al.*, 2000) it was found that the main characteristics of the tsunami, namely wave pattern, wave amplitude and energy, are mostly influenced by the time history of the Froude number. Practically, tsunamis generated in oceans of different bathymetry by slides running with different speeds may be very similar provided that their associated Froude-number time-histories are the same. Taking advantage of this property, we will make the convenient hypothesis that the sea has constant depth, since this simplifies the analysis but does not narrow the range of applicability of our study. Mathematically, tsunami generation may be viewed as an initial value problem of differential equations of hyperbolic type. Restricting the attention to 1D systems, the general solution can be given by applying the Duhamel theorem (Tinti *et al.*, 2000). In this paper the general Duhamel formula for water elevation is first specialised to the case of a rigid-body slide, which leads to expressions that are more convenient since they are simpler to evaluate and more suitable to physical interpretation. Then, systematic use of the new solving expressions will be made to explore the causative dynamical relationship between the motion of the underwater body and the features of the tsunami generated.

2. The model

Tsunami generation by landslides may be studied by means of the shallow water approximation, implying that all involved variables are independent from the vertical co-ordinate z. Under the further assumption that motion is 1D, namely with all variables supposed to be uniform along the transversal co-ordinate axis y, water waves in a flat ocean can be shown to be governed by the following system of equations (Tinti *et al.*, 2000):

$$\partial_t \xi + \partial_x (uh) = \partial_t h_s \tag{2.1}$$

$$\partial_t u + u \partial_x u + g \partial_x \xi = 0 \tag{2.2}$$

where the following notation has been used: $\xi(x,t)$ is the elevation of the water free surface above the mean sea level, $u(x,t)$ is the velocity of the fluid particle along the axis x, $h(x,t)$ is the instantaneous water depth, $h_s(x,t)$ is the height of the slide and g is the vertical component of the gravity acceleration. In eqs. (2.1) and (2.2) symbols ∂_x and ∂_t denote partial differentiation with respect to space and time respectively. Note that, due to the passage of the slide h_s and the displacement of the sea surface $\xi(x,t)$, water depth changes with position and time according to the following definition:

$$h(x,t) = H - h_s(x,t) + \xi(x,t) \tag{2.3}$$

where H is the undisturbed ocean floor, supposed to be constant. Altogether, eqs. (2.1) and (2.2) form a closed system of two nonlinear differential equations in the unknowns ξ and u, that must be completed by the initial conditions of still water

$$\xi(x,0) = u(x,0) = 0. \tag{2.4}$$

Once the system is solved, ancillary unknowns such as vertical velocity and pressure can be also computed by means of explicit expressions

$$w(x,z,t) = -z\partial_x u + \partial_t \xi + \partial_x(u\xi) \qquad -H+h_s < z < \xi \tag{2.5}$$

$$\mathrm{p}(x,z,t) = \rho g(\xi - z) \qquad -H+h_s < z < \xi \tag{2.6}$$

both showing a linear dependence on depth z. Here ρ designates water density, supposed to be constant throughout the fluid. If the landslide height and water wave amplitude are small compared to ocean floor depth H, system (2.1)-(2.6) can be reduced to the linear form

$$\partial_t \xi + H\partial_x u = \partial_t h_s \tag{2.7}$$

$$\partial_t u + g\partial_x \xi = 0 \tag{2.8}$$

$$\xi(x,0) = u(x,0) = 0 \tag{2.9}$$

with the additional expressions

$$w(x,z,t) = -z\partial_x u + \partial_t \xi \qquad -H < z < 0 \tag{2.10}$$

$$\mathrm{p}(x,z,t) = -\rho g z \qquad -H < z < 0. \tag{2.11}$$

Note that pressure is perfectly hydrostatic and is totally unaffected by slide or water motion in linear theory.

Let us concentrate on the differential problem (2.7)-(2.9) and introduce typical scales such as λ, τ, for space-time co-ordinates x and t respectively. If we introduce further scales for all other quantities, such as d for wave amplitude and slide height, D for water depth and c for phase velocity, then new dimensionless co-ordinates and variables can be defined

$$x' = x/\lambda \qquad y' = y/\lambda \qquad t' = t/\tau \tag{2.12}$$

$$\xi' = \xi/d \quad u' = uD/cd \quad h_s' = h_s/d \quad H' = H/D. \tag{2.13}$$

Scales τ and c satisfy the following relationships:

$$\tau = \lambda/c \qquad c = (gH)^{1/2}. \tag{2.14}$$

As is known, c is the propagation speed of free long waves in flat oceans. Due to the above scaling rules, the equivalent propagation speed in the dimensionless space x' and t' is equal to 1. Furthermore, observing that the depth scale D for a flat ocean of depth H is trivially equal to H, it follows that the corresponding dimensionless sea depth is also 1. The vertical velocity w and the pressure p can be coherently made dimensionless by means of the transformations

$$w' = w\lambda/cd \qquad \mathrm{p}' = \mathrm{p}/\rho g D. \tag{2.15}$$

On making use of definitions (2.12)-(2.15), system (2.7)-(2.9) can be rewritten in non-dimensional variables, and takes the form

$$\partial_t \xi + \partial_x u = \partial_t h_s \tag{2.16}$$

$$\partial_t u + \partial_x \xi = 0 \tag{2.17}$$

$$\xi(x,0) = u(x,0) = 0 \tag{2.18}$$

where primes have been dropped for simplicity. Likewise, eqs. (2.10) and (2.11) can be rewritten in dimensionless form as

$$w(x,z,t) = -z\partial_x u + \partial_t \xi \quad -1 < z < 0 \qquad (2.19)$$

$$\mathrm{p}(x,z,t) = -z \qquad -1 < z < 0. \qquad (2.20)$$

This is the basic system in terms of which generation of waves by underwater bodies can be explored. From a mathematical viewpoint, eqs. (2.16)-(2.18) are a hyperbolic system of linear differential equations describing an initial-value, or Cauchy, problem with forcing, where the forcing term is $\partial_t h_s$ in eq. (2.16).

The set of eqs. (2.16)-(2.18) can be easily transformed to a single second-order differential equation for the unknown water elevation ξ

$$\partial^2_{tt}\xi - \partial^2_{xx}\xi = \partial^2_{tt} h_s \qquad (2.21)$$

where symbols ∂^2_{tt} and ∂^2_{xx} have the obvious meaning of double partial differentiation with respect to time and space respectively, complemented by the additional initial conditions

$$\xi(x,0) = 0 \qquad (2.22)$$

$$\partial_t \xi(x,0) = \partial_t h_s(x,0) \qquad (2.23)$$

and water velocity u can be calculated by means of the explicit expression

$$u = -\int_0^t \partial_x \xi(x,\mathrm{q})\,\mathrm{dq}. \qquad (2.24)$$

It can be shown that problem (2.21)-(2.23) admits a general solution valid for an arbitrary forcing function $h_s(x,t)$ and that, by virtue of Duhamel theorem, this solution can be given the form (Tinti *et al.*, 2000)

$$\xi(x,t) = \frac{1}{2}\int_{x-t}^{x+t} \partial_t h_s(\chi,0)\mathrm{d}\chi + \frac{1}{2}\int_0^t \mathrm{dq} \int_{x-(t-\mathrm{q})}^{x+(t-\mathrm{q})} \mathrm{d}\chi \partial^2_{\mathrm{qq}} h_s(\chi,\mathrm{q}). \qquad (2.25)$$

To be rigorous, it is worth recalling that application of the Duhamel theorem imposes some restrictions on $h_s(x,t)$ that has to be continuous together with its derivatives up to the third order. More precisely, $\partial^2_{\mathrm{qq}} h_s(\chi,\mathrm{q})$ and $\partial^2_{\mathrm{qqx}} h_s(x,\mathrm{q})$ are required to be continuous functions, but these limitations are immaterial for the purposes of this study and do not hamper the relevance of the solving expression (2.25). This formula is quite useful because it can be exploited to investigate tsunami generation by underwater slides of any shape and moving with arbitrary time-history. With only few exceptions, the evaluation of the integrals in eq. (2.25) must be computed numerically. Computation time grows along with the length of the intervals of integration, that for any given position x grows either linearly (first integral) or quadratically (second integral) with time. This paper focuses its attention on tsunami generation by submarine slides such as slumps of coherent material, that often move with little deformation or as rigid bodies. Under this hypothesis formula (2.25) can be further manipulated and given a simpler expression that has the double advantage of being more suitable for numerical evaluation and handier for physical interpretation.

