The impact of *a* and *b* value uncertainty on loss estimation in the reinsurance industry

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Abstract
In the reinsurance industry different probabilistic models are currently used for seismic risk analysis. A credible loss estimation of the insured values depends on seismic hazard analysis and on the vulnerability functions of the given structures. Besides attenuation and local soil amplification, the earthquake occurrence model (often represented by the Gutenberg and Richter relation) is a key element in the analysis. However, earthquake catalogues are usually incomplete, the time of observation is too short and the data themselves contain errors. Therefore, a and b values can only be estimated with uncertainties. The knowledge of their variation provides a valuable input for earthquake risk analysis, because they allow the probability distribution of expected losses (expressed by Average Annual Loss (AAL)) to be modelled. The variations of a and b have a direct effect on the estimated exceeding probability and consequently on the calculated loss level. This effect is best illustrated by exceeding probability *versus* loss level and AAL *versus* magnitude graphs. The sensitivity of average annual losses due to different a to b ratios and magnitudes is obvious. The estimation of the variation of a and b and the quantification of the sensitivity of calculated losses are fundamental for optimal earthquake risk management. Ignoring these uncertainties means that risk management decisions neglect possible variations of the earthquake loss estimations.

Key words *completeness – seismic risk – b-value – average annual loss – insurance*

1. Introduction

In seismic risk analysis probabilistic models are used to estimate possible losses (*e.g.*, Cornell, 1968; Shah and Dong, 1991; Schmid and Schaad, 1995). A successful loss estimation of insured and reinsured values depends on the seismic hazard analysis, on the vulnerability functions, on local site effects (*cf.* Aki and Irikura, 1991) and on the capability to apply insurance structures and conditions. Besides attenuation functions, earthquake occurrence models are a key element of the hazard analyses. In order to study the impact of the uncertainty of the occurrence models on loss estimates, a- and b-values of the Gutenberg-Richter relation (Gutenberg and Richter, 1956) were varied based on different assumptions on the completeness of the earthquake catalogue.

In this study, losses due to earthquakes are modelled for a realistic insurance portfolio in Japan. In a first step, historical earthquake data were analysed to study completeness intervals for different observation periods and magnitude ranges. Twenty occurrence models that differ in the derived a- and b-values of the Gutenberg-Richter relation could be derived: $\log N = a - bM$ (N: cumulative number of earthquakes; a: seismic activity; b: slope of cumulative magnitude distribution (see table II).

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In a second step, for each of the 20 different occurrence models, a stochastic event set was generated with the expected annual rate of occurrence. For each stochastic event the site intensity is calculated taking into account the distance from site to epicentre, magnitude of the event and the local soil conditions. The calculated site intensity is then transformed into damage ratios depending on construction classes (Shah and Dong, 1991). IRAS© Japanese model (1997) was used for these calculations. Based on the losses for each stochastic event and on their occurrence probability, Average Annual Losses (AAL) have been calculated to demonstrate the sensitivity of the occurrence model to loss estimates. AAL, also called risk premium, burning cost or pure premium, corresponds to the sum product of event losses and the event occurrence probabilities that is equal to the expectation of the loss probability distribution (see formula (4.1)) (*cf.* Benjamin and Cornell, 1970).

