

A note on the topographic distortion of magnetotelluric impedance

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Abstract

Magnetotelluric surveys are prone to interpretation errors in the presence of rough topography, which may need to be compensated for. In the present work we assume that the surface of the Earth is simulated by a single-valued, twice differentiable function $f(x, y)$. By appropriately expanding the surface magnetic field, we obtain the distorted magnetotelluric impedance tensor in terms of an expansion depending on the external radii of curvature of $f(x, y)$ at the observation point and on skin depth. Based on first principles, an analytic estimation of the topographic corrections of the magnetotelluric impedance tensor is obtained.

Key words *magnetotellurics – topography*

1. Introduction

A problem commonly appearing in magnetotelluric (MT) surveys is topographic distortion (Schnegg *et al.*, 1983, 1986; Chouteau and Bouchard, 1988; Fischer, 1989), especially in the case of measurements carried out in mountainous areas (*e.g.*, Draget *et al.*, 1980; Kurtz *et al.*, 1986; Vallianatos, 1995). Interpretation errors may occur if the distortion is not taken into account, or compensated for. Telluric currents flowing parallel to the surface of the Earth, converge beneath valleys and diverge beneath hills. Therefore the current

density J and the associated MT field tend to increase under a valley and decrease under a hill. The reduction of this distortion is therefore an important step toward the correct detection and interpretation of subsurface conductivity distributions. Several studies exist on the effect of topography on MT measurements, most of which adopt a numerical modeling approach (Jiracek, 1973; Harinarayana and Sarma, 1982; Wannamaker *et al.*, 1985, 1986; Jiracek *et al.*, 1986; Xu and Zhou, 1997). Under static conditions, the topographic effects have also been calculated analytically, using Schwarz-Christoffel transformations (Thayer, 1975; Harinarayana and Sarma, 1982; Harinarayana, 1986). In the present short work, we present a formulation to analytically estimate the topographic effect, to first and second order correction terms. The proposed approach is based on the simulation of the earth surface by a single-valued, twice differentiable function and an appropriate expansion of the magnetic field. The estimated corrections depend on frequency and the external radii of curvature of the earth surface at the observation point.

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2. Mathematical formulation

As physical basis for the analysis of any magnetotelluric problem, consider Maxwell's equations for an isotropic, source-free region. In the frequency domain, with the constituent relations $\mathbf{D} = \varepsilon \mathbf{E}$, $\mathbf{B} = \mu \mathbf{H}$, $\mathbf{J} = \sigma \mathbf{E}$ incorporated, these are (see for example Rokityansky, 1982):

$$\nabla \times \mathbf{H} = (\sigma - i\omega\varepsilon)\mathbf{E} = \sigma^* \mathbf{E}$$

$$\nabla \times \mathbf{E} = i\omega\mu \mathbf{H}$$

$$\nabla \cdot \mathbf{H} = 0 \quad \text{and} \quad \nabla \cdot \mathbf{E} = 0.$$

By combining the curl equations and using the vanishing divergences, it can be seen (Jackson, 1975) that both fields satisfy the homogeneous wave equation (Helmholtz equation) (Wait, 1987)

$$\nabla^2 \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} + k^2 \begin{bmatrix} \mathbf{E} \\ \mathbf{H} \end{bmatrix} = 0$$

where the propagation constant is $k^2 = i\omega\mu\sigma^*$.

The MT impedance, can be expressed as a rank 2 tensor. If x and y are orthogonal measurement axes on the Earth surface, the tangential to the surface electric field variations $\mathbf{E}_\parallel = (E_x, E_y)^T$ is directly related to the variations of the two magnetic fields $\mathbf{H}_\parallel = (H_x, H_y)^T$ by the frequency domain relation

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} Z_{xx} & Z_{xy} \\ Z_{yx} & Z_{yy} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \end{bmatrix}$$

or (2.1)

$$\mathbf{E}_\parallel = \bar{\mathbf{Z}} \cdot \mathbf{H}_\parallel$$

where Z_{ij} are the elements of the impedance tensor (Kaufman and Keller, 1981).

