Effects of transient water mass redistribution associated with a tsunami wave on Earth’s pole path

Anna Rita Pisani, Antonio Piersanti, Daniele Melini, Alessio Piatanesi and Gaia Soldati
Istituto Nazionale di Geofisica e Vulcanologia, Sezione di Sismologia e Tetttonofisica, Roma, Italy

Abstract
We have quantified the effects of a water mass redistribution associated with the propagation of a tsunami wave on the Earth’s pole path and on the Length-Of-Day (LOD) and applied our modeling results to the tsunami following the 2004 giant Sumatra earthquake. We compared the result of our simulations on the instantaneous rotational axis parameters variation with the preliminary instrumental evidence on the pole path perturbation (which has not been confirmed) registered just after the occurrence of the earthquake. The detected perturbation in the pole path showed a step-like discontinuity that cannot be attributed to the effect of a seismic dislocation. Our results show that the tsunami induced instantaneous rotational pole perturbation is indeed characterized by a step-like discontinuity compatible with the observations but its magnitude is almost one hundred times smaller than the detected one. The LOD variation induced by the water mass redistribution turns out to be not significant because the total effect is smaller than current measurements uncertainties.

Key words Earth rotation – pole path variation – tsunami wave – Sumatra earthquake

1. Introduction

Due to the action of various internal geophysical processes the Earth rotates about an axis not aligned with its figure axis; this induces the Planet to wobble as it rotates. The Earth wobbles over a broad range of frequencies in response to a variety of forcing mechanisms; besides, it is characterized by a few discrete frequencies at which it naturally wobbles in the absence of forcing (i.e. Chandler wobble) that are function of its internal structure. In absence of an excitation source, these natural wobbles would freely decay due to the action of dissipation processes (Gross, 2003). Since its discovery, several processes have been investigated to determine whether or not they could be excitation mechanism(s) of the Chandler wobble, for instance atmospheric processes, continental water storage, core-mantle interactions and earthquakes (Gross, 2000). The predominant mechanisms were found to be concerned with the periodical redistribution of atmospheric, oceanic and hydrologic masses (Gross, 2003).

This work is aimed at studying the perturbation to the instantaneous rotational pole due to a transient phenomenon that, to our knowledge, has not been still investigated, i.e. the transient water mass redistribution associated with the propagation of a tsunami wave. Tsunamis are generally caused by disturbances associated mostly with earthquakes, submarine landslides, submarine volcanic eruptions. Not all earthquakes produce tsunamis because in order to generate them, earthquakes must occur at a shallow depth, have a large seis-
mic moment and produce a significant displacement of the sea floor.

In this work we take into account the propagation of a tsunami wave following a giant earthquake.

This phenomenon involves both permanent solid mass redistribution due to the earthquake and transient redistribution of water mass due to the tsunami propagation. Theoretically, the rotational effect due to a permanent dislocation associated with a seismic event is characterized by a step-like perturbation modeled by an excitation function of the form \( \Psi_e(t) = \Delta \Psi H(t) \), where \( H(t) \) is the Heaviside step function and \( \Delta \Psi \) is the magnitude of the excitation. The resulting effect obtained from such an excitation is a shift of the mean pole of rotation while the instantaneous rotational pole describes a continuous curve with a sudden change in the curvature radius at the time \( t=0 \) when the perturbation is «turned on» (Lambeck, 1980). Such an effect on the Earth rotation has never been observed, not even in recent earthquakes (i.e. the 26th December 2004 Sumatra earthquake) (Gross and Chao, 2006), with the exception of a preliminary observation by Bianco et al. (2005a,b) who evidenced a step-like discontinuity in the pole path instead of the expected change in curvature radius in correspondence of the Sumatra earthquake.

The effect on the pole motion associated with a tsunami wave propagation is quite different from that produced by a permanent solid mass redistribution because it is the effect of a transient phenomenon due to the water mass redistribution. The rotational theory predicts that a transient phenomenon can be modeled by an excitation function with a delta-like temporal dependence that will give a step-discontinuous solution in the pole path (Lambeck, 1980), as it will be shown in Section 2; the effect produced by such excitation could qualitatively explain the step-like discontinuity evidenced in Bianco et al. (2005a,b) data.