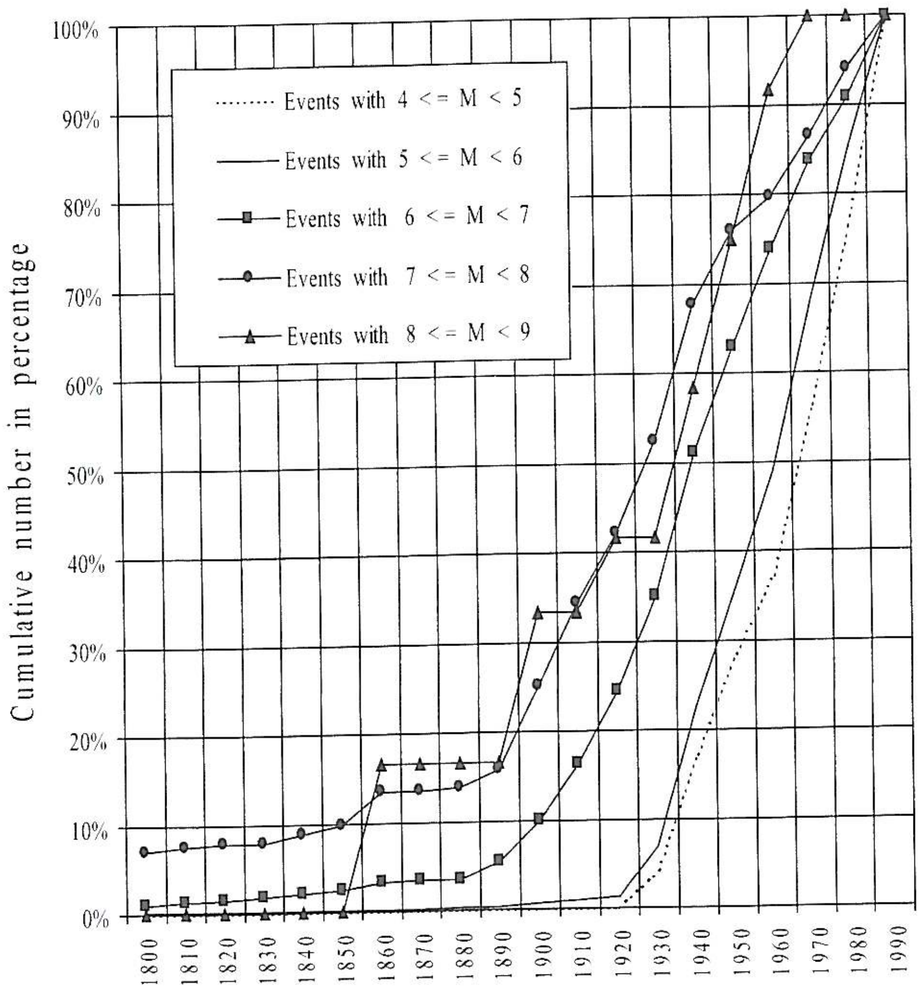

Fig. 1. Cumulative number of historical events *versus* year of occurrence.

2. Completeness of historical earthquake data

The historical earthquake catalogue used in this study (JMA, 1990) covers events dated from 1715 to 1990 and contains more than 20000 events . To model earthquake occurrence in time, it is assumed that they follow a Poisson process with constant recurrence rate λ:

$$P(N = n, t) = e^{-\lambda \cdot t} \cdot (\lambda \cdot t)^n/(n!) \qquad (2.1)$$

where $P(N = n, t)$ is the probability of n events during t years.

The cumulative number *versus* year graphs (fig. 1) show for five magnitude intervals the point in time from when the data is assumed to be complete. Table I gives the time period of complete observation for different magnitude ranges. Figure 1 indicates that for the catalogue used events with magnitude smaller than 5 are complete since 1920. Events with magnitude between 6 and 8 are complete since 1890 and events with magnitude larger than 8 have been completely documented since 1850.

Table I. Starting year of complete earthquake observations for different magnitude ranges.

Magnitude range	Complete since
$4 \le M < 5$	1920
$5 \le M < 6$	1920
$6 \le M < 7$	1890
$7 \le M < 8$	1890
$8 \le M < 9$	1850

Table II. Assumed parameters and calculated a and b of the 20 models derived.

