The separation of the geomagnetic field originated in the core, in the asthenosphere, and in the crust

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Abstract

The separation of the field produced by different internal sources can be accomplished by means of the so-called spatial spectrum of the geomagnetic field of internal origin. It is shown how such a rationale, when suitably interpreted, allows to recognize the field that is originated by electric currents that flow either on the Inner-Core Boundary (ICB), or on the Core-Mantle Boundary (CMB), or on the Asthenosphere-Lithosphere Boundary (ALB). It appears crucial, however, to rely on satellite measurements alone, because ground-based and ship- and air-borne records are severely perturbed by the crustal field. Therefore, it is shown, on the basis of a critical reconsideration of a few key-papers in the literature, that the best approach is to avoid mixing together all kinds of measurements. Satellite data are best suited for recognizing the dynamo field, while ground-based, ship- and air-borne records, which are measured much closer to crustal sources, are best suited, after subtraction of the satellite-derived dynamo field, for inferring the geomagnetic anomalies that are to be associated with crustal sources alone.

Key words *dynamo – spatial spectrum – white spectrum – Lowes' law – Nevanlinna's law – Hamilton's principle – geomagnetic anomalies – inner core – outer core – asthenosphere*

1. Introduction

The magnetic field of the Earth is usually represented (*e.g.*, Chapman and Bartels, 1940; Parkinson, 1983) by a potential

$$V(r,\theta,\phi)=\sum_{n=1}^{N} \mathrm{a}\left(\frac{\mathrm{a}}{r}\right)^{n+1}\cdot \sum_{m=0}^{n} P_n^m(\theta)\left(g_n^m\cos\phi+h_n^m\sin\phi\right) \tag{1.1}$$

where θ and ϕ are geocentric colatitude and longitude, r is the radial coordinate, a is the (spherical) Earth radius, $P_n^m(\theta)$ is an associated Legendre polynomial with Schmidt's normalisation, and the entire sum is called Spherical Harmonic Expansion (SHE), g_n^m and h_n^m are suitable coefficients, named Gauss' elements of terrestrial magnetism, the two summation indices n and m are called degree and order respectively, and the maximum degree N being considered depends on the quality and amount of the observational database. The papers that date back a few decades ago define V as the potential of the magnetic field $\boldsymbol{H}$, while the most recent literature defines V as the potential of the induction field $\boldsymbol{B}$. Therefore, when comparing different papers with each other, one should assess what definition they implicitly adopt. The SH functions are distinguished according to their geometric figure. They are: either i) zonal (for $m = 0$) when they have no longitudinal dependence, or ii) sectorial when they have no latitudinal dependence, and look

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almost like a peeled orange, or iii) tesseral recalling a composition by means of *tesserae* within a mosaic, and their figure looks like a golf-ball with a varying total number of valleys and hills (the depth and height of which depend on the corresponding value of g_n^m and h_n^m). Such a formalism holds as long as the Earth-air currents can be neglected (see Section 3). Moreover, concerning the set $\{g_n^m; h_n^m\}$, any most general and arbitrary values are applicable, as all this is a matter of a mere mathematical definition.

The physics of the Earth dynamo is expressed by the set $\{g_n^m; h_n^m\}$, *i.e.* by the dependence of g_n^m and h_n^m *versus* n and m. The dependence *versus* m can be shown to be basically related mainly to the choice of the frame of reference, unlike the dependence *versus* n which is invariant with respect to it (*e.g.*, Parkinson, 1983). Several such laws were discussed and tentatively proposed in the literature; but, for brevity, they cannot be reviewed here (in preparation). Only a few key-papers are recalled, in order to focus on the main target of concern. They are discussed in Section 2, and we report their original figures to give a better feeling of their historical relevance. Section 3 focuses on their respective interpretations, and Section 4 emphasises the specific aspects that are directly related to the leading item of the present paper.

2. The observational evidence

I – Lowes (1966) considered the average square field intensity at ground over the entire Earth surface

$$E = \sum_{n=1}^{N} E_n \tag{2.1a}$$

$$E_n = \sum_{m=0}^{n} E_n^m \tag{2.1b}$$

$$E_n^m = (n + 1)\,[(g_n^m)^2 + (h_n^m)^2] \tag{2.1c}$$

Such a quantity is equal, apart from a constant factor, to the average magnetic field density at ground, that was considered by Mauersberger (1956). Lowes (1974) seems to have been the very first author to give the crucial observational evidence that the plot of [log E_n] *versus* n displays 2 straight lines (fig. 1). It soon became customary to associate them to the crustal and core fields, respectively, for the reasons explained in Section 3. He evaluated: i) for the crustal field

$$E_n \approx 150. \cdot \exp(-0.004 \cdot n) \equiv P^{(2)} \cdot [Q^{(2)}]^n \equiv$$