Using a synthetic numerical simulation of the 2004 Indian Ocean tsunami, we performed a forward modeling of the expected effect of the water mass transient redistribution on the pole path and length-of-day (LOD), that will be discussed in Section 3. We explicitly computed the time dependent excitation functions, the pole path and the LOD variations induced by the tsunami wave. The pole path variations turn out to have the same temporal dependence of the step-like signal observed by Bianco et al. (2005a,b), but their magnitude is too small to explain the observed data, besides the LOD variations turn out to be smaller than the uncertainty in the measurements.

2. Model formulation

The equations of motion for a deformable Earth come from the solution of the Liouville equation (Munk and MacDonald, 1960) and have the following form:

\[
m = i \sigma_0 (m - \Psi) \quad (2.1)
\]

\[
m_3 = \Psi_3. \quad (2.2)
\]

Equation (2.1) describes the wobble, \( \sigma_0 \) is the frequency of the free oscillation (Chandler wobble), and \( m = m_1 + i m_2 \) represents the instantaneous rotational pole path in the complex plane. Equation (2.2) deals with the LOD; \( \Psi = \Psi_1 + i \Psi_2 \) and \( \Psi_3 \) are the «modified excitation functions» containing the perturbation associated with any prescribed event.

\[
\Psi = k_w \bar{\psi} \quad (2.3)
\]

\[
\Psi_3 = k_{LOD} \psi_3 \quad (2.4)
\]

where \( \bar{\psi} = \psi_1 + i \psi_2 \) is a vector quantity and \( k_w \) and \( k_{LOD} \) are the «transfer functions» and arise from the correction for the elastic deformation associated with the centrifugal forces due to Earth rotation; their values depend on whether the event does or does not load the Earth (Munk and MacDonald, 1960; Lambeck, 1980). The solutions of eq. (2.1) for the wobble and of the eq. (2.2) for the LOD variations are

\[
m(t) = m_0 e^{i \omega_0 \omega} - i \sigma_0 e^{i \omega_0 \omega} \int_{-\infty}^{t} \Psi(\tau) e^{-i \omega_0 \omega} d\tau \quad (2.5)
\]

\[
\frac{\Delta \Lambda}{\Lambda_0} = -m_3 = -\Psi_3 \quad (2.6)
\]

where \( m_0 \) is an arbitrary complex constant and
\( \Lambda_0 \) is the nominal LOD (e.g., 86400 s) (Munk and MacDonald, 1960; Lambeck, 1980). In particular, once the excitation function \( \Psi \) has been determined, the effect of a perturbation to the instantaneous pole of rotation at any time \( t \) can be quantified from eq. (2.5) neglecting the term of free nutation \( m(t) \) and solving the partial solution of the pole motion equation

\[
m(t) = -i \sigma \omega e^{i \omega t} \int_{-\infty}^{t} \Psi(\tau) e^{-i \omega \tau} d\tau \quad (2.7)
\]

while the variation of the LOD can be directly obtained from eq. (2.6). In order to estimate the perturbation to the instantaneous pole of rotation and to the LOD due to a transient water mass displacement occurred during the propagation of a tsunami, we have to calculate first the time dependent excitation functions. For the sake of simplicity we will omit the time dependence in the excitation functions during the model formulation.