3. Water waves by rigid-body slides

If the underwater slide is assumed to move as a rigid body, then its motion can be univocally described through the instantaneous slide velocity $v(t)$. Notice that in the dimensionless space, the dimensionless body velocity is the ratio of the body speed to the free-wave celerity c, consistently with scaling rules (2.12)-(2.15), a ratio that is known as Froude number. Therefore, v(t) will hereafter be indifferently denoted by slide velocity or Froude number. For a rigid body the slide function $h_s(x,t)$ may be written in the special form

$$h_s(x,t) = h_s(\sigma(x,t)) \qquad \sigma(x,t) = x - s(t) \qquad (3.1)$$

where $s(t)$ is the body displacement at time t, and s(0) may be assumed to be equal to zero with no loss of generality. Coherently, body velocity and acceleration may be defined as

$$v(t) = \mathrm{d}_t s(t) \qquad a(t) = \mathrm{d}_t v(t) \tag{3.2}$$

where symbol d_t denotes total derivation with respect to time. In the special case that the slide moves with constant speed, the argument of h_s simplifies to $\sigma = x - vt$. Let us now see how solution (2.25) can be transformed for a rigid-body slide with kinematics given by eqs. (3.1) and (3.2), and let us start tackling the first integral, whose manipulation is rather straightforward. Considering that

$$\partial_t h_s(x,t) = -v(t)\, \mathrm{d}_\sigma h_s(\sigma(x,t)) \tag{3.3}$$

we obtain

$$\frac{1}{2}\int_{x-t}^{x+t} \partial_t h_s(\chi,0)\mathrm{d}\chi =$$

$$= -\frac{1}{2}\int_{x-t}^{x+t} v(0)\mathrm{d}_\sigma h_s(\chi - s(0))\mathrm{d}\chi = \tag{3.4}$$

$$= -\frac{1}{2}\int_{x-t}^{x+t} v(0)\mathrm{d}_\sigma h_s(\chi)\mathrm{d}\sigma$$

with the last equality deriving from the fact that $\mathrm{d}\chi = \mathrm{d}\sigma$ from (3.1). Since $v(0)$ is constant, the last integral in (3.4) can be easily calculated

$$\frac{1}{2}\int_{x-t}^{x+t} \partial_t h_s(\chi,0)\, \mathrm{d}\chi =$$

$$= -\frac{1}{2} v(0)\, h_s(x+t) + \frac{1}{2} v(0)\, h_s(x-t). \tag{3.5}$$

The above expression is suitable for a ready interpretation. Remembering that the dimensionless velocity of free waves is 1, formula (3.5) turns out to be the sum of two constant-amplitude free water waves travelling in opposite directions: the one propagating forward, that is toward positive x, is positive, while the other is negative. Manipulation of the double integral in expression (2.25) is more lengthy. First of all, bearing in mind that

$$\partial^2_{tt} h_s(\sigma(x,t)) = -a(t)\mathrm{d}_\sigma h_s(\sigma(x,t)) + v^2(t)\mathrm{d}^2_{\sigma\sigma} h_s(\sigma(x,t)) \tag{3.6}$$

we can obtain

$$\frac{1}{2}\int_0^t \mathrm{d}q \int_{x-(t-q)}^{x+(t-q)} \mathrm{d}\chi \partial^2_{qq} h_s(\chi - s(q)) =$$

$$= -\frac{1}{2}\int_0^t \mathrm{d}q a(q) \int_{x-(t-q)}^{x+(t-q)} \mathrm{d}\chi\, \mathrm{d}_\sigma h_s(\sigma(\chi, q)) + \tag{3.7}$$

$$+\frac{1}{2}\int_0^t \mathrm{d}q v^2(q) \int_{x-(t-q)}^{x+(t-q)} \mathrm{d}\chi\, \mathrm{d}^2_{\sigma\sigma} h_s(\sigma(\chi, q)).$$

Then, on remembering that $\mathrm{d}\chi = \mathrm{d}\sigma$ and after posing

$$\begin{aligned} \alpha(x,t,q) &= x-(t-q)-s(q) \\ \beta(x,t,q) &= x+(t-q)-s(q) \end{aligned} \tag{3.8}$$

the right-hand-side member (r.h.m) of equality (3.7) can be rewritten in the following form:

$$\begin{aligned} \text{r.h.m.} = &-\frac{1}{2}\int_0^t \mathrm{d}q\, a(q) \int_{\alpha(x,t,q)}^{\beta(x,t,q)} \mathrm{d}\sigma\, \mathrm{d}_\sigma h_s(\sigma) + \\ &+\frac{1}{2}\int_0^t \mathrm{d}q v^2(q) \int_{\alpha(x,t,q)}^{\beta(x,t,q)} \mathrm{d}\sigma\, \mathrm{d}^2_{\sigma\sigma} h_s(\sigma) \end{aligned} \tag{3.9}$$

where the internal integrals can be easily calculated

$$\begin{aligned} \text{r.h.m.} = &-\frac{1}{2}\int_0^t \mathrm{d}q\, a(q)\, h_s(\beta(x,t,q)) + \\ &+\frac{1}{2}\int_0^t \mathrm{d}q\, a(q)\, h_s(\alpha(x,t,q)) + \\ &+\frac{1}{2}\int_0^t \mathrm{d}q v^2(q)\, \mathrm{d}_\beta h_s(\beta(x,t,q)) - \\ &-\frac{1}{2}\int_0^t \mathrm{d}q v^2(q)\, \mathrm{d}_\alpha h_s(\alpha(x,t,q)). \end{aligned} \tag{3.10}$$

Notice that in passing from the original double integral in formula (2.25) to the above expression (3.10) containing only simple integrals, a remarkable advantage has already been obtained in terms of numerical computation time. Let us now concentrate on the third and fourth integrals of eq. (3.10), that can be further manipulated. Beginning from the third one, since

$$\partial_q h_s(\beta(x,t,\mathrm{q})) = -(1+v(q))\,\mathrm{d}_\beta h_s(\beta(x,t,\mathrm{q})) \quad (3.11)$$

we can write it in the form

$$\text{third int.} = -\frac{1}{2}\int_0^t \mathrm{dq}\,\frac{v^2(\mathrm{q})}{1+v(\mathrm{q})}\,\partial_{\mathrm{q}} h_s(\beta(x,t,\mathrm{q})) \tag{3.12}$$

that can be further transformed by means of an integration by parts. After some computations, it can be combined together with the first integral of eq. (3.10), and their sum can be written as

$$\text{first int.} + \text{third int.} =$$

$$= -\frac{1}{2}\int_0^t \mathrm{dq}\,\frac{a(\mathrm{q})}{(1+v(\mathrm{q}))^2}\,h_s(\beta(x,t,\mathrm{q})) - \frac{1}{2}\frac{v^2(t)}{1+v(t)}\,h_s(x-s(t)) + \frac{1}{2}\frac{v^2(0)}{1+v(0)}\,h_s(x+t). \tag{3.13}$$

The fourth integral can be processed in an analogous way. After considering that

$$\partial_q h_s(\alpha(x,t,\mathrm{q})) = (1-v(q))\,\mathrm{d}_\alpha h_s(\alpha(x,t,\mathrm{q})) \quad (3.14)$$

we can first obtain

$$\text{fourth int.} = -\frac{1}{2}\int_0^t \mathrm{dq}\,\frac{v^2(\mathrm{q})}{1+v(\mathrm{q})}\,\partial_{\mathrm{q}} h_s(\alpha(x,t,\mathrm{q})). \tag{3.15}$$

After carrying out some more calculations, it can be added to the second integral of eq. (3.10) resulting in the following expression:

$$\text{second int.} + \text{fourth int.}$$

$$= \frac{1}{2}\int_0^t \mathrm{dq}\,\frac{a(\mathrm{q})}{(1-v(\mathrm{q}))^2}\,h_s(\alpha(x,t,\mathrm{q})) - \frac{1}{2}\frac{v^2(t)}{1-v(t)}\,h_s(x-s(t)) + \frac{1}{2}\frac{v^2(0)}{1-v(0)}\,h_s(x-t). \tag{3.16}$$