Next, consider an Earth interface (air-Earth interface) described by a profile function $f(x, y)$, which is assumed to be single valued and twice differentiable. The vector

normal to the surface is (Lipschutz, 1989; Morse and Feshbach, 1963)

$$\hat{\mathbf{N}} = \Phi(x, y) [\hat{\mathbf{z}} - \nabla f(x, y)]$$

where

$$\Phi(x, y) = \left[1 + (\nabla f(x, y))^2 \right]^{-\frac{1}{2}}.$$

In order to generalize eq. (2.1), we note that

$$H_x = (\hat{\mathbf{N}} \times \mathbf{H})_y$$

and (2.2)

$$H_y = -(\hat{\mathbf{N}} \times \mathbf{H})_x.$$

Combining eqs. (2.1) and (2.2), gives

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} -Z_{xy} & Z_{xx} \\ -Z_{yy} & Z_{yx} \end{bmatrix} \begin{bmatrix} (\hat{\mathbf{N}} \times \mathbf{H})_x \\ (\hat{\mathbf{N}} \times \mathbf{H})_y \end{bmatrix}$$

or equivalently (2.3)

$$E_{\parallel, i} = C_{ij}^g (\hat{\mathbf{N}} \times \mathbf{H})_j$$

where the sum over j is implied (Einstein notation). In the case of a simple one dimensional (1D) structure $\mathbf{C}_{ij}^g = \mathbf{Z}_o \mathbf{C}_{ij}$ where C_{ij} are the elements of a «normalized» surface impedance tensor. It is straightforward to see that for a flat surface $C_{ij} = \delta_{ij}$. We note that \mathbf{E}_\parallel and $\hat{\mathbf{N}} \times \mathbf{H}$ have only components in the tangent plane, so that the normalized surface impedance tensor \mathbf{C} , being a surface tensor in curvilinear coordinates, is represented by a 2×2 matrix. Hereafter, in order to simplify the mathematical formulation, we assume one dimensional structure with a surface profile function $f(x, y)$.

For z approaching $f(x, y)$ from below, the component of \mathbf{E} tangential to the surface is

$$E_{//} = E - \hat{N}(\hat{N} \cdot E)$$

$$= \frac{1}{\sigma - i\omega\mu} \left\{ \nabla \times \mathbf{H} - \hat{N} [\hat{N} \cdot (\nabla \times \mathbf{H})] \right\}.$$

We introduce the coordinate variables $u = x$, $v = y$, $w = [z - f(x, y)]/D$ where $D(\omega) = ilk(\omega)$, in which the Helmholtz vector equation for \mathbf{H} is

$$\left[\frac{\partial^2}{\partial w^2} - \Phi^2 - D\Phi^2 \left(\nabla^2 f + 2\nabla f \cdot \nabla \right) \frac{\partial}{\partial w} + D^2\Phi^2 \left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} \right) \right] \mathbf{H} = 0$$

Expanding \mathbf{H} in powers of $D(\omega)$ (see Appendix)

$$\mathbf{H} = \sum_{i=0}^{\infty} D^i \mathbf{h}_{(i)}$$

we obtain the recursive relation

$$\left(\frac{\partial^2}{\partial w^2} - \Phi^2 \right) \mathbf{h}_{(n)} =$$

$$= \Phi^2 (\nabla^2 f + 2\nabla f \cdot \nabla) \frac{\partial}{\partial w} \mathbf{h}_{(n-1)} -$$

$$- \Phi^2 \left(\frac{\partial^2}{\partial u^2} + \frac{\partial^2}{\partial v^2} \right) \mathbf{h}_{(n-2)}.$$

For $n = 0$, the solution that vanishes at $\text{Re}(w) \rightarrow -\infty$ is $\mathbf{h}_{(0)} = \mathbf{a}_{(0)}(u, v) \exp(\Phi w)$. Inserting this in the above recursive relation, we obtain

the equation for the next term

$$\left(\frac{\partial^2}{\partial w^2} - \Phi^2 \right) \mathbf{h}_{(1)} = (\mathbf{W} + \mathbf{V}w) \exp(\Phi w).$$

With a solution that vanishes at $\text{Re}(w) \rightarrow -\infty$ (Bronson, 1973; Morse and Feshbach, 1963)

$$\mathbf{h}_{(1)} = \left[\mathbf{a}_{(1)} + \left(\frac{\mathbf{W}}{2\Phi} - \frac{\mathbf{V}}{4\Phi^2} \right) w + \frac{\mathbf{V}}{4\Phi} w^2 \right] \exp(\Phi w)$$

where

$$\mathbf{V} = -2\Phi^6 (\nabla f \cdot \nabla \nabla f \cdot \nabla f) \mathbf{a}_{(0)}$$

$$\mathbf{W} = \Phi^3 \left[\nabla^2 f - 2\Phi^2 \nabla f \cdot \nabla \nabla f \cdot \nabla f + 2(\nabla f \cdot \nabla) \right] \mathbf{a}_{(0)}.$$

Continuing in way, we expand the magnetic field in terms that include the expression

$$\sum_i D^i \alpha_{(i)}.$$

The arbitrariness of the latter term is partially removed by the remaining Maxwell equation $\nabla \cdot \mathbf{H} = 0$.

