Variation of the feedback coefficient with $R_{12}$ and the geographic latitude in 1-h ahead forecast of $f_0 F_2$

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Abstract
The «prediction» and «forecast» of the critical frequency of the $F_2$ layer ($f_0 F_2$) is an important issue for frequency planning in short wave radio communications. In this context, «prediction» is used for the determination of monthly median values of $f_0 F_2$ for each hour, while «forecast» denotes the determination of hourly values. In a previous paper we proposed a «sliding window» technique for prediction combined with «feedback» for forecast (Bilge and Tulunay, 2000). In the present paper we obtain the variation of the feedback coefficient with $R_{12}$ and geographic latitude.

1. Introduction
The prediction and forecasting of the ionospheric critical frequency $f_0 F_2$ is crucial in planning HF communication and for radar and navigation systems. The monthly median values of $f_0 F_2$ for each hour can be considered as a first approximation to the data and tabulated values for these medians provide a good guideline for frequency planning. However the monthly medians fail to follow short term irregular variations and forecasting methods are needed. The «feedback» method that is widely used in control engineering and signal processing areas was applied 1-h ahead of forecast of $f_0 F_2$ in a previous paper (Bilge and Tulunay, 2000). In the present paper we investigate the dependency of the feedback coefficient on physical parameters such as the 12-month smoothed sunspot number $R_{12}$ and the geographic latitude.

The monthly median values for each hour depend mainly on solar activity and season. Various sophisticated models using different indices for solar activity have been proposed (Smith and King, 1981; Kane, 1992a,b; Alberca et al., 1999) but for practical purposes it is preferable to use $R_{12}$ as the only physical parameter, because of its availability, reliability and predictability (Bradley, 1994). The well-known hysteresis and saturation effects characterize the dependency of $f_0 F_2$ on $R_{12}$. Namely, $f_0 F_2$ changes linearly with $R_{12}$ for low and medium solar activity and then reaches saturation. In addition, various periodic components of the $f_0 F_2$ variation are modulated by $R_{12}$. The saturation effect is usually dealt with by using a parabolic fit to the data. More sophisticated functional fits using square roots.
and higher order polynomials (Bilge and Tulunay, 1998) give better fits but they are not stable for long-term prediction. The character of the dependency of \( f_o F_2 \) on \( R_n \) is different in rising and falling phases of a solar cycle and it also changes from solar cycle to solar cycle. These differences are important for understanding the mechanism underlying ionospheric processes, however for prediction purposes, non-stationarity of the data can be overlooked by using short term past data to build the models for prediction.

We have noted that the dependency of \( f_o F_2 \) on \( R_n \) is more or less linear over a time span of 2 to 4 years and we developed a «sliding window» technique to predict the monthly median \( f_o F_2 \) using immediately past 48 month data within 3-4% error, compared with 6-7% errors based on 20 year models (Baykal, 1998).

Prediction by «sliding windows» and forecast by «feedback» was proposed in a previous work (Bilge and Tulunay, 2000), and tested in the framework of COST 251 action Stanisławska et al., 1999). The feedback coefficient is the key parameter in single step feedback, and it is determined by a one-dimensional optimization. The aim of the present work is to study the dependency of the optimal feedback coefficient on \( R_n \) and geographic latitude.

We used those data described in detail in Mizrahi et al. (2001), provided in the COST 251 CD-ROM, namely data from 48 stations between 1958 and 2000. Eliminating those stations with less regular data coverage, we based our study on 13 stations arranged in 3 groups according to their latitudes. The first group includes the stations Lycele, Kiruna, Arkhangelsk, Uppsala and Leningrad, which lie nearly above 60N line. The second, mid latitude group includes the stations in the 50N-57N band, i.e. Slough, Juliusruh, Moscow, Pruhonice, Kiev. The stations Tortosa, Rome and Sofia in the 40N-45N band are arranged in the third group (Mizrahi et al., 2001).

Our preliminary investigations have shown that the feedback coefficients for forecast based on predicted or actual monthly medians differ in general by about 0.1 and the interrelations are the same. Thus for simplicity of data processing we based our forecast on actual monthly medians.

2. Forecasting with feedback

The estimation of the actual hourly values of \( f_o F_2 \) is called «forecasting», and it is dealt with by means of neural network (Tulunay et al., 2000) and «feedback» methods (Bilge and Tulunay, 2000), in addition to more standard auto-correlation techniques (Stanisławska et al., 1999). The feedback method is a standard tool in control engineering, for maintaining a constant output or equivalently for following or tracking a given signal. At each step, an appropriate multiple or combination of the measurement error is «fed back», to modify the controlling signal. Here, the error is the difference between the measurement and the prediction of monthly medians. An appropriate multiple of the error in the \( i \) th step is added with the reverse sign to correct the prediction at the \( i + 1 \)st step. That is, if the measurement and prediction at the time \( t_i \) are respectively \( f_o F_2 (t_i) \) and \( f_o F_2 \text{ pred}(t_i) \), then the error at stage \( t_i \) is

\[
E(t_i) = f_o F_2 (t_i) - f_o F_2 \text{ pred}(t_i).
\]

As the predicted value is available at the stage \( t_{n+1} \), the forecast at the hour \( t_{n+1} \), denoted by \( f_o F_2 ^* \) is obtained from

\[
f_o F_2 ^* (t_{n+1}) = f_o F_2 \text{ pred}(t_{n+1}) - k E(t_i)
\]

where \( k \) is the feedback coefficient.