Now the final expression for the r.h.m. of eq. (3.10) can be obtained by summing up the contributions of all four integrals together, which results in

$$\begin{aligned} \text{r.h.m.} = &-\frac{1}{2}\int_0^t \mathrm{dq}\,\frac{a(\mathrm{q})}{(1+v(\mathrm{q}))^2}\,h_s(\beta(x,t,\mathrm{q})) + \\ &+\frac{1}{2}\int_0^t \mathrm{dq}\,\frac{a(\mathrm{q})}{(1-v(\mathrm{q}))^2}\,h_s(\alpha(x,t,\mathrm{q})) + \\ &+\frac{v^2(t)}{v^2(t)-1}\,h_s(x-s(t)) + \\ &+\frac{1}{2}\frac{v^2(0)}{1+v(0)}\,h_s(x+t) + \frac{1}{2}\frac{v^2(0)}{1-v(0)}\,h_s(x-t). \end{aligned} \tag{3.17}$$

At this point it is straightforward to obtain the final form of the solving formula (2.25) in case of a rigid body, since it is sufficient to add expression (3.5) to the above expression (3.17), which gives

$$\begin{aligned} \xi(x,t) = &\ \frac{1}{2}\int_0^t \mathrm{dq}\,\frac{a(\mathrm{q})}{(1+v(\mathrm{q}))^2}\,h_s(\beta(x,t,\mathrm{q})) + \\ &+\frac{1}{2}\int_0^t \mathrm{dq}\,\frac{a(\mathrm{q})}{(1-v(\mathrm{q}))^2}\,h_s(\alpha(x,t,\mathrm{q})) + \\ &+\frac{v^2(t)}{v^2(t)-1}\,h_s(x-s(t)) - \frac{1}{2}\frac{v(0)}{1+v(0)}\,h_s(x+t) + \\ &+\frac{1}{2}\frac{v(0)}{1-v(0)}\,h_s(x-t). \end{aligned} \tag{3.18}$$

This formula is the solving expression of the problem in case of sliding rigid bodies. Mathematically, it is perfectly equivalent to eq. (2.25), but it has some important advantages. It contains only first integrals, that can be calculated much faster than double integrals on a computer, and, most importantly, it has a structure that enables us to gain a deeper physical insight on the generation process. It tells us that the wave field generated by the sliding body can be interpreted as the sum of five distinct contributions, corresponding to the five terms of the sum. The last two terms depend on the initial velocity of the body $v(0)$, and both are null if the body starts from rest, as is normally assumed. They are free waves propagating in opposite directions: the backward-going wave is always a trough, whereas the wave travelling forward is a crest or a trough depending on the sign of the denominator $1-v(0)$. If $v(0)$ is smaller than 1 (subcritical condition), it is positive, otherwise (supercritical condition) it is negative. The third term is a wave moving exactly together with the slide, and may be therefore seen as a forced wave, with amplitude depending only on the instantaneous Froude number. It is a crest or a trough respectively for supercritical or subcritical flow. This wave exists only as long as the slide is in motion, being identically zero when and after the slide stops. The first and second terms are integrals that may be viewed as superposition of free waves generated at different times. For example, the first integral represents the superposition of free waves $h_s(\beta(x,t,\mathrm{q}))$ propagating backward. They are generated by the slide at the time q at the corresponding current position of the slide x-$s(\mathrm{q})$, with amplitude depending on the weighing factor in the integral, namely $a(\mathrm{q})/(1+v(\mathrm{q}))^2$. Due to the negative sign preceding the integral, these waves are troughs if body acceleration is positive, and crests if the body decelerates. Extension of the integral over the interval $[0,t]$ means that all free waves produced until time t contribute to form the water elevation at time t. A similar interpretation can be provided for the second integral of formula (3.18), representing the superposition of free waves travelling forward. Notice that, also in this case, wave sign depends on body acceleration: crests and troughs correspond respectively to accelerating and decelerating phases of body motion.

It is worth observing that expression (3.18) also holds when the slide moves with constant speed. In this case, body acceleration is zero and consequently both integrals of eq. (3.18) vanish identically, which results in a very simple expression. The resulting wave can be written as

$$\xi(x,t) = \xi_F(x,t) + \xi_+(x,t) + \xi_-(x,t) \qquad (3.19)$$

where

$$\xi_F(x,t) = \frac{v^2}{v^2-1} h_s(x-vt) \qquad v \neq 1 \qquad (3.20)$$

$$\xi_+(x,t) = -\frac{1}{2}\frac{v}{v-1} h_s(x-t) \qquad v \neq 1 \qquad (3.21)$$

$$\xi_-(x,t) = -\frac{1}{2}\frac{v}{v+1} h_s(x+t) \qquad (3.22)$$

and the total wave $\xi(x,t)$ is the sum of three waves, *viz.* the forced wave $\xi_F(x,t)$, the advancing free wave $\xi_+(x,t)$ and the regressing free wave $\xi_-(x,t)$.

The original linear problem (2.16)-(2.20) is mathematically well posed and always admits a solution that is provided by formula (2.25), having general validity. However, solution (3.18) as well as the special solution (3.19)-(3.22) hold when the Froude number differs from 1. The amplitude of waves travelling forward (free waves and forced wave) diverges as body velocity approaches free wave celerity, which is a case that is known as the critical regime. Technically, the differential relationship (3.14) is no longer valid if $v(t) = 1$, and consequently the transformation of the fourth integral (3.15) is not allowed. To avoid this divergence problem, we can assume that body speed is always smaller than 1, which is the most common case for real slides, since water resistance prevents the body gaining supercritical velocity. We can also assume that the slide always moves with velocity larger than 1 within the time interval of interest, but this would restrict the ambit of our study unacceptably. We must then revert to the prior derivation of formula (3.18) to provide an expression of general validity, which can be obtained if transformations (3.14) to (3.16) are not performed. In this case, after some calculations, it is possible to write down the resulting expression for water elevation $\xi(x,t)$, as follows:

$$\xi(x,t) = -\frac{1}{2}\int_0^t \mathrm{dq}\frac{a(\mathrm{q})}{(1+v(\mathrm{q}))^2} h_s(\beta(x,t,\mathrm{q})) +$$

$$+\frac{1}{2}\int_0^t \mathrm{dq}\, a(\mathrm{q})\, h_s(\alpha(x,t,\mathrm{q})) +$$

$$-\frac{1}{2}\int_0^t \mathrm{dq}\, v^2(\mathrm{q})\, \mathrm{d}_\alpha\, h_s(\alpha(x,t,\mathrm{q})) + \qquad (3.23)$$

$$-\frac{1}{2}\frac{v^2(t)}{1+v(t)} h_s\,(x-s(t)) +$$

$$-\frac{1}{2}\frac{v(0)}{1+v(0)} h_s\,(x+t) + \frac{1}{2}v(0) h_s(x-t).$$

This formula holds for any value of body speed. Like expression (3.18), it contains only simple integrals that can be readily computed. Expression (3.18) is, however, simpler and more convenient to use. Therefore, it is preferable to use formula (3.18) whenever permitted, and (3.23) in all other cases. When the underwater body moves at constant critical speed, formula (3.23) reduces to the same expression derived by means of the theory of characteristics by Tinti and Bortolucci (2000)

$$\xi(x,t) = \xi_F(x,t) + \xi_+(x,t) + \xi_-(x,t) \qquad (3.24)$$

with

$$\xi_F(x,t) = -\frac{1}{2} t\, \mathrm{d}_\sigma h_s(x-t) \qquad v=1 \qquad (3.25)$$

$$\xi_+(x,t) = \frac{1}{4} h_s(x-t) \qquad v=1 \qquad (3.26)$$

$$\xi_-(x,t) = -\frac{1}{4} h_s(x+t) \qquad v=1. \qquad (3.27)$$

Notice that the forced wave (3.25) grows unlimited as time goes by, suggesting that slide forcing at critical speed is able to supply energy to the water with no limit, like forcing an elastic mechanical system excited in resonance conditions. Notice further that since forced and free wave (3.25) and (3.26) advance exactly with the same unit speed, they never separate, and practically what can be observed is only their combination at any one time.