$$\equiv 150. \cdot (0.996)^n \ (\mathrm{nT})^2 \tag{2.2a}$$

$$(25 \le n \le 500)$$

and ii) for the core field

$$E_n \approx 4.0 \cdot 10^9 \exp(-1.5 \cdot n) \equiv$$

$$\equiv 4.0 \cdot 10^9 \cdot (4.5)^{-n} \equiv P^{(1)} \cdot [Q^{(1)}]^n \equiv \tag{2.2b}$$

$$\equiv 4.0 \cdot 10^9 \cdot (0.22)^n \ (\mathrm{nT})^2 \ \ (n \le 11)$$

where the constants $P^{(1)}$, $Q^{(1)}$, $P^{(2)}$, and $Q^{(2)}$ conform to the symbols used here below (see (4.1)). Since his units are $(\mathrm{nT})^2$, he used a SHE of the potential of $\boldsymbol{B}$, not of $\boldsymbol{H}$. Malin *et al.* (1983) and Kerridge and Barraclough (1985) call such E_n the «Lowes parameter», Harrison *et al.* (1986) the «Lowes-Mauersberger function», and Backus *et al.* (1996) the «Mauersberger-Lowes spectrum». It is usual to call fig. 1 the spatial spectrum of the geomagnetic field.

Lowes (1974) obtained such an impressive and remarkable result following a former suggestion by Bullard (1967) on the basis of a spatial Fourier analysis of the intensity of the geomagnetic field that had been carried out by Alldredge *et al.* (1963) approximately along one full circle of the Earth. A striking feature is the fact that such a SHE extended up to $n = 500$, which is very large even when it is compared with the most recent SHE evaluated by means of satellite data (*e.g.*, the MAGSAT analysis, discussed here below, extends up to $n = 23$, while the more detailed analysis by Cain *et al.* (1989) extends up to $n = 63$). Alldredge *et al.* (1963) considered observational records along a great circle, almost equatorial; hence, they could provide a comparatively less reliable estimate of the zonal terms. Since the minimum spatial wavelength is $2\pi/N$, they had to use a suitable

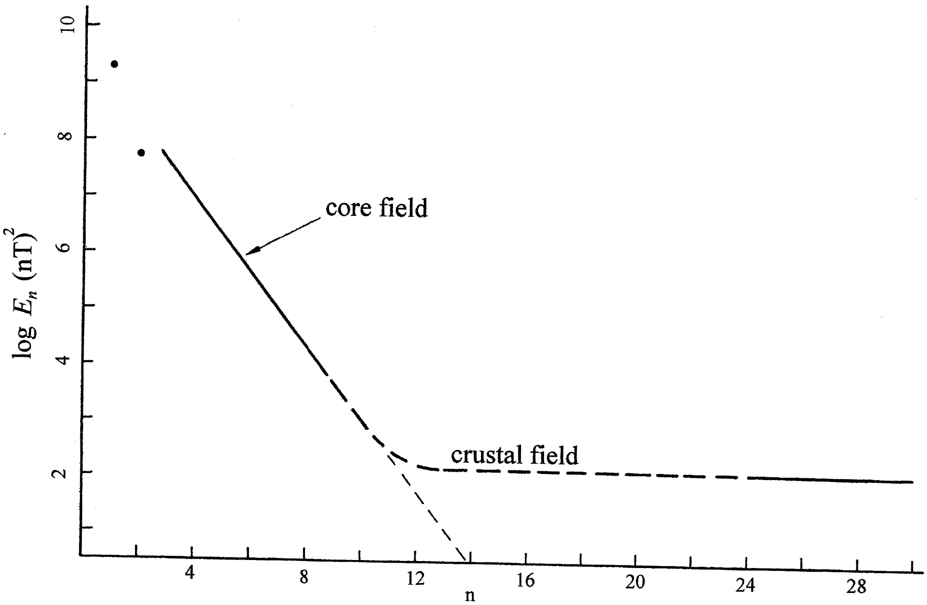

Fig. 1. «Idealized surface power spectrum, 1965». This is the historical figure (here redrawn) and captions by Lowes (1974), showing for the first time the 2 Lowes lines, based on a clever expansion carried out on the geomagnetic records measured approximately along one full and almost equatorial great circle of the Earth (see text).

dense database in order to fit the requirements of the Nyquist limit. The use of a Fourier analysis along one full circle has been the object of some discussion, as reviewed by Cain *et al.* (1989; and references therein). Such an entire methodological aspect is, however, of little concern for the purposes of the present study. The greatest merit of such an analysis was suggesting the existence of such 2 straight lines in fig. 1.

One major exception, however, is concerned with the point for $n = 1$ that does not fit into either one of such 2 lines, being $\sim 80\%$ higher than expected. When this fact was noticed, it was generally claimed that the dipole field (*i.e.*, $n = 1$) follows its own law, and it cannot be likened to other terms. Such an anomalous behaviour, however, has an intriguing implication that was first noted by Nevanlinna (1987), who assumed that a third line ought to be drawn through the 2 points with $n = 1$ and $n = 2$ (fig. 2). He also supposed that such a third line has to be treated just like the other two. Although at a first glance this may appear speculative, his choice was supported by the «white spectrum» argument discussed in Section 3.