This method has been applied to data from Rome, Poitier and Uppsala ionosonde stations over 1986-1990 (Bilge and Tulunay, 2000). The comparison of the monthly median (prediction) data, actual \( f_o F_2 \), and predicted \( f_o F_2 \) is shown in fig. 1, for a typical storm time disturbance followed by quiet days, Rome, 16-23 April 1958.

In one-step feedback, there is a single parameter to be adjusted, namely the feedback constant \( k \). The «best» feedback constant is found by applying feedback with \( k \) ranging in a certain interval and computing the error with respect to a certain norm. The value of \( k \) corresponding to a local minimum in the chosen norm of the error is called \( k^* \).
Variation of the feedback coefficient with $R_n$ and the geographic latitude in 1-h ahead forecast of $f,F_2$

We have run the feedback algorithm applied to actual monthly medians for a feedback constant in the range 0.2-1.1. We used the $l_2$ norm of the error as our performance criterion. However preliminary investigations have shown that RMS error also leads to same values for optimal feedback constant.

3. Results

The optimal feedback coefficient was computed by minimizing the $l_1$ norm of the error between the hourly values and monthly medians for each hour, i.e. constant feedback is used during a year. A total of 369 samples were analyzed. The feedback constant ranges all times between 0.5 and 1.0. The stations are arranged into three main groups according to their latitudes. The percentage of occurrences of the $k^*$ values in each group is given in table I. The first group includes Lycele, Kiruna, Arkhangelsk, Uppsala and Leningrad; the second group includes Slough, Juliusruh, Moscow, Pruohonice, Kiev and the third group includes Tortosa, Rome and Sofia stations. The number of samples in these groups is respectively 147, 151 and 71.

Thus the optimal feedback coefficient ranges mostly in the 0.7-0.9 interval, hence the value $k = 0.8$ used in Bilge and Tulunay (2000) is typical. There is a tendency of $k^*$ to be lower at lower latitudes. The graph of the feedback constants for all stations versus years is given.

Fig. 1. Comparison of the monthly medians, hourly data and forecast results. Data show a typical storm condition in Rome 1958 data for 8 days from April 16 to April 23. The forecast by feedback follows well the depression at the 3rd and 4th days but fails to follow closely the midday and midnight fluctuations.
in fig. 2. We note that the extreme value $k = 1$ occurs during 1958 and 1959 corresponding to an extremely high solar activity. On the other hand, the extreme low value $k = 0.5$ occurs for low $R_{12}$ (with an exception of 1969, for Sofia). We can thus conclude that the value of the optimal feedback constant increases with $R_{12}$.

The optimal feedback coefficients and corresponding forecast errors are shown in figs. 3a-f for selected stations. In these graphs, $R_{12}$ is drawn to scale, while $k^*$ is multiplied by 100 and the errors are multiplied by 10 for convenient display. The variation of $k^*$ follows the $R_{12}$ variation quite regularly for Rome, Slough, Juliusruh, Uppsala, Leningrad and Lycele stations, oscillating between 0.6-0.8 for Rome, and between 0.7-0.9 at higher latitudes. The variation of $k^*$ does not follow $R_{12}$ as regularly at

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**Table 1.** The percentage of occurrences of the feedback constants for all stations and years.

<table>
<thead>
<tr>
<th>Feedback constant $k^*$</th>
<th>0.5</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latitude: 40N-43N %</td>
<td>9.86</td>
<td>15.49</td>
<td>26.76</td>
<td>36.62</td>
<td>9.86</td>
<td>1.41</td>
</tr>
<tr>
<td>Latitude: 50N-55N %</td>
<td>0.00</td>
<td>2.65</td>
<td>11.26</td>
<td>46.36</td>
<td>39.07</td>
<td>0.66</td>
</tr>
<tr>
<td>Latitude: 59N-68N %</td>
<td>0.68</td>
<td>0.68</td>
<td>14.29</td>
<td>53.06</td>
<td>30.61</td>
<td>0.68</td>
</tr>
<tr>
<td>All stations</td>
<td>2.17</td>
<td>4.34</td>
<td>15.45</td>
<td>47.15</td>
<td>30.08</td>
<td>0.81</td>
</tr>
</tbody>
</table>

**Fig. 2.** A plot of the feedback constants versus years for all stations.
Variation of the feedback coefficient with $R_{ij}$ and the geographic latitude in 1-h ahead forecast of $f_{ij}$