4. Energy of the tsunami

Energy of the tsunami excited by a submarine slide satisfies the following dimensionless equation in a 1D flat ocean in the shallow water linear approximation (Tinti and Bortolucci, 2000)

$$\partial_t \varepsilon + \partial_x(\xi u) = \xi \partial_t h_s \qquad (4.1)$$

where ε is the density of total tsunami energy per unit width and unit length of the ocean. The term on the right-hand-side member of this equation plays the role of an energy source or sink: if positive, it injects energy into the water, whereas when it is negative it subtracts energy from the water. If the total tsunami energy per unit ocean width is denoted by E, *i.e.* if

$$\mathrm{E}(t) = \int_{-\infty}^{+\infty} \varepsilon(x,t)\, \mathrm{d}x \qquad (4.2)$$

in the light of (4.1) we can write

$$\mathrm{d}_t \mathrm{E}(t) = \int_{-\infty}^{+\infty} \xi(x,t)\, \partial_t h_s(x,t)\, \mathrm{d}x\,. \qquad (4.3)$$

Hence, on making use of relationship (3.3), the total energy per unit width that the sliding rigid body transfers to the water since the beginning of its motion may be written as

$$\mathrm{E}(t) = -\int_0^t \mathrm{dq}\, v(\mathrm{q}) \int_{-\infty}^{+\infty} \mathrm{d}x\, \xi(x,\mathrm{q})\, \mathrm{d}_\sigma h_s(x,\mathrm{q})\,. \qquad (4.4)$$

The above space integral is extended over the entire x axis, which could create difficulties for proper numerical computations, but indeed its integrand differs from zero only in correspondence of the space interval that is instantaneously occupied by the body, an interval that is finite since the body has finite extension. This makes this formula suitable for easy numerical evaluation. It is worth recalling that in the above formula (4.4) the energy per unit width is dimensionless, and that in agreement with scaling laws (2.12)-(2.15), the corresponding dimensional quantity can be obtained by multiplying E by $\rho g \lambda d^2$, which represents therefore the scale of the energy density for our problem.

5. Motion of the slide

In cases where shallow water theory can be applied, the main near-field characteristics of tsunamis excited by rigid bodies in oceans with different bathymetries are similar under the provision that the bodies have the same Froude number time histories (Tinti and Bortolucci, 2000). In virtue of this property, the theory developed in previous sections for a flat ocean can also be useful to investigate the generation of waves in a sea of variable depth. In order to obtain Froude number curves that are typical of real cases, let us focus on the motion of the slide. It can be studied by means of the classical Newtonian equations governing rigid-body dynamics. In 1D cases bodies sliding over the bottom floor can be analysed by only one equation describing the body centre-of-mass displacement from its initial position. The body moves under the action of gravity, and its motion is chiefly influenced by the buoyancy force exerted by the ocean water, by the bottom friction and by the resistant drag force, that is often assumed as proportional to the square of the body speed (Tinti *et al.*, 1999a,b). Over an incline with constant slope θ, slide kinematics can be expressed by means of a very simple law (Watts, 1998), that is

$$S(t) = S_0 \ln\left[\cosh\left(\frac{A_0}{U_t}t\right)\right], \quad S_0 = \frac{U_t^2}{A_0} \tag{5.1}$$

$$U(t) = U_t \tanh\left(\frac{A_0}{U_t}t\right) \tag{5.2}$$

$$A(t) = A_0 \operatorname{sech}^2\left(\frac{A_0}{U_t}t\right). \tag{5.3}$$

Notice that all variables in expressions (5.1)-(5.3) are dimensional: S, U and A are respectively the body displacement, velocity and acceleration measured along the slope at the dimensional time t. The meaning of the constants U_t and A_0 can be straightforwardly deduced from the equations: the body moves with initial acceleration A_0, running faster and faster until it approaches the terminal constant velocity U_t. The quantity S_0 represents a typical distance covered by the body to approach the asymptotic constant-speed regime. The Froude number associated with the motion laws (5.1)-(5.3) *versus* dimensionless time t is

$$v(t) = \frac{U(\tau t)\cos\theta}{\sqrt{g(H_0 + S(\tau t)\sin\theta)}} \tag{5.4}$$

where H_0 is the water depth corresponding to the initial position of the body, and it is clear that the Froude-number time history (5.4) depends upon the depth parameters H_0 and θ in addition to the kinematic parameters A_0 and U_t. The time scale τ can be conveniently taken to be equal to λ/c according to expression (2.14), with λ being the horizontal slide length and c being the free-wave phase speed corresponding to a typical depth of the generation region, namely $H_0 + 0.5\, S_0 \sin\theta$. Furthermore it is easy to see that

$$v(0) = 0 \qquad d_t v(0) = \frac{A_0 \tau \cos\theta}{\sqrt{gH_0}} \tag{5.5}$$

and that

$$v(t) = \sqrt{\frac{U_t}{g\,\tau t \sin\theta}}\cos\theta \quad \text{for large } t. \tag{5.6}$$

Typically the Froude number increases rapidly until it reaches its maximum value, say v_{Max}, and then decreases more slowly, going to zero according to a $t^{-1/2}$ power decay law.

6. Numerical experiments and results

Numerical experiments were performed using Froude number curves satisfying expression (5.4) with a broad range of kinematic parameters covering field cases as well as small-scale laboratory trials with blocks sliding in water tanks. A set of 11 curves denoted by C1 to C11, plotted *versus* dimensionless time, are shown in fig. 1, and are divided into three subsets. The corresponding relevant parameters are listed in