In the following we call *Nevanlinna's law* such a line between $n = 1$ and 2, and *first* and *second Lowes' law* the other 2 lines for $n = 3, \ldots, 13$ and $n = 14, \ldots$, respectively.

II – A substantial improvement was achieved by the MAGSAT data (epoch 1979.85), according to the analysis by Langel and Estes (1982) (fig. 3), who found the values (with the same symbols fitting into (2.2a, b) and (4.1))

$$P^{(0)} = 5.68 \cdot 10^{10} \qquad Q^{(0)} = 0.0329 \tag{2.3a}$$

$$P^{(1)} = 1.349 \cdot 10^{9} \qquad Q^{(1)} = 0.270 \tag{2.3b}$$

$$P^{(2)} = 37.1 \qquad Q^{(2)} = 0.974 \tag{2.3c}$$

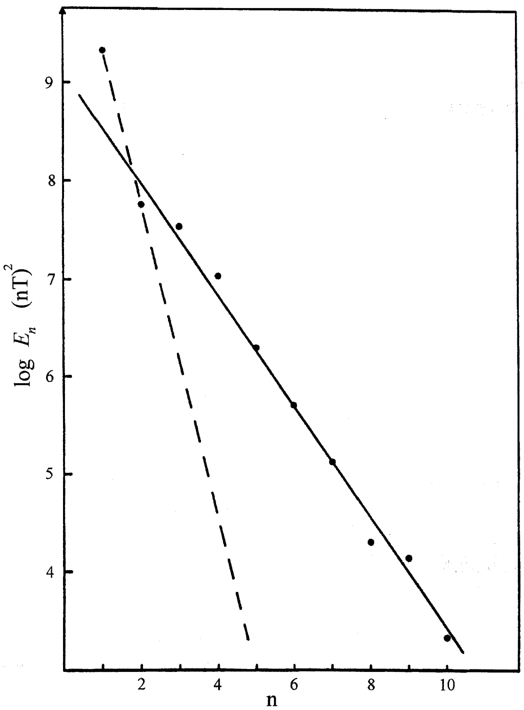

Fig. 2. «The mean-square of the total field (***B***) as a function of the harmonic degree (n) as calculated from the IGRF 1980... The slope of the regression line (solid line) gives the radius (3300 km) of the apparent source shell in the core. The dashed line drawn through the two first harmonics represents the spatial magnetic spectrum due to sources from the inner core». This is the first historical figure hypothesizing the Nevanlinna line. Figure (redrawn) and captions after Nevanlinna (1987).

III – Cain *et al.* (1989) (figs. 4 and 5) drew a Lowes' plot by means of a clever combination of MAGSAT's data with several other standard ground-based, ship- and air-borne data (see their paper for details). They evaluated the 2 Lowes lines, and discussed an upper limit for n in order to obtain significant results, by considering the noise level. The Nyquist frequency ought to be associated with $n = 60$, although points in fig. 4 seem to stabilize beyond $n \sim 45$, while the average of all points above $n = 50$ gave a noise level ~ 0.091 nT^2 per degree, that they adopted for their analysis and interpolation over fig. 4. They plotted 2 Lowes's lines for $n = 2, 3, ..., 15$ and $n = 16, 17, ..., 50$, respectively, after subtracting for every point the aforementioned guessed noise.

They also correctly carried out a suitable analytical continuation downward from ~ 420 km height until the Earth's surface, by applying a correction factor $[(a + 420)/a]^{2(n+2)}$, basically derived from (1.1) after some algebra not here given. They obtained fig. 5, where the noise was subtracted.

3. Interpretation

I – A premise is needed dealing with some mathematical, rather than physical, items. The potential (1.1) holds only provided that air-Earth currents can be neglected. This was, and still is, generally agreed since the beginning of our century, and apparently was never re-discussed anew (Fukushima, 1989). Berdichevskiy and Faynberg (1972, 1974) computed a model by the vector potential ***A***, but they concluded that their apparent regions of out-flowing and in-flowing currents crossing the Earth's surface were the consequence of noise in their database. The problem can now, perhaps, be better tackled in terms of a wealthier database, and also of different kinds of potentials (*e.g.*, Stratton, 1941; Backus *et al.*, 1996). Provided that such air-Earth currents can be neglected, the magnetostatic potential (1.1) is a solution of Laplace's equation, a classic item in mathematics (see *e.g.*, the extensive account by Sansone, 1950, 1955). In this respect, the following facts have to be emphasised (their formal support can be found *e.g.*, in Courant and Hilbert, 1953).

I-1 – Laplace's equation allows for one unique solution in a region contoured by a surface over which its boundary value is specified (Dirichlet's problem). It is called «harmonic function» and it can be expressed in terms of orthogonal polynomials. Such an expansion is unique, whenever a specific form has been chosen for such orthogonal polynomials. By this the geomagnetic field can be extrapolated in space outside the Earth's surface, as long as the