The excitation function \( \Psi \), corresponding to a tsunami wave is given by the sum of two main contributions

\[
\Psi^{(\text{wobble})}_m = \Psi_m + \Psi_v \quad (2.8)
\]

\[
\Psi^{(\text{LOD})}_m = \Psi_m + \Psi_v \quad (2.9)
\]

\( \Psi_m \) and \( \Psi_v \) are associated with the perturbation to the Earth static mass distribution caused by the variation of the sea surface («matter» term); in this case the event loads the Earth, so the transfer functions in eq. (2.3) and in eq. (2.4) assume the following values \( k_m = 1.03 \) and \( k_{\text{LOD}} = 0.7 \) (Munk and MacDonald, 1960; Lambeck, 1980). \( \Psi_m \) and \( \Psi_v \) are associated with the angular momentum exchange between the horizontal flowing water and the solid Earth («velocity» term); in this case there is no loading so the transfer functions in eq. (2.3) and in eq. (2.4) assume the following values \( k_m = 1.47 \) and \( k_{\text{LOD}} = 1.0 \) (Munk and MacDonald, 1960; Lambeck, 1980). Therefore the total excitation functions for the perturbation to the pole path (2.7) and to the LOD (6) can be written as follows:

\[
\Psi^{(\text{wobble})}_m = 1.03 \Psi^{(\text{wobble})}_m + 1.47 \Psi^{(\text{wobble})}_v \quad (2.10)
\]

\[
\Psi^{(\text{LOD})}_m = 0.7 \Psi^{(\text{LOD})}_m + \Psi^{(\text{LOD})}_v \quad (2.11)
\]

where \( \Psi_m \) and \( \Psi_v \) are vector quantities. In spherical coordinates the excitation functions corresponding to a water mass element \( dm = \rho_v dV \) located at latitude \( \theta \) and longitude \( \phi \) read («matter» term) (Lambeck, 1980):

\[
\left( \psi_1 \right)^{(\text{wobble})}_m = - \int_{V_c} \rho_v C \omega r^2 \cos \theta \sin \theta \left( \frac{\cos \theta}{\sin \theta} \right) dV \quad (2.12)
\]

\[
\psi^{(\text{LOD})}_m = - \int_{V_c} \rho_v C r^2 \cos^2 \theta dV \quad (2.13)
\]

where the integration is carried out over the wave volume \( V_w \), while \( C \) is the axial inertia moment of the Earth and \( A \) the equatorial one. The excitation functions corresponding to a mass \( dm = \rho_v dV \) moving with tangential velocity \( v_w = (v_{w1}, v_{w2}) \) read («velocity» term) (Lambeck, 1980)

\[
\left( \psi_1 \right)^{(\text{wobble})}_v = \int_{V_c} \frac{2 \rho_v}{\Omega (C - A)} r v_{w1} \sin \theta \left( - \cos \phi \right) dV + \int_{V_c} \frac{2 \rho_v}{\Omega (C - A)} r v_{w2} \sin^2 \theta \left( - \cos \phi \right) dV \quad (2.14)
\]

\[
\psi^{(\text{LOD})}_v = - \int_{V_c} \frac{\rho_v}{\Omega C} r v_{w1} \cos \phi dV \quad (2.15)
\]

where \( \Omega \) is the mean Earth angular velocity, \( V_0 \) is the total oceanic volume involved in the tsunami propagation and can be evaluated considering an unperturbed oceanic surface since the water volume \( V_w \), corresponding to the tsunami wave perturbation, is negligible with respect to the oceanic volume, i.e. \( V_w \ll V_0 \).

The excitation functions resulting from the integration of eqs. ((2.12)-(2.15)) that we are analysing will represent by definition a transient phenomenon, in fact the wave volume \( V_w \) in eqs. ((2.12)-(2.13)) and the velocity fields in eqs. ((2.14)-(2.15)) are identically null before and after the tsunami propagation, so that the excitation functions will be non-zero only during the tsunami propagation. Mathematically it is possible to prove that a delta excitation shows a step function temporal dependence in the partial solution of the wobble equation as it fol-
lows. If we consider a time-scale much greater than the characteristic times of tsunami propagation, we can approximate the transient excitation functions with a Dirac delta, i.e. \( \Psi(t) = \Delta \Psi \delta(t) \). By substituting this excitation function in eq. (2.7) we obtain

\[ m(t) \sim -i \sigma e^{i \omega t} \Delta \Psi H(t). \quad (2.16) \]

In the time scale in which we perform our simulation the trend of the excitation associated with the propagation of the tsunami wave shows a deltalike temporal dependence which represents a step-like discontinuity in the pole path. Since a global scale tsunami propagation typically occurs on timescales of the order of 20 h, we can expect, after carrying out the numerical integration, to find a pole path that will approximate a step discontinuity on time-scales much larger than 20 h.