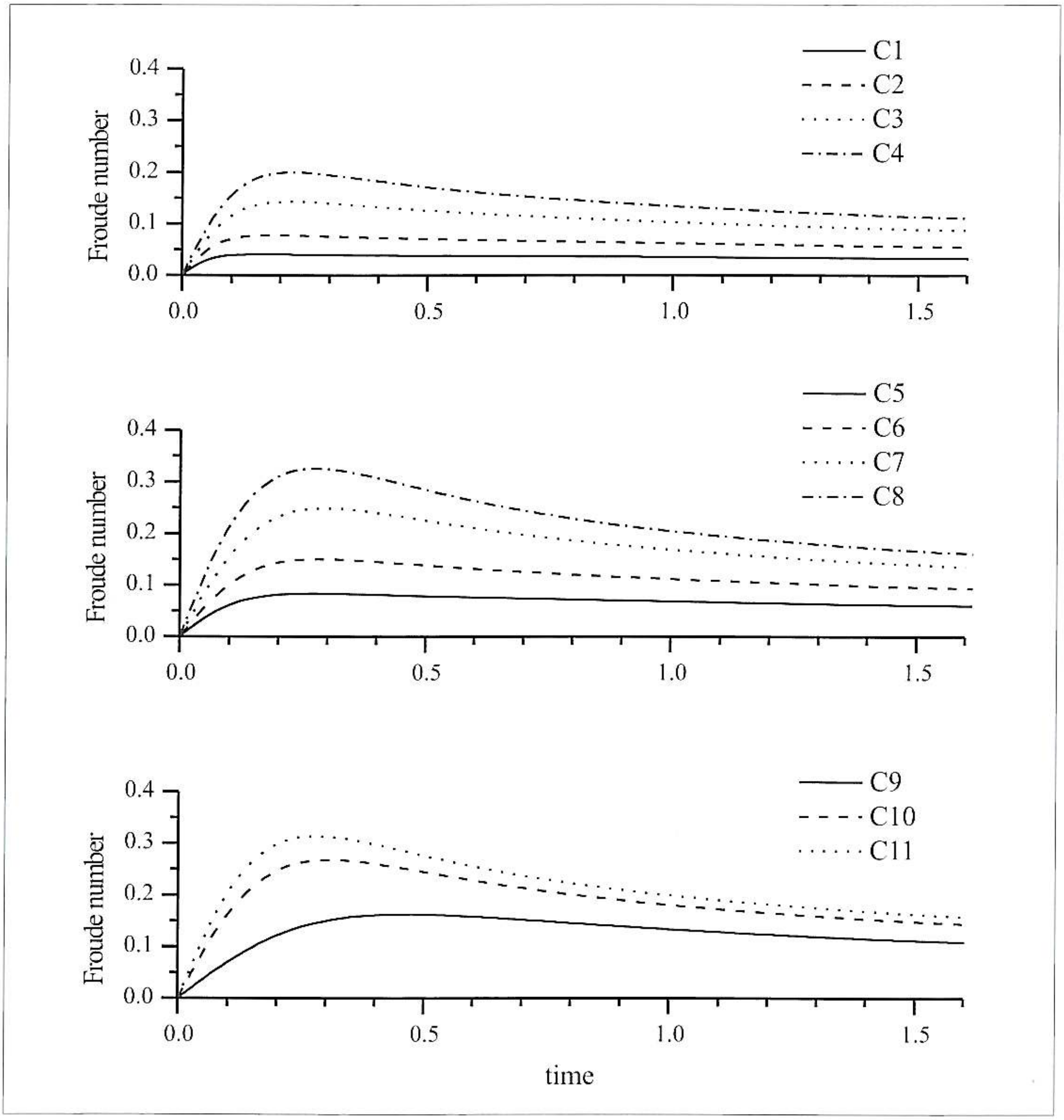

Fig. 1. Froude number curves *versus* dimensionless time used in numerical experiments, computed by means of formula (5.4). The corresponding parameters are listed in table I.

table I. Subsets C1-C4 and C5-C8 correspond to the value of initial acceleration A_0 and peak velocity U_t that are typical of real large-scale underwater slides running on steep slopes (Harbitz, 1992; Tinti *et al.*, 1999a,b; Heinrich *et al.*, 1999) and differ only as regards the ocean depth in the generation region: in the order of 750 m for the former group and of 150 m for the latter. The last subset C9-C11 corresponds to smaller values of A_0 and U_t, that are typical of wavemaker

Table I. Parameters used to compute Froude number curves according to formula (5.4). Values of U_t and A_0 of curves C1-C8 may be considered typical of submarine landslides in natural environment, if U_t is viewed as the peak velocity. For curves C9-C11 these values are taken from small-scale hydraulic experiments (Watts, 1998). For all curves the incline angle θ is equal to 45°.

	U_t (m/s)	A_0 (m/s^2)	τ (s)	H_0 (m)
C1	5.00	1.00	87.95	750
C2	10.00	1.50	88.80	750
C3	20.00	2.00	91.47	750
C4	30.00	2.50	94.57	750
C5	5.00	1.00	40.24	150
C6	10.00	1.50	42.06	150
C7	20.00	2.00	47.43	150
C8	30.00	2.50	53.16	150
C9	0.37	0.83	1.35	0.12
C10	0.65	1.95	1.42	0.12
C11	0.80	2.41	1.48	0.12

blocks sliding in hydraulic tanks with water depth in the order of 0.1-1 m (Watts, 1998). All curves exhibit a similar trend, and it is worth observing that Froude number is much smaller than the critical value 1 at any time, which means that expression (3.18) is adequate to compute water elevation for all the cases proposed here, and most importantly for natural slides originating from gravitational instability of underwater masses. Higher Froude numbers, possibly even in excess of 1, are usually expected only for subaerial slides generated at some altitude above the mean sea level on very steep coastal slopes, and entering the sea at very great speed. The landslide body used for numerical experiments has the symmetrical bell-shaped profile given in the Appendix with aspect ratio 0.01. The solution we have computed fulfils the system of eqs. (2.16)-(2.20) that is the linear shallow-water approximation to the system (2.1)-(2.6). For all the results presented here, we checked the validity of the approximation even *a posteriori*, that is after we solved system (2.16)-(2.20) we computed all terms of system (2.1)-(2.6) that are discarded in the approximated system (2.16)-(2.20), and checked that they are typically small enough to be neglected, if the conditions that d is much smaller than D (required by linearisation) and D is much smaller than λ (shallow-water waves) are satisfied. For example, restricting to eq. (2.16), we first checked that all terms $\partial_t\xi$, $\partial_x u$ and $\partial_t h_s$ have the same order of magnitude, and then that the discarded non-linear terms $\partial_x(uh_s)$ and $\partial_x(u\xi)$ are negligibly small.

Waves produced by the moving body have the typical pattern shown in fig. 2 where the specific case corresponding to curve C8 has been taken as a useful illustrative example. Wave profiles are graphed at different times during the generation phase. It may be seen that at the very initial stage a double wave, one crest and one trough, is formed growing in amplitude and involving an increasingly larger region. Progressively, the double wave splits into two different wave systems. One travels along the same direction as the slide with a leading positive wave, while the other propagates backward. The latter system is dominated by the leading trough moving at the free-wave phase speed, whereas the advancing perturbation is more complex, since it is due to the joint contribution of progressing free waves and a forced wave, travelling at different speeds. The back-going trough attains the largest amplitude, Ξ_-, at a time that may be conveniently denoted by T_-, that is slightly smaller than the corresponding time, say T_+, at which the wave going forward reaches its maximum Ξ_+. Observe that Ξ_+ is substantially larger than Ξ_-. Complete separation of the two wave systems propagating in opposite directions occurs at a larger time, say T_G: this can be taken as the scale of the generation time, since wave formation is the dominant process until T_G, while wave propagation prevails for later times. The last panel of fig. 2 displays the total energy per unit width of the tsunami *versus* time, calculated by means of formula (4.4). It grows approximately until T_G, and afterwards it decreases slowly, which means that energy flows back from the wave system to the decelerating landslide body with the forcing term in eq. (4.1) being negative. It is interesting to point out that the maximum value of energy is in the order of Ξ_+^2, which is not surprising since the energy