After extensive algebra and differential geometry, we can write

$$\mathbf{E}_{//} = (\sigma - i\omega\epsilon)^{-1} \left[\frac{\hat{N}}{D} \times \mathbf{a}_{(0)} + \hat{N} \times \mathbf{a}_{(1)} + \right.$$

$$+ K_m \hat{N} \times \mathbf{a}_{(0)} + \nabla \times \mathbf{a}_{(0)} + \Phi \hat{N} \times$$

$$\left. \times (\nabla f \cdot \nabla) \mathbf{a}_{(0)} - \hat{N} (\hat{N} \cdot \nabla \times \mathbf{a}_{(0)}) \right]$$

where

$$K_m = \frac{1}{2} \left(\frac{1}{R_{\min}} + \frac{1}{R_{\max}} \right) = \frac{1}{2} \text{Tr}(S) =$$

$$= \frac{1}{2} \Phi (\nabla^2 f - \Phi^2 \nabla f \cdot \nabla \nabla f \cdot \nabla f)$$

the mean surface curvature and S is the extrinsic curvature tensor of the surface (Lipschutz, 1989). Finally, by applying vector calculus we obtain at $w = 0$

$$E_{//} = (\sigma - i\omega\varepsilon)^{-1} \left[\left(\frac{1}{D} - K_m \right) \hat{N} \times \mathbf{H} + \right. \\ \left. + \Phi \mathbf{P} \cdot \nabla \nabla f \cdot (\hat{N} \times \mathbf{H}) \right] \quad (2.4)$$

where $\mathbf{P} = \bar{\mathbf{I}} - \hat{N} \cdot \hat{N}$ is the projection operator in the tangent plane, which in matrix terms is given as

$$\mathbf{P} = \begin{pmatrix} 1 - \Phi^2 \left(\frac{\partial^2 f}{\partial^2 x} \right)^2 & -\Phi^2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} & \Phi^2 \frac{\partial f}{\partial x} \\ -\Phi^2 \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} & 1 - \Phi^2 \left(\frac{\partial^2 f}{\partial^2 y} \right)^2 & \Phi^2 \frac{\partial f}{\partial y} \\ \Phi^2 \frac{\partial f}{\partial x} & \Phi^2 \frac{\partial f}{\partial y} & 1 - \Phi^2 \end{pmatrix}$$

Since the operator \mathbf{P} acts in the same way as the identity $\bar{\mathbf{I}}$ applied on $\hat{N} \times \mathbf{H}$, eq. (2.4) could be written as

$$E_{//} =$$

$$= \frac{1}{\sigma^* D} \left[\bar{\mathbf{I}} + D(\Phi \mathbf{P} \cdot \nabla \nabla f \cdot \mathbf{P} - K_m \mathbf{P}) \right] (\hat{N} \times \mathbf{H}) =$$

$$= Z_0 \left[\bar{\mathbf{I}} + D(S - K_m \mathbf{P}) \right] (\hat{N} \times \mathbf{H}) \quad (2.5)$$

where $S = \Phi \mathbf{P} \cdot \nabla \nabla f \cdot \mathbf{P}$ is the extrinsic curvature tensor of the surface (Lipschutz, 1989). A comparison of eqs. (2.3) and (2.5) leads to the conclusion that the normalized impedance tensor $\mathbf{C} = (C_{ij})$ has the form $\mathbf{C} = \bar{\mathbf{I}} + D\mathbf{C}^{(1)}$, where $\mathbf{C}^{(1)}$ is simply the traceless part of the extrinsic curvature tensor of the surface S , given by the expression $\mathbf{C}^{(1)} = S - K_m \mathbf{P}$. We note that the eigenvalues of $\mathbf{C}^{(1)}$ are 0 and $\pm \Delta$ where

$$\Delta = \frac{1}{2} \left(\frac{1}{R_{\min}} - \frac{1}{R_{\max}} \right).$$

Since the aforementioned first order correction is expressed by terms that include the second spatial derivatives of $f(x,y)$, it is apparent that the topographic profile with linear slope introduces a topographic correction given by a second order term.