To evaluate the «matter» and «velocity» terms of the excitation functions (2.12)-(2.13) and (2.14)-(2.15) we estimated the water vertical displacement and the horizontal velocity, as a function of time, by means of a numerical model, in the whole area interested by the propagation of the tsunami.

Tsunamis propagate in the sea as gravity waves and since their wavelength is much larger than the sea depth, they are considered as long waves propagating in shallow waters. The model for tsunami propagation is based on the following shallow-water equations (Mei, 1983; Mader, 2004)

\[
\frac{\partial (z + h)}{\partial t} + \nabla \cdot (v(z + h)) = 0 \quad (2.17)
\]

\[
\frac{\partial v}{\partial t} + (v \cdot \nabla) v = g \nabla z + C \quad (2.18)
\]

where \( z \) is the water elevation above the undisturbed sea level, \( v \) the depth averaged horizontal velocity vector, \( h \) the sea depth, \( g \) the gravity acceleration and \( C \) represents the Coriolis forces. This system is completed by boundary conditions of pure wave reflection along the coastlines and of full wave transmission at the open ocean. We assume that the initial tsunami wave is set in motion by the instantaneous transmission of the sea bed displacement to the overlying water mass. Once the seismic fault geometry has been specified, we compute the coseismic deformation of the sea floor using an elastic dislocation model (Okada, 1985). Equations ((2.17)-(2.18)) are solved numerically by means of a finite difference method in a staggered grid of 2 arcminutes of spatial resolution (Mei, 1983; Mader, 2004).

Once the sea depth \( h(\theta, \phi) \), the water elevation \( z(\theta, \phi) \) and the horizontal velocity vector \( v(\theta, \phi) \) have been determined, the excitation functions \( \psi_m \) and \( \psi_v \) can be obtained through a numerical integration of eqs. ((2.12) and (2.15)) by substituting

\[
dV = z(\theta, \phi) r^2 \cos \theta d\theta d\phi
\]

in eqs. ((2.12) and (2.13)) and

\[
dV = \left[h(\theta, \phi) + z(\theta, \phi)\right] r^2 \cos \theta d\theta d\phi \approx h(\theta, \phi) r^2 \cos \theta d\theta d\phi
\]

in eqs. ((2.14)-(2.15)). In these last two integrations we dropped \( z \) because it is negligible with respect to the bathymetry \( h \). Afterwards, the variation in the pole path can be computed by numerically integrating eq. (2.7) while the LOD variation is directly obtained from eq. (2.6).

3. Application to the Sumatra event

To study the effect of the transient water mass redistribution on the pole path we applied our theoretical model to the tsunami wave following the devastating megathrust earthquake of December 26, 2004 off the west coast of Northern Sumatra.

The Sumatra earthquake raised a big debate about its potential effect on Earth rotation. Earth scientists have tried to associate Earth rotational instabilities and seismic activity for 40 years. The attempts concerned both permanent instabilities, associated with global seismic activity as well as single detected irregularities in the pole path, associated with single earthquakes (Dahlen, 1971, 1973; Alfonso et al., 1997; Soldati et al., 2001). The Sumatra earthquake is the largest event since the giant 1960
Chile earthquake, with coseismic effects affecting many geophysical observables, among which the Earth rotation and gravitational field. In particular, a decrease of the LOD of 6.8 $\mu$s, a shift in the position of the mean rotation pole of 2.32 mas (milliarcseconds) towards 127°E longitude, a decrease of the Earth’s oblateness $J_2$ of $2.37 \times 10^{-11}$ and of the Earth’s pear-shapedness $J_3$ of $0.63 \times 10^{-11}$ (Boschi et al., 2006; Gross and Chao, 2006) are expected on the basis of current models; at the same time the accuracy of pole position detecting techniques have drastically improved in the last decades, yielding an increase in accuracy by more than two orders of magnitude since 1960. Therefore, this event represents a unique opportunity to test the theoretical models about the impact of earthquakes on the secular pole motion. Recent studies do not show evidences in the Earth rotation observations of the modeled coseismic effects (Gross and Chao, 2006), otherwise preliminary geodetic observations revealed a step discontinuity in the instantaneous rotational pole path in correspondence with the Sumatra event (Bianco et al., 2005a,b) that however has not been confirmed. According to the rotational theory the observed step-like discontinuity cannot be explained only in terms of a solid mass redistribution associated with a coseismic permanent deformation. As we discussed before, a step-like discontinuity in the pole path may be associated to such as the one corresponding to the tsunami wave propagation.