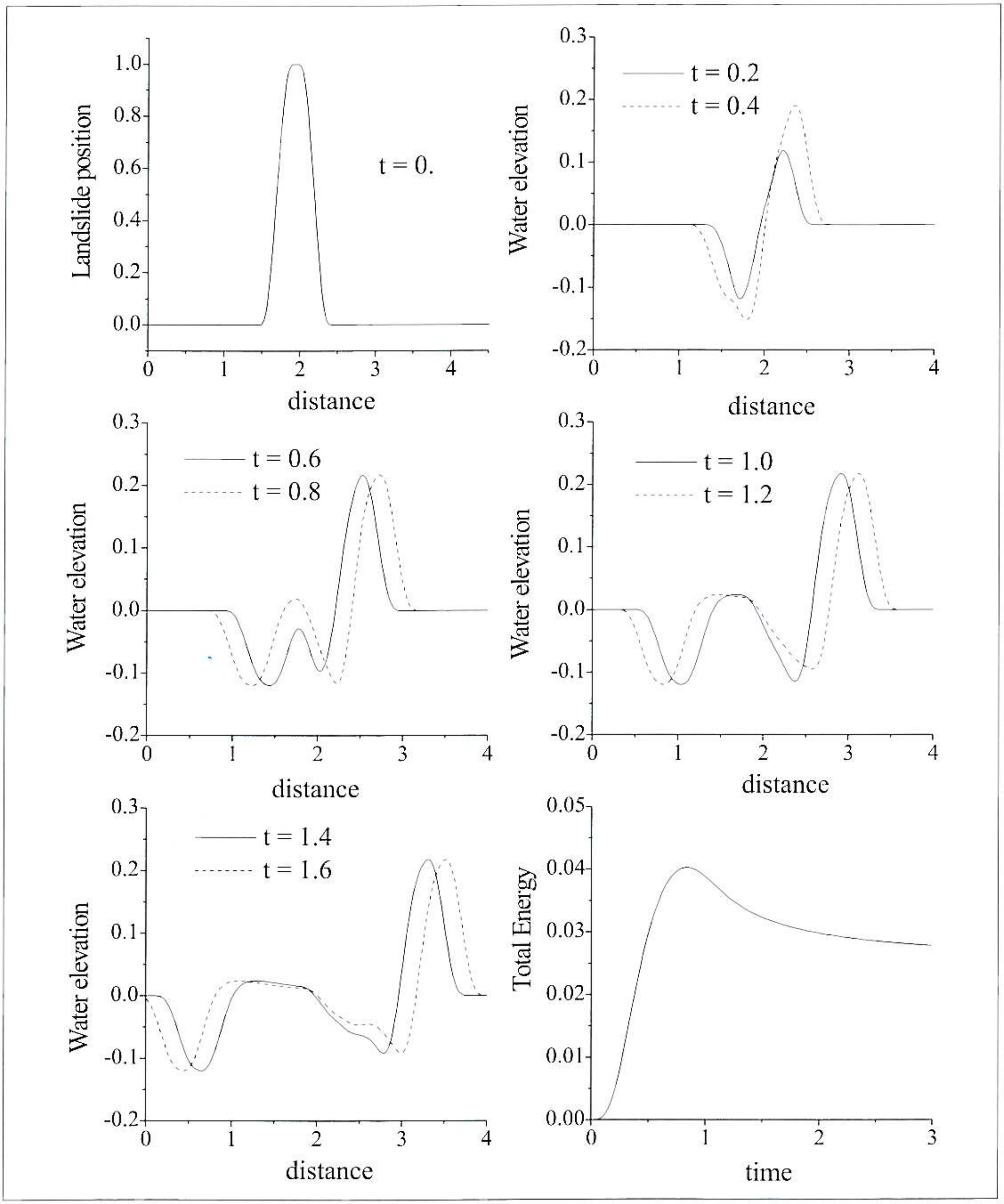

Fig. 2. Landslide profile satisfying the expression given in the Appendix (upper-left-corner panel). It is a symmetrical slide with unitary dimensionless height. Water elevation profiles calculated at different times, with wave formation prevailing at earlier instants and wave propagation dominating at later times (intermediate panels). Total energy of the water waves in dimensionless units *versus* time (lower-right-corner panel).

conveyed by a long water wave scales as the square of its amplitude.

A better understanding of the evolution of the wave pattern can be gained by considering distinctly the values taken by the various terms in the solving eq. (3.18). Since supposedly $v(0)$ equals zero for all curves C1-C11 displayed in fig. 1, only the first three terms of eq. (3.18) have to be evaluated. They are here designated by T1-T3 following the order in the formula. Figure 3 shows snapshots of waves associated with these terms at different times. The first integral T1 represents the back-going wave, while T2 and T3 are associated with the advancing system: the integral T2 is mainly responsible for the leading crest, while the following trough is importantly influenced also by term T3 that is always negative. Wave T3 is very simple to interpret: it is a trough with the same profile as the landslide, moving at the same speed as the landslide, and having an amplitude governed by the factor $v^2/(1-v^2)$. Focusing on T1, it can be observed that it is formed by a trough, followed by a crest, with the trough largely prevailing in amplitude. The time T_G is the time taken by the backward travelling trough of T1 to separate from the advancing forced trough T3. Observe further that the trough length tends to diminish with time, remaining smaller than, but close to, the landslide length. On the other hand the following crest of T1 has a progressively increasing length, since it covers the region going from the landslide front, moving forward, to the rear part of the leading trough of T1, moving backward. Hence its length tends to grow unlimitedly. Similar considerations also hold for the wave T2: the leading crest is largely predominant; its length is approximately equal to, but smaller than, unit; the following trough has a length linearly growing with time. It is worth observing that the linear growth of the length of the secondary waves of T1 and T2 is not a paradox, since in the long term it is accompanied by a corresponding amplitude decrease, so that the total wave energy remains limited. As a matter of fact, it has been observed from the energy panel in fig. 1 that wave energy diminishes for large times. It is important to underline that in correspondence with the current position of the slides all waves T1-T3 are present and interfere at any time. Indeed, the slide position is the one where the forcing term $\xi\,\partial_t h_s$ differs from zero and where a tsunami is continuously generated, which implies that rigorously, it would be not correct to distinguish a prior generation phase from a subsequent propagation phase, since both generation and propagation are processes occurring continuously. Equally incorrect would be the statement that waves are separate after a certain time, since they interfere in the creation region at any one time. Practically, however, the slide progressive deceleration weakens the generation process and makes the perturbation produced in the generation region negligibly small after time T_G.

It is interesting to recall that also in case of constant Froude number the solution for the water elevation ξ can be written as the superposition of three waves, as expressed by eqs. (3.19)-(3.22), that may be put in correspondence with the above waves T1-T3. The pulse ξ_F corresponds exactly to T3, and the leading pulses of T1 and T2 may be associated with the pulses ξ_- and ξ_+. Solution (3.19) does not exhibit the secondary, continuously lengthening, pulses characterising T1 and T2, and hence full separation of waves can occur: the time T_G defined above is here the time when ξ_- separates from ξ_F, and another time, say T_S, can be defined as the one at which full separation of ξ_+ from ξ_F occurs. Later than T_S, generation process ceases completely since the integral energy contribution (4.3) vanishes.

The leading pulses are quite important since they produce the first impact of the tsunami and, as has been seen, are the largest oscillations. It is therefore quite relevant to find a way to estimate the amplitude of these frontal waves from the kinematic properties of the motion of the sliding body. It has already been observed that amplitudes Ξ_- and Ξ_+ are almost constant after the respective times T_- and T_+, which means that the movement of the body at later times has very little effect on leading wave amplitudes that are practically already governed by free wave propagation. In order to estimate the amplitudes, let us first consider a way to estimate the times T_- and T_+, starting with T_-. For this purpose, let us consider the linearly growing distance between