Model	a-value for T = 1 year	b-value	Observation period (years)	Magnitude range		Completeness applied
1	5.30	0.81	275	4.25	8.75	no
2	4.67	0.70	275	4.25	8.25	no
3	5.79	0.87	275	4.75	8.75	no
4	5.04	0.75	275	4.75	8.25	no
5	5.84	0.83	100	4.25	8.75	no
6	5.30	0.73	100	4.25	8.25	no
7	6.33	0.90	100	4.75	8.75	no
8	5.72	0.79	100	4.75	8.25	no
9	6.19	0.87	40	4.25	8.75	no
10	6.00	0.83	40	4.25	8.25	no
11	6.59	0.92	40	4.75	8.75	no
12	6.41	0.89	40	4.75	8.25	no
13	6.18	0.87	100	4.25	8.75	yes
14	5.67	0.78	100	4.25	8.25	yes
15	6.68	0.94	100	4.75	8.75	yes
16	6.11	0.84	100	4.75	8.25	yes
17	6.02	0.88	140	4.25	8.75	yes
18	5.52	0.79	140	4.25	8.25	yes
19	6.53	0.94	140	4.75	8.75	yes
20	5.97	0.85	140	4.75	8.25	yes

3. Model definition

With respect to the completeness intervals, for different observation periods and various magnitude ranges, a- and b-values were calculated. Table II summarises the assumptions and the corresponding a- and b-values for the 20 models.

Models 1 to 4 are based on the original historical catalogue without any changes. Models 5 to 8 and 9 to 12 consider only events since 1890 and 1950, respectively, but do not apply any adjustments for incomplete data. The time period of complete observation was considered in the models 13 to 20. The models 13 to 16 use data starting from 1890; the models 17 to 20 use data since 1850 in order to include the strong earthquakes in 1854. For each of these five model groups four magnitude ranges were used to calculate a- and b-values: 4.25 to 8.75; 4.25 to 8.25; 4.75 to 8.75, and 4.75 to 8.25. As a general tendency, neglecting moderate earthquakes increases the a- and b-values, on the other hand, neglecting strong earthquakes reduces the a- and b-values. Additionally, the shorter the observation period, the larger a and b are. Considering the completeness intervals (models 13 to 20) increases a and b. The comparison of the 20 a and b pairs among themselves shows a large dispersion, with a mean of 5.89 and a standard deviation of 0.53 for a and a mean of 0.84 and a standard deviation of 0.07 for b, respectively.

Given the various a- and b-parameters, a stochastic event set has been generated to represent not only the frequency-magnitude distribution (as determined by a- and b-values), but also the geographic seismicity pattern. The event sets used in this study were derived from the area source model as given in the IRAS© Japanese model (1997), whereby the occurrence probability of each individual event is derived from the a- and b-values.

4. Average annual loss analysis

The sensitivity of loss estimates due to different (a, b)-models is best illustrated by calculating average annual losses. The Average Annual Loss (AAL) is defined as the sumproduct of the occurrence probability of each stochastic event and the corresponding event loss

$$\mathrm{AAL} = \Sigma_i \,(\mathrm{OP}_i \cdot \mathrm{Loss}_i) \tag{4.1}$$

OP_i = probability of at least one occurrence of event i; Loss_i = estimated loss of event i.

Figure 2 shows the sensitivity of AAL due to different models. The calculated AALs show a relatively large variation due to different combinations of completeness, observation period and magnitude ranges. The average AAL is 1838 million Yen with a standard deviation of 622 million Yen for the insurance portfolio used. Models 1 to 4 cause the smallest losses, because they cover the largest time period and neglect the incomplete data (compare table II). The a- and b-values are slightly below the average and produce small AALs. Models 5 to 16 show similar AALs. Model 17 to 20 show AALs smaller than the average. The large difference between