In our theoretical model, the time dependent excitation functions have been evaluated by means of a numerical tsunami model whose outputs are the vertical displacement and the horizontal velocity field of the propagating tsunami wave as described in Section 2. The numerical tsunami simulation has been carried out in a time window of 16 h ($i.e.$ 57600 s) and on a spatial domain covering the whole Indian Ocean area interested by the propagation of the tsunami ($i.e.$ about $10^8$ km$^2$). The tsunami source geometry is the one obtained by Bao et al. (2005) (see table I). The volume $V_0$ on which the integration of eq. (2.14) and eq. (2.15) has been carried out takes into account the Indian Ocean bathymetry from the ETOP02 dataset (Smith and Sandwell, 1997) available at http://www.ngdc.noaa.gov/mgg/global/global.html.

Figure 1a,b shows three snapshots of the modeled tsunami, taken at 4 h interval. The bathymetric complexities of the Indian Ocean, such as trenches, ridges and seamounts, affect the tsunami propagation and induce strong re-

<table>
<thead>
<tr>
<th>Long (°)</th>
<th>Lat (°)</th>
<th>Width (km)</th>
<th>Length (km)</th>
<th>Strike (°)</th>
<th>Rake (°)</th>
<th>Dip (°)</th>
<th>Slip (m)</th>
<th>Depth (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S01</td>
<td>94.610</td>
<td>2.375</td>
<td></td>
<td>300</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S02</td>
<td>93.902</td>
<td>2.918</td>
<td></td>
<td>315</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S03</td>
<td>93.359</td>
<td>3.625</td>
<td></td>
<td>330</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S04</td>
<td>93.018</td>
<td>4.449</td>
<td></td>
<td>345</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S05</td>
<td>92.823</td>
<td>5.326</td>
<td></td>
<td>350</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S06</td>
<td>92.667</td>
<td>6.212</td>
<td></td>
<td>350</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S06</td>
<td>92.435</td>
<td>7.077</td>
<td>200</td>
<td>340</td>
<td>90</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S06</td>
<td>92.165</td>
<td>7.934</td>
<td></td>
<td>345</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S09</td>
<td>91.970</td>
<td>8.811</td>
<td></td>
<td>350</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S10</td>
<td>91.853</td>
<td>9.702</td>
<td></td>
<td>355</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S11</td>
<td>91.853</td>
<td>10.598</td>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S12</td>
<td>91.970</td>
<td>11.488</td>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S13</td>
<td>92.165</td>
<td>12.366</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
fraction of the wavefields. In the open sea, the tsunami waves hardly exceed an amplitude of 10 cm while the induced particle velocity is generally smaller than 1 mm/s. On the contrary, when the waves enter in very shallow waters and approach the coastline, the water elevation may grow up to several meters and the velocity may be as large as few m/s.

The temporal evolution of $\Psi_1$ and $\Psi_2$ components and of $|\Psi_t|$ are reported in figs. 2 and 3. Figure 3 shows that the maximum variation of $\Psi_t$ during the propagation of the tsunami wave exceeds $2 \times 10^{-8}$. The curves plotted between the dotted lines evidence the delta-like temporal evolution of the perturbation associated with the simulated tsunami, out of the simulated range the curves have been smoothly cut off in order to mimic an excitation that «turns on» and «switches off». If we compare our result to that obtained by the coseismic effect of the Sumatra earthquake in Gross and Chao (2006) it can be seen that this value is two times greater than the one associated with the seismic excitation function $\Psi_e$ that has been estimated to be about $1 \times 10^{-8}$ (Gross and Chao, 2006). This of course does not imply that the pole path perturbation induced by the tsunami is more significant, due to the different temporal dependence between $\Psi_e$ and $\Psi_t$ as explained before.