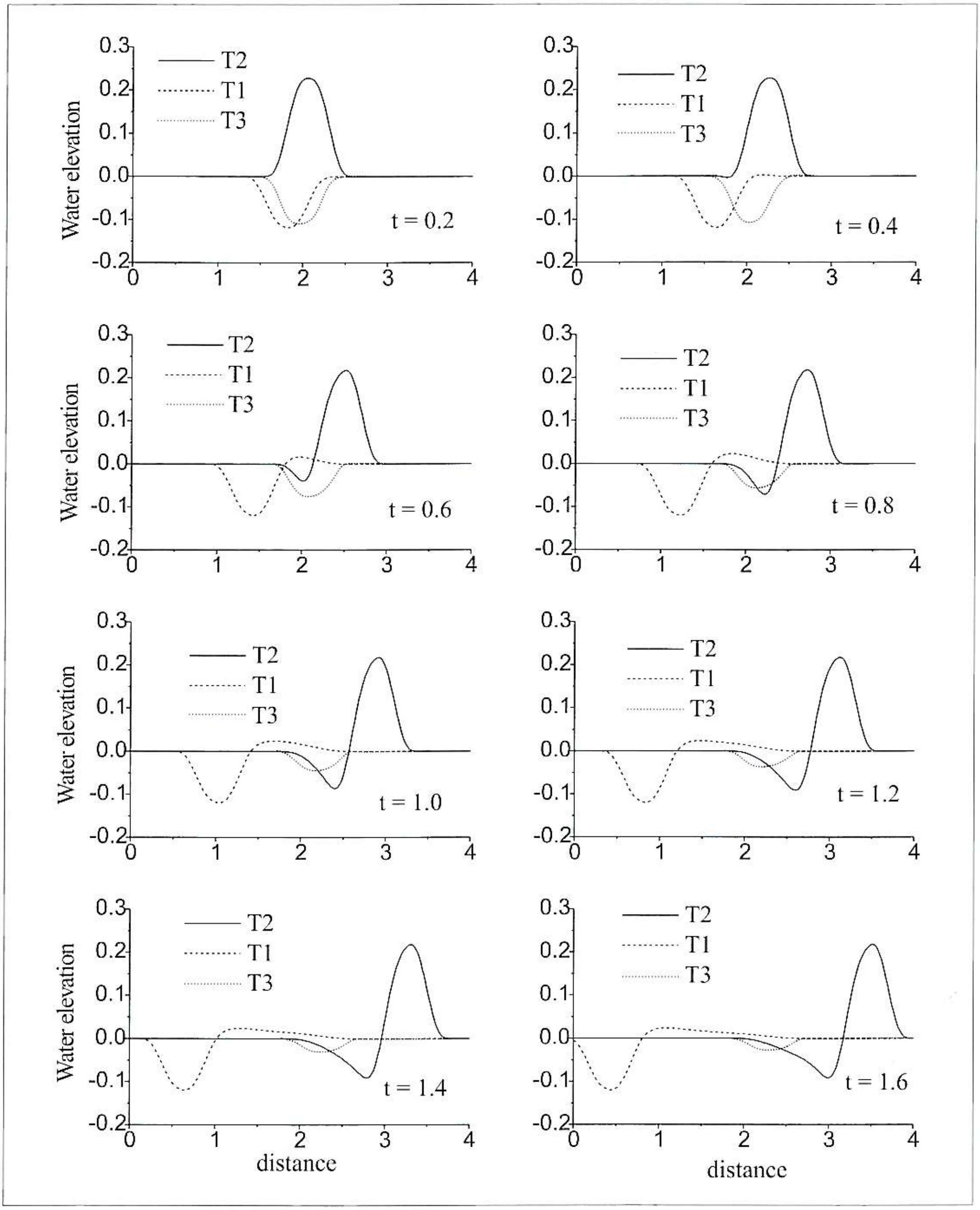

Fig. 3. Profiles of the waves corresponding to the first three terms of expression (3.18), that are designated as T1, T2 and T3 (see text). Their superposition gives the total wave field graphed in fig. 2.

the front of the back-going wave, travelling at unitary speed, and the rear of the advancing wave system that moves at the instantaneous speed $v(t)$. The time T_- is the time when this distance equals the distance separating the initial point of the slide from the position of its maximum. For the symmetrical slide considered in this paper, this latter distance is equal to 1/2 in dimensionless units. Mathematically, the above conditions may be expressed by means of the following equation for T_-:

$$T_- + \int_0^{T_-} v(t)\,\mathrm{d}t = \frac{1}{2}. \tag{6.1}$$

With analogous considerations applied to the growing distance between the front of the forced wave and the rear of the advancing leading pulse, it is easy to derive that the time T_+ must fulfil the corresponding equation

$$T_+ - \int_0^{T_+} v(t)\,\mathrm{d}t = \frac{1}{2}. \tag{6.2}$$

Since amplitudes Ξ_- and Ξ_+ are mostly dominated by the body kinematics, namely by the Froude number curves, until the respective times T_- and T_+, we tested two possible hypotheses: amplitudes depend upon a characteristic value of the wavemaker body speed, that is (1) the maximum value of the Froude number until the above times (which practically corresponds to the absolute maximum v_{Max} of the Froude number curve), or (2) the mean value of the Froude number until these times. In this second hypothesis, after defining

$$v_- = \frac{1}{T_-}\int_0^{T_-} v(t)\,\mathrm{d}t$$

and equivalently

$$v_+ = \frac{1}{T_+}\int_0^{T_+} v(t)\,\mathrm{d}t$$

it can be easily derived that

$$v_- = -\left(1 - \frac{1}{2T_-}\right) \tag{6.3}$$

$$v_+ = +\left(1 - \frac{1}{2T_+}\right). \tag{6.4}$$

We then computed the estimated amplitudes of

Table II. Characteristic times T_- and T_+ and corresponding velocities v_- and v_+ are evaluated by means of eqs. (6.1)-(6.4). The estimate ratios K, corresponding to hypothesis 1 ($K_-(v_{\mathrm{Max}})$ and $K_+(v_{\mathrm{Max}})$) and to hypothesis 2 ($K_-(v_-)$ and $K_+(v_+)$) are computed by using eqs. (6.5) and (6.6).

	v_{Max}	T_-	v_-	T_+	v_+	$K_-(v_{\mathrm{Max}})$	$K_-(v_-)$	$K_+(v_{\mathrm{Max}})$	$K_+(v_+)$
C1	0.040	0.483	0.036	0.519	0.036	1.02	0.92	1.02	0.92
C2	0.077	0.469	0.068	0.536	0.068	1.02	0.91	1.03	0.90
C3	0.142	0.447	0.120	0.568	0.121	1.03	0.88	1.05	0.88
C4	0.200	0.430	0.165	0.599	0.166	1.03	0.88	1.08	0.87
C5	0.083	0.468	0.069	0.537	0.070	1.03	0.87	1.03	0.86
C6	0.150	0.446	0.121	0.571	0.125	1.02	0.85	1.04	0.84
C7	0.249	0.420	0.195	0.625	0.204	1.03	0.84	1.07	0.83
C8	0.325	0.398	0.256	0.680	0.264	1.02	0.85	1.11	0.83
C9	0.162	0.450	0.112	0.569	0.122	1.07	0.77	1.05	0.76
C10	0.267	0.415	0.207	0.639	0.218	1.03	0.83	1.07	0.82
C11	0.313	0.402	0.246	0.670	0.255	1.03	0.85	1.10	0.83

the leading waves by means of the following formulas:

$$\Xi_{-}^{E}(v) = \frac{1}{2}\frac{v}{v+1} \tag{6.5}$$

$$\Xi_{+}^{E}(v) = \frac{1}{2}\frac{v}{1-v} \tag{6.6}$$

that derive from the amplitude factors of the waves of the constant Froude-number case. The estimates were compared with the corresponding amplitudes, Ξ_{-} and Ξ_{+}, resulting from the computation of the water elevation through the solving formula (3.18), that may be considered to be the exact solution, apart from small numerical errors that are immaterial for the present purposes. Table II shows the ratios $K_{-}(v) = \Xi_{-}^{E}(v)/\Xi_{-}$ and $K_{+}(v) = \Xi_{+}^{E}(v)/\Xi_{+}$ computed for the various values of the characteristic slide velocity, that are v_{Max} (hypothesis 1), and v_{+} and v_{-} (hypothesis 2). It may be seen that hypothesis-1 values slightly overestimate the amplitude (a few percent), whereas evaluations based on hypothesis 2 provide underestimates in the order of 10-30 percent. Observe that this can be considered a very good result, since the joint use of estimates deriving from hypotheses 1 and 2 provides a very expedite tool to bracket the amplitudes of the leading waves within a reasonable expectation interval.

7. Conclusions

The 1D shallow-water theory for basins of constant depth has been used to study tsunami generation by underwater moving landslides, that do not deform during their motion. The instant ratio of the body speed to the free-wave phase celerity was defined as the Froude number and was seen to play an essential role in determining the characteristics of the resulting waves. In dimensionless unit, the Froude number coincides with the body speed $v(t)$. In the case of subcritical regime ($v < 1$), the solution to the problem has been given a simple form expressing the water elevation as the sum of five terms, two of which imply the computation of integrals (see eq. (3.18)). The more general solution also covering supercritical and critical flows provides the water elevation as the sum of six terms, three of which are integrals (see eq. (3.23)). Numerical experiments to evaluate the main characteristics of the generated waves in relation to the property of the wavemaker were carried out using values of the kinematic parameters of the landslide that are typical of hydraulic experiments and of real cases. It has been emphasized that usually flow regime is largely subcritical. Under these circumstances the slide produces two main systems of waves, both characterised by leading pulses moving at the free-wave phase speed. The pulse advancing in the same direction as the slide is positive, whereas the regressing pulse is negative. These pulses are both followed by pulses of opposite sign. A third negative pulse moves exactly together with the slide at the same speed. It was found that the amplitude of the leading waves remains approximately constant after the formation of the frontal part of the pulses (after times T_{-} and T_{+}), and that it can be estimated by means of formulas (6.5) and (6.6) that are essentially based on wave generation by bodies moving at constant speed (see eqs. (3.19)-(3.22)). It is known that in largely subcritical flows constant-speed slides are poorly effective in producing tsunamis, since the ratio of wave amplitude to slide height is rather low (Tinti and Bortolucci, 2000). The scarce tsunamigenic efficiency of slow bodies is essentially confirmed by the present analysis, but this does not imply that the produced waves are small. Natural submarine slides consisting of sedimentary bodies as thick as tens to hundreds of meters are likely capable of producing catastrophic waves even if their velocity is much smaller than the critical speed for ocean basins.