Following the same procedure and keeping second order terms in our approximations, we reach an expression for the normalized impedance tensor

$$\mathbf{C} = \bar{\mathbf{I}} + D\mathbf{C}^{(1)} - \left(\frac{D^2}{4} \right) \bar{\mathbf{I}} \text{Tr} \left[(\mathbf{C}^{(1)})^2 \right]$$

where

$$\text{Tr} \left[(\mathbf{C}^{(1)})^2 \right] = 2\Delta^2 =$$

$$= \Phi^2 \text{Tr} \left[(\nabla \nabla f)^2 \right] - 2\Phi^4 (\nabla f \cdot \nabla \nabla f)^2 +$$

$$+ \Phi^6 (\nabla f \cdot \nabla \nabla f \cdot \nabla f)^2 - 2K_m^2.$$

According to the aforementioned mathematical formulation, it is obvious that a crucial parameter in our expansion is the dimensionless product

$$D(\omega) \frac{\partial^2 f}{\partial x_i \partial x_j}$$

where $x_1 = x$ and $x_2 = y$. It is expected that when they are small the retention of only the first few terms in the expansion should yield inaccurate results. Alternatively, we can say that our approximation should be valid as long as $k_T D(\omega)$ is much smaller than unity, where k_T is the wavenumber that characterizes the roughness of our terrain.

3. Concluding remarks

In the present preliminary work we present a formulation to analytically approximate the topographic distortion of the magnetotelluric impedance tensor up to first and second order correction terms. The proposed approach is based on the simulation of the Earth's surface with a single-valued, twice differentiable function and an appropriate expansion for the magnetic field. The estimated corrections depend on frequency and on

the external radii of curvature of the Earth's surface at the observation point, leading to accurate results, as the dimensionless product

$$D(\omega) \frac{\partial^2 f}{\partial x_i \partial x_j} \text{ is small.}$$

The formulation does not make any assumption about the type of topographic profile (*i.e.* slopes, height). Applying the presented first principles approach, we estimate the correction terms as functions of the lateral second derivatives of the topographic profile. In this way we have obtained a general expression for the topographic correction of the surface MT impedance tensor up to first and second order correction terms. The first order correction $C^{(1)}$ term is given by the traceless part of the extrinsic surface curvature tensor S , while the second order correction is just a scalar given in terms of trace of $C^{(1)}$.

Appendix

Consider a time-harmonic plane wave incident from free space into a homogeneous earth surface described by a terrain function $f(x,y)$. In formulating the problem one assumes that the topography acts as a scatterer. Since the currents flowing parallel to the earth's surface converge beneath valleys and diverge beneath hills, it is straightforward to describe the scattering from the topography by a volume distribution of currents induced inside the «effective topographic scatterer». Maxwell's equations corresponding to this representation of the electric field is (Verdin and Bostick, 1992; Habashy *et al.*, 1993)

$$\nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k_b^2 \mathbf{E}(\mathbf{r}) = Q_T(\mathbf{r}) \mathbf{E}(\mathbf{r}) \quad (\text{A.1})$$

where the wavenumber $k_b^2 = i\omega\mu\sigma_b$ and the topography factor $Q_T(\mathbf{r})$ are the terms describing the scattering of currents due to the presence of topography.

The solution to eq. (A.1) can be expressed with a dyadic Green's function which is governed by the appropriate boundary conditions and satisfies the equation

$$\nabla \times \nabla \times \bar{\mathbf{G}}(\mathbf{r}) - k_b^2 \bar{\mathbf{G}}(\mathbf{r}) = \bar{\mathbf{I}} \delta(\mathbf{r} - \mathbf{r}') \quad (\text{A.2})$$

where $\bar{\mathbf{I}}$ is the identity dyadic. For a homogeneous background, the solution to eq. (A.2) is (Kong, 1986)

$$\bar{\mathbf{G}}(\mathbf{r}, \mathbf{r}') = \left[\bar{\mathbf{I}} + \frac{1}{k_b^2} \nabla \nabla \right] g(\mathbf{r}, \mathbf{r}'), \text{ where } g(\mathbf{r}, \mathbf{r}') \text{ is the scalar, Green's function which satisfies}$$

$$\nabla^2 g(\mathbf{r}, \mathbf{r}') + k_b^2 g(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}') \text{ and is given by } g(\mathbf{r}, \mathbf{r}') = \frac{e^{ik_b |\mathbf{r} - \mathbf{r}'|}}{4\pi |\mathbf{r} - \mathbf{r}'|} = \frac{e^{-\frac{|\mathbf{r} - \mathbf{r}'|}{D}}}{4\pi |\mathbf{r} - \mathbf{r}'|}.$$