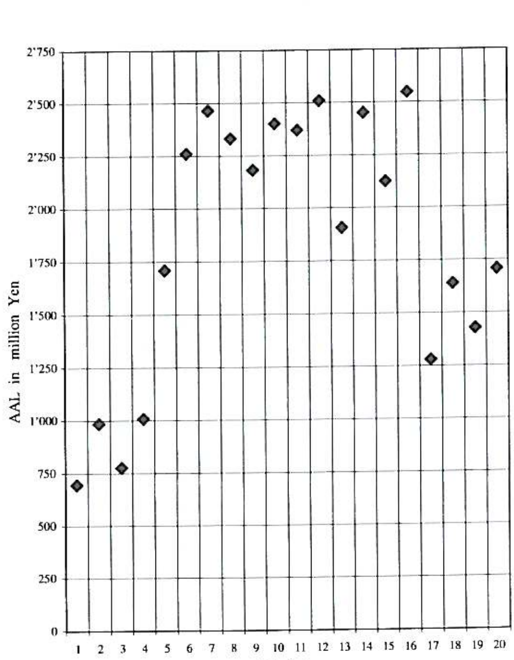

Fig. 2. Average annual losses based on different occurrence models (see table II for model parameter).

models 13 to 16 and models 17 to 20 has to be explained with the longer observation period used in the last four models and with the different completeness intervals applied. The strong dependency of AAL on *a*- and *b*-values shows the importance of historical data records and their correct treatment and application.

Not all of the 20 models derived are of the same quality. The two most reasonable models are number 10 and 13. Model 10 covers only 40 years (1950 to 1990) and therefore does not pose the problem of incomplete data. This model neglects the strong events, but gives more credit to the short-term trend of seismicity. The resulting AAL corresponds to the average AAL plus one standard deviation. Model 13 yields an AAL slightly larger than the average value. This model takes the whole magnitude range into account and covers all events of the last 100 years. Applying the completeness adjustments assures a correct treatment of the earthquake catalogue.

5. Exceeding probability analysis

Exceeding Probability (EP) curves are another way to present loss estimates that are used in the reinsurance industry for pricing issues.

The EP curves are based on the same analysis as the AAL calculations, but instead of adding up the product of event probability and loss (*cf.* formula (4.1)), the events are sorted by the size of their loss (in descending order) and the exceeding probability is calculated as follows:

$$EP_{i+1} = 1-(1-OP_{i+1})\cdot(1-EP_i) \text{ or} \tag{5.1}$$

$$EP_i = 1-\Pi_{j<=i}(1-OP_j) \tag{5.2}$$

OP_i = probability of at least one occurrence of event i; $1-OP_j$ = non-occurrence probability of event $j <= i$.

Table III. Event loss table for model 16, giving 25 most severe events sorted by the size of their loss. The table shows for these events the magnitude, the calculated loss, the occurrence probability and the exceeding probability to the corresponding loss level.

Magnitude	Event loss (million Yen)	Occurrence probability of model 16	Exceeding probability
8.5	34707	0.00144	0.00144
8.1	22682	0.00491	0.00634
8.0	20955	0.00177	0.00810
8.5	19890	0.00260	0.01067
8.0	17835	0.00134	0.01200
8.0	16832	0.00311	0.01507
8.0	14812	0.00159	0.01664
7.2	14648	0.01233	0.02877
7.5	11617	0.00091	0.02965
7.5	10953	0.00267	0.03225
8.5	10234	0.00490	0.03699
7.6	10054	0.00461	0.04143
8.0	8817	0.00253	0.04385
7.5	8407	0.00141	0.04519
8.0	8257	0.00241	0.04750
7.5	7743	0.00197	0.04937
8.5	7461	0.00433	0.05348
7.5	7082	0.00927	0.06225
7.5	6630	0.00395	0.06596
7.5	5729	0.00180	0.06764
6.7	5501	0.04293	0.10766
7.0	4966	0.00667	0.11362
8.0	4840	0.01163	0.12393
7.0	4659	0.00237	0.12601