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Appendix

Slides studied in this paper are supposedly rigid bodies whose motion can be consequently described unambiguously by means of the displacement of anyone of their points, for example their centre of mass. They do not change shape during motion, and their kinematics can be uniquely identified by their initial profile and their velocity function. In dimensional space, the slide has horizontal length L_s, and its initial position is between the horizontal extremes x_i and $x_f = x_i + L_s$. The mathematical expression of the slide profile at the initial time is given by

$$h_s(x) = 0 \qquad x \notin [x_i, x_f] \tag{A.1}$$

$$h_s(x) = \frac{A}{4\pi^2}\left\{\frac{1}{2}\kappa^2 (x - x_i)^2 + \cos[\kappa(x - x_i)] - 1\right\} \qquad x \notin [x_i, x_i + L_s/4] \tag{A.2}$$

$$h_s(x) = \frac{A}{4\pi^2}\left\{-\frac{1}{2}\kappa^2 (x - x_i)^2 + \cos[\kappa(x - x_i)] + 4\pi\kappa(x - x_i) + 1 + 4\pi^2\right\}$$
$$x \notin [x_i + L_s/4, x_i + 3L_s/4] \tag{A.3}$$

$$h_s(x) = \frac{A}{4\pi^2}\left\{\frac{1}{2}\kappa^2 (x - x_i)^2 + \cos[\kappa(x - x_i)] + 8\pi\kappa(x - x_i) - 1 + 32\pi^2\right\}$$
$$x \notin [x_i + 3L_s/4, x_f] \tag{A.4}$$

where

$$\kappa = \frac{8\pi}{L_s} \tag{A.5}$$

and A is the maximum height of the slide. In the experiments carried out in the paper, the slide aspect ratio, that is the ratio A/L_s is taken to be 0.01. Notice that in dimensionless units, in accordance with scaling laws (2.12)-(2.15) the slide height and length are both equal to 1.

REFERENCES

ASSIER RZADKIEWICZ, S., C. MARIOTTI and PH. HEINRICH (1997): Numerical simulation of submarine landslides and their hydraulic effects, *ASCE, J. Waterw., Harbors Coastal Eng. Div.*, **123**, 149-157.

BONDEVIK, S., J.I. SVENDSEN and J. MANGERUD, (1997): Tsunami sedimentary facies deposited by the Storegga tsunami in shallow marine basins and coastal lakes, Western Norway, *Sedimentology*, **44**, 1115-1131.

DAVIES, H. (1998): *The Sissano Tsunami*, Univ. Papua New Guinea, Port Moresby.

DAWSON, A.G., D. LONG and D.E. SMITH (1988): The Storegga slides: evidence from Eastern Scotland of a possible tsunami, *Mar. Geol.*, **82**, 271-276.

FINE, I.V., A.B. RABINOVICH, E. KULIKOV, R.E. THOMSON and B.D. BORNHOLD (1999): Numerical modelling of landslide-generated tsunamis with application to the Skagway Harbor tsunami of November 3, 1994, in *International Conference on Tsunamis, Paris, France, May 26-28, 1998*, published by Commissariat à l'Energie Atomique (CEA), 212-223.

HAMPTON, M.A., H.J. LEE and J. LOCAT (1996): Submarine landslides, *Rev. Geophys.*, **34**, 33-59.

HARBITZ, C.B. (1992): Model simulation of tsunamis generated by Storegga slides, *Mar. Geol.*, **104**, 1-21.

HARBITZ, C.B. and A. ELVERHØI (1999): On tsunami characteristics and submarine dynamics, in *International Conference on Tsunamis, Paris, France, May 26-28, 1998*, published by Commissariat à l'Energie Atomique (CEA), 257-266.

HEINRICH, PH., S. GUIBOURG, A. MANGENEY and R. ROCHE (1999): Numerical modeling of a landslide-generated tsunami following a potential explosion of the Montserrat volcano, *Phys. Chem. Earth*, **24**, 163-168.

IWASAKI, S. (1982): Experimental study of a tsunami generated by a horizontal motion of a sloping beach, *Bull. Earthquake Res. Inst.*, **57**, 239-262.

JOHNSGARD, H. and G. PEDERSEN (1996): Slide generated waves in near-shore regions. A Lagrangian description, *Phys. Chem. Earth*, **21**, 45-49.

KAJIURA, K. (1970): Tsunami source, energy and the directivity of wave radiation, *Bull. Earthquake Res. Inst., Univ. Tokyo*, **48**, 835-869.

KAWATA, Y., B.C. BENSON, J.C. BORRERO, H.L. DAVIES, W.P. DE LANGE, F. IMAMURA, H. LETZ, J. NOTT and C.E. SYNOLAKIS (1999): Tsunami in Papua New Guinea was as intense as first thought, *Eos, Trans. Am. Geophys. Un.*, **80**, 101, 104-105.

LANDER, J.F., P.A. LOCKRIDGE and M.J. KOZUCH (1993): *Tsunamis Affecting the West Coast of the United States*, U.S. Department of Commerce, National Oceanic and Atmospheric Administration, National Environmental Satellite, Data and Information Service, National Geophysical Data Centre, Boulder, Colorado, December 1993, pp. 242.

NODA, E.K. (1970): Water waves generated by landslides, *ASCE, J. Waterw., Harbors Coastal Eng. Div.*, **96**, 835-858.

NODA, E.K. (1971): Water waves generated by a local surface disturbance, *J. Geophys. Res.*, **76**, 7389-7400.

PELINOVSKY, E. (1996): (1996): *Tsunami Waves Hydrodinamics*, Institute of Applied Physics, Nizhny Novgorod, Russia, pp. 273 (in Russian).

PELINOVSKY, E. and A. POPLAVSKY (1996): Simplified model of tsunami generation by submarine landslides, *Phys. Chem. Earth*, **21**, 13-17.

SABATIER, P.C. (1983): On water waves produced by ground motions, *J. Fluid Mech.*, **126**, 27-58.

SOLOVIEV, S.L., CH.N. GO, KH.S. KIM, O.N. SOLOVIEVA and N.A. SHCHETNIKOV (1997): *Tsunamis in the Mediterranean Sea, 2000 B.C.-1991 A.D.*, Russian Academy of Sciences, National Geophysical Committee, Institute of Oceanology, Scientific and Coordination Centre «Tsunami», Moskow, pp. 139.

TAPPIN, D.R., T. MATSUMOTO, P. WATTS, K. SATAKE, G.M. MCMURTRY, M. MATSUYAMA, Y. LAFOY, Y. TSUJI, T. KANAMATSU, W. LUS, Y. IWABUCHI, H. YEH, Y. MATSUMOTU, M. NAKAMURA, M. MAHOI, P. HILL, K. CROOK, L. ANTON and J.P. WALSH (1999): Sediment slump likely caused 1998 Papua New Guinea tsunami, *Eos, Trans. Am. Geophys. Un.*, **80**, 329, 334 and 340.

TINTI, S. and A. MARAMAI (1996): Catalogue of tsunamis generated in Italy and in Côte d'Azur, France: a step towards a unified catalogue of tsunamis in Europe, *Ann. Geofis.*, **39** (6), 1253-1299 (Errata Corrige, *Ann. Geofis.*, **40** (3), 781).

TINTI, S. and E. BORTOLUCCI (2000): Energy of water waves induced by submarine landslides, *Pure Appl. Geophys.*, **157**, 281-318.

TINTI, S., E. BORTOLUCCI and A. ARMIGLIATO (1999a): Numerical simulation of the landslide-induced tsunami of 1988 in Vulcano island, Italy, *Bull. Volcanol.*, **61**, 121-137.

TINTI, S., C. ROMAGNOLI and E. BORTOLUCCI (1999b): Modeling a possible Holocenic landslide-induced tsunami at Stromboli volcano, Italy, *Phys. Chem. Earth*, **24**, 423-429.

TINTI, S., E. BORTOLUCCI and C. CHIAVETTIERI (2000): Tsunami excitation by submarine landslides in shallow-water approximation, *Pure Appl. Geophys.* (in press).

WATTS, P. (1998): Wavemaker curves for tsunamis generated by underwater landslides, *ASCE, J. Waterw., Harbors Coastal Eng. Div.*, **124**, 127-137.

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