Thus the solution to eq. (A.1) is given by (Yaghjian, 1980; Habashy *et al.*, 1993; Chave and Smith, 1994; Kong, 1986)

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_b(\mathbf{r}) + \left(\bar{\mathbf{I}} + \frac{1}{k_b^2} \nabla \nabla \right) \left(\int_{V_t} d\mathbf{r}' g(\mathbf{r}, \mathbf{r}') Q_T(\mathbf{r}') \mathbf{E}(\mathbf{r}') \right) \quad (\text{A.3})$$

where V_t is the effective volume related to the topography and supports the appearance of $Q(r)$. Applying Ampere's law to (A.3) gives an analogous integral equation for the magnetic field

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_b(\mathbf{r}) + \frac{1}{i\omega\mu} \nabla \times \left(\int_{V_t} d\mathbf{r}' g(\mathbf{r}, \mathbf{r}') Q_T(\mathbf{r}') \mathbf{E}(\mathbf{r}') \right). \quad (\text{A.4})$$

We seek to obtain \mathbf{H} as an expansion of D . In order to accomplish this we make the following change in variables $x' = x + Dq_1$, $y' = y + Dq_2$. Introducing the dimensionless variables q_1 and q_2 the scalar Green's functions

$$g(\mathbf{r}, \mathbf{r}') = \frac{e^{-\frac{|\mathbf{r}-\mathbf{r}'|}{D}}}{4\pi|\mathbf{r}-\mathbf{r}'|} \quad \text{where}$$

$$R^2 = \left[(x - x')^2 + (y - y')^2 + (f(x, y) - f(x', y'))^2 \right]^{1/2} \quad \text{possesses the expansion}$$

$$\begin{aligned} g(\mathbf{r}, \mathbf{r}') &= \frac{1}{D} \frac{e^{-r_0}}{r_0} \left\{ 1 - \frac{D}{2} \left(\frac{1}{r_0} + \frac{1}{r_0^2} \right) f_3(q_1, q_2) + \right. \\ &+ D^2 \left[-\frac{1}{2} \left(\frac{1}{r_0} + \frac{1}{r_0^2} \right) g_4(q_1, q_2) + \frac{1}{8} \left(\frac{1}{r_0^2} + \frac{1}{r_0^3} + \frac{1}{r_0^4} \right) f_3^2(q_1, q_2) + \frac{1}{8} \left(\frac{1}{r_0^3} + \frac{1}{r_0^4} \right) h_4(q_1, q_2) \right] \left. \right\} + O(D^3) = \\ &= \frac{1}{D} \frac{e^{-r_0}}{r_0} \left(\sum_{n=0}^{\infty} K_n D^n \right) \end{aligned} \quad (\text{A.5})$$

where (see Morse and Feshbach, 1963; Nietro-Vesperinas, 1991)

$$r_0(q_1, q_2) = \left[q_1^2 + q_2^2 + (f_1(x, y)q_1 + f_2(x, y)q_2)^2 \right]^{1/2}$$

$$f_1(x, y) = \partial f / \partial x \quad f_2(x, y) = \partial f / \partial y$$

$$f_3(q_1, q_2) = (f_i q_i)(f_{ij} q_i q_j)$$

$$g_4(q_1, q_2) = \frac{1}{3}(f_i q_i)(f_{ijk} q_i q_j q_k) + \frac{1}{4}(f_{ij} q_i q_j)^2$$

$$h_4(q_1, q_2) = (f_i q_i)^2 (f_{ij} q_i q_j).$$

When the latter expansion is substituted into eq. (A.5) we obtain

$$\mathbf{H}(\mathbf{r}) = \mathbf{H}_b(\mathbf{r}) + \nabla \times \left(\int_{V_t} d\mathbf{r}' \frac{e^{-r_0}}{r_0} \left(\sum_{n=0}^{\infty} K_n D^{n+1} \right) \mathcal{Q}_T(\mathbf{r}') \mathbf{E}(\mathbf{r}') \right). \quad (\text{A.6})$$

From the preceding equation we see that the magnetic field can be expanded in powers of the parameter $D = ik$ according to $\mathbf{H} = \left(\sum_{n=0}^{\infty} \mathbf{h}_{(n)} D^n \right)$.

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