From the numerical integration of eq. (2.7) we obtained the temporal evolution of the pole position shown in fig. 4. Figure 5 shows the evolution of the instantaneous pole of rotation (polodia) during the 16 h of simulation of the tsunami. As can be seen from fig. 4, on timescales typical of pole position measurements (1 day), the prop-
Effects of transient water mass redistribution associated with a tsunami wave on Earth's pole path

Fig. 2. Temporal evolution of $\Psi_1$ and $\Psi_2$ components of the total excitation function.

Fig. 3. Temporal evolution of $|\Psi_t|$.

Fig. 4. Temporal evolution of $m_1$ and $m_2$ components of the instantaneous pole of rotation.

Fig. 5. Polar motion during the 16 h of the tsunami propagation.
agitation of the tsunami wave induces a step-like perturbation in the pole path with a magnitude of about 0.02 mas.

Although recent studies of Gross and Chao (2006) based on geodetic measurements seem not to detect signals on the Earth rotation in correspondence with the Sumatra earthquake, the preliminary observations of Bianco et al. (2005a,b) based on laser ranging and GPS techniques reveals a westward step discontinuity of about 1.5-2.0 mas in the instantaneous rotational pole path that, in our opinion, is an evidence which is worth considering. According to what we obtained the observed step-like discontinuity could be related to an excitation with a delta-like temporal dependence due to transient mechanisms. We compared our modeled step discontinuity with that observed and even if the curves have the same temporal dependence, our result is almost one hundred times smaller than the observed, therefore too small to be significant in the excitation of the pole path.

Figure 6 shows the evolution of LOD variations during the propagation of the simulated tsunami obtained from eq. (2.6). The plot evidences a transient perturbation of LOD in agreement with the transient nature of a water mass redistribution. The transient corresponds to a LOD decrease as in the case of static coseismic deformation and has a peak of 8.8 μs four hours after the beginning of the tsunami propagation, greater than the value obtained as a result of solid mass redistribution (6.8 μs) (Gross and Chao, 2006). Anyway, these transient variations occur in a too short time window to be evidenced by current measurement techniques. We note also that, since the LOD changes can currently be determined with an accuracy of about 20 μs (Gross and Chao, 2006) and the modeled LOD changes are 2.3 times smaller than the uncertainty in the measurements, the changes due to a water mass redistribution could be too small to be detected.

4. Conclusions

In this work we studied the effect of a perturbation to the Earth rotation due to a transient phenomenon associated with the propagation of a tsunami wave that has been never investigated yet.

We developed a theoretical model about the effect of a transient water mass redistribution on the instantaneous pole of rotation and on the LOD and applied it to the tsunami following the Sumatra earthquake.

The effect produced by the modeled water mass redistribution on the pole path is qualitatively in agreement with the observed step-like discontinuity in the pole path but its magnitude is one hundred times smaller than that assessed in geodetic data. The geodetic observations however have still to be confirmed (or not), but unfortunately no work on this topic is available in the literature yet.

The LOD variation caused by the modeled tsunami is a transient decrease with a peak value greater than the coseismic effect due to the static dislocation, but its time scale is too short to be detected by current measurement techniques and its amplitude is smaller than current measurement uncertainties.

Therefore we have to discard the perturbation effect of the tsunami as a possible explanation of the step-like pole path observed in available geodetic data.

REFERENCES


BAO, L.F., A. PIATANESI, Y. LU, H. T. HSU and X. H. ZHOU
Effects of transient water mass redistribution associated with a tsunami wave on Earth’s pole path


(received July 05, 2007; accepted December 21, 2007)