Fig. 3a,b. Values of the feedback constant $k$ (*) versus years for stations Rome (a) and Slough (b). A graph of $R_{ij}$ is shown for comparison. The corresponding errors are shown with (+). The $R_{ij}$ is drawn to scale, while $k$ values ranging between 0.5 and 1.0 are multiplied by 100 and percentage errors ranging between 6%-14% are multiplied by 10 for convenient display. These figures are typical examples of regular variations with $R_{ij}$ at low, mid and high latitudes.
Fig. 3c,d. Values of the feedback constant \( k \) (*) versus years for stations Uppsala (c) and Sofia (d). A graph of \( R_{12} \) is shown for comparison. The corresponding errors are shown with (+). The \( R_{12} \) is drawn to scale, while \( k \) values ranging between 0.5 and 1.0 are multiplied by 100 and percentage errors ranging between 6%-14% are multiplied by 10 for convenient display. Figure 3c is a typical example of regular variations with \( R_{12} \) at low, mid and high latitudes. Figure 3d shows some irregularities.
Variation of the feedback coefficient with $R_{ij}$ and the geographic latitude in 1-h ahead forecast of $f_i F_j$.

Fig. 3e-f. Values of the feedback constant $k$ (*) versus years for stations Moscow (e) and Kiruna (f). A graph of $R_{ij}$ is shown for comparison. The corresponding errors are shown with (+). The $R_{ij}$ is drawn to scale, while $k$ values ranging between 0.5 and 1.0 are multiplied by 100 and percentage errors ranging between 6%-14% are multiplied by 10 for convenient display. These figures show some irregularities.
Aysê H. Bilge, Eti Mizrahi and Yurdanur Tulunay

Sofia, Pruhonice, Moscow, Arkhangelsk and Kiruna, while data for Tortosa are scarce. We have shown the graphs for Rome, Slough, Uppsala, in figs. 3a-c, and the graphs for Sofia, Moscow and Kiruna in figs. 3d,f, as typical examples of regular and irregular behavior in each latitude band.

An observation of these graphs also reveals that the errors tend to be higher for low feedback constant \( k \). A plot of the forecast errors versus the optimal feedback coefficient \( k^* \) given in fig. 4 confirms this observation, as the plot has a negative slope. The errors range mostly between 6% and 12%. Furthermore, we also observed that the error is not too sensitive to \( k \); in the range \([k^* - 1, k^* + 1]\).

In order to quantify the dependency of \( k^* \) on the latitude and on \( R_{12} \), we grouped the yearly samples from all stations, according to the latitudes of the stations as in table I, and the \( R_{12} \) value of the corresponding years as given in table II.

The average value of \( k^* \) corresponding to each group of latitude and \( R_{12} \) range is given in table III.

We note that, except for the second column, the average \( k^* \) in the highest latitude range is slightly lower than the value in the mid latitude range. This result is similar to the behavior of the variation of the upper deciles of the negative deviations with \( R_{12} \), presented in Mizrahi et al. (2001) and the «On the day-to-day variation of MUF over Europe» reported in Kouris et al. (2000). These parameters increase with latitude but then decrease slightly after 60N. As the feedback compensates for the disturbances, it is

Fig. 4. A plot of the feedback error versus feedback constant.
Variation of the feedback coefficient with $R_{12}$ and the geographic latitude in 1-h ahead forecast of $f_0F_2$.

It is plausible that we need higher feedback constants when the fluctuations in the data have larger amplitudes, which also explains the correlated latitude dependency of $k^*$ and the upper deciles.

### REFERENCES


TULUNAY, E., C. OZKAPTAN and Y. TULUNAY (2000): Temporal and spatial forecasting of the $f_0F_2$ values up to twenty four hours in advance, Phys. Chem. Earth (C), 25, 281-285.

### Table II. The breakdown of years 1958-1998 into $R_{12}$ ranges.

<table>
<thead>
<tr>
<th>$R_{12}$</th>
<th>Years</th>
</tr>
</thead>
</table>

### Table III. Average values of the optimal feedback coefficient in different latitude groups and $R_{12}$ ranges.

<table>
<thead>
<tr>
<th>Latitude: 40N-43N</th>
<th>$R_{12}$: 0-29</th>
<th>$R_{12}$: 30-59</th>
<th>$R_{12}$: 60-109</th>
<th>$R_{12}$: 140-189</th>
</tr>
</thead>
<tbody>
<tr>
<td>%</td>
<td>6.588</td>
<td>6.824</td>
<td>7.680</td>
<td>7.917</td>
</tr>
<tr>
<td>Latitude: 50N-55N</td>
<td>%</td>
<td>7.750</td>
<td>8.111</td>
<td>8.380</td>
</tr>
<tr>
<td>Latitude: 59N-68N</td>
<td>%</td>
<td>7.543</td>
<td>8.162</td>
<td>8.275</td>
</tr>
</tbody>
</table>

Table III. The breakdown of years 1958-1998 into $R_{12}$ ranges.