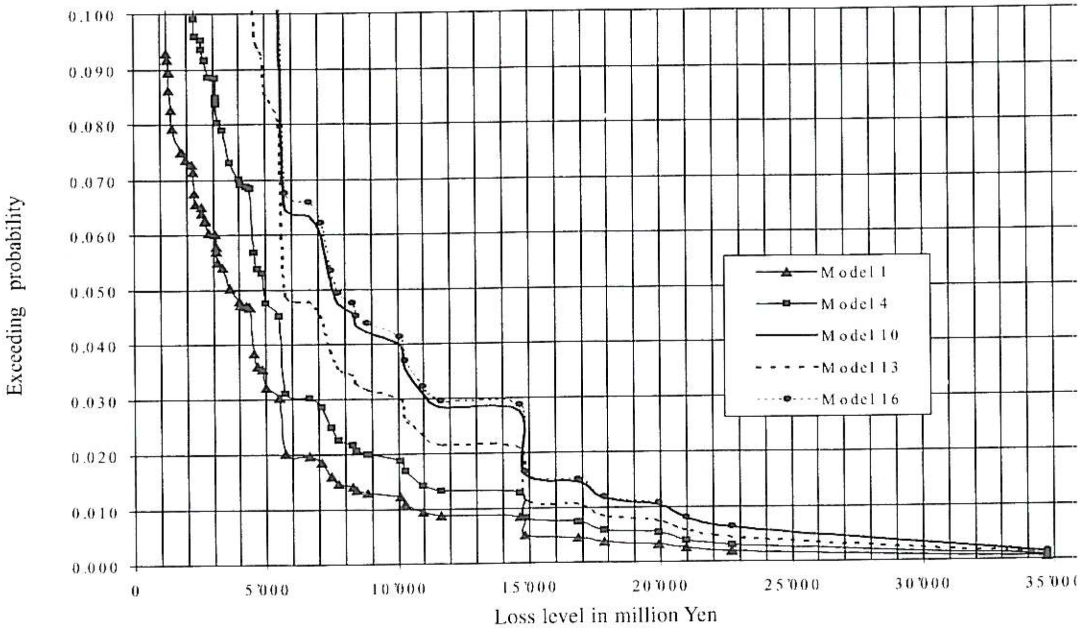

Fig. 3. Exceeding probability curves of five representative occurrence models.

The exceeding probability is the probability that there occurs a loss greater or equal to the associated loss (*e.g.*, there is a probability of 0.1 that a loss greater than 5501 million Yen will occur in a year (*cf.* table III)).

Exceeding Probability (EP) curves for five representative models (1, 4, 10, 13 and 16) (compare table II and fig. 2) are shown in fig. 3. Model 1 produces the smallest AAL, model 16 stands for the largest AAL. For an EP of 1% the loss varies between 10500 million and 20500 million Yen. For an EP of 10% the damage varies between 1200 million and 5500 million Yen for the models 1 and 16. The relative and the absolute loss differences between two models are strongly dependent on the EP values. The larger the probability, the bigger the relative difference in losses. The loss ratios, representing the relative differences, are between 2 and 4. The absolute loss differences increase with smaller probabilities. For example: the loss differences for an EP of 10% is about 4300 million Yen and for to an EP of 1% it is about 10000 million Yen.

6. Discussion and conclusions

The various analyses showed that there i[s] no correlation between AAL and the a- an[d] b-values. The obtained AALs are not proportiona[l] to either the a-values nor to the b-values. How[ever], a strong correlation exists between AAL[s] and the ratio of a to b and *vice versa* (fig. 4). Th[e] correlation coefficient is between 0.931 for th[e] ratio a to b. Provided that a similar portfolio i[s] analysed, the ratio of a to b can be used as a firs[t] measure for AAL estimations.

The question often arises in the reinsuranc[e] industry, whether the more frequent moderat[e] events or the less frequent strong events caus[e] larger AAL. The distribution of AAL with re[-]spect to magnitude (fig. 5) demonstrates thi[s] dependency.

Figure 5 shows clearly that events with mag[-]nitudes smaller than 6 contribute to the tot[al] AAL with less than 5%. The largest contribu[-]tion (55%) stems from events with magnitude[s] ranging from 6.5 to 8. Surprisingly, the strong[-]est events with magnitudes larger than 8 con[-]

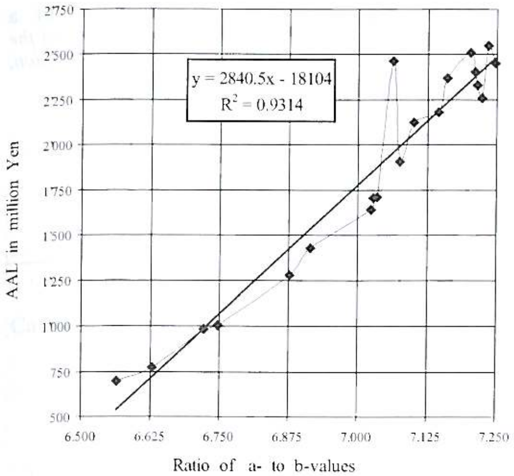

Fig. 4. Average annual loss *versus* a to b ratio.

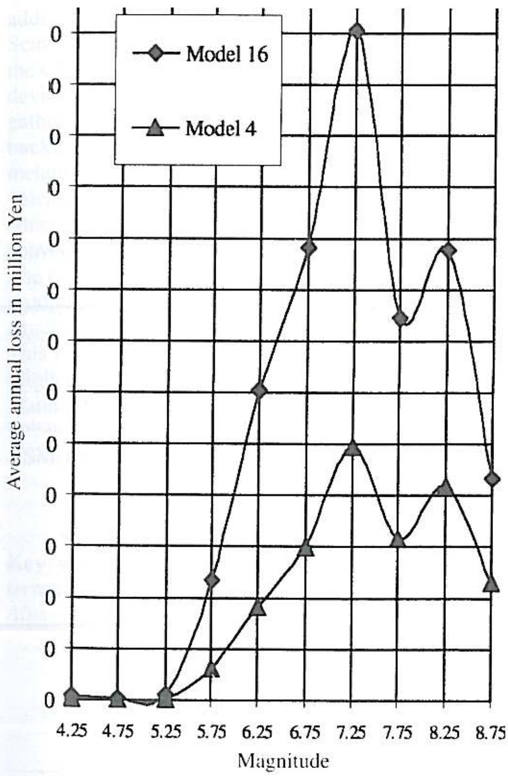

Fig. 5. Average annual loss distribution with respect to magnitude. (Total AAL of model 4 = 1006 Mio Yen and AAL of model 16 = 2548 Mio Yen).

tribute to the total AAL with only 25% (model 16) and 32% (model 4).

Three main conclusions can be drawn, but it has to be stressed that these conclusions are only valid for the studied portfolio and the Japanese seismicity patterns:

1) Since historical data records are often incomplete, it is important that the probabilistic model accounts for this deficiency. The definition and application of several completeness intervals, the use of different observation periods and magnitude ranges gives an estimate of the variations of the modelled losses, which is important for successful risk management in the insurance and reinsurance industry.

2) AALs show a strong dependency on the ratio of a- to b-values. The higher the ratio, the larger the AAL. Hence, this ratio can serve as first measure for AAL estimations, if similar insurance portfolios are evaluated in Japan.

3) Independently of the model used, the contribution of the moderate earthquakes (magnitude range 4 to 6) to the AAL is negligible. The contribution of the strongest earthquakes with magnitudes larger than 8 amounts to not more than approximately 30% of the total AAL; earthquakes in the magnitude range 6.5 to 8 are the dominant events for the total AAL. We assume that in regions with similar seismicity and comparable building quality, the strongest earthquakes contribute in a similar size to the total AAL found in this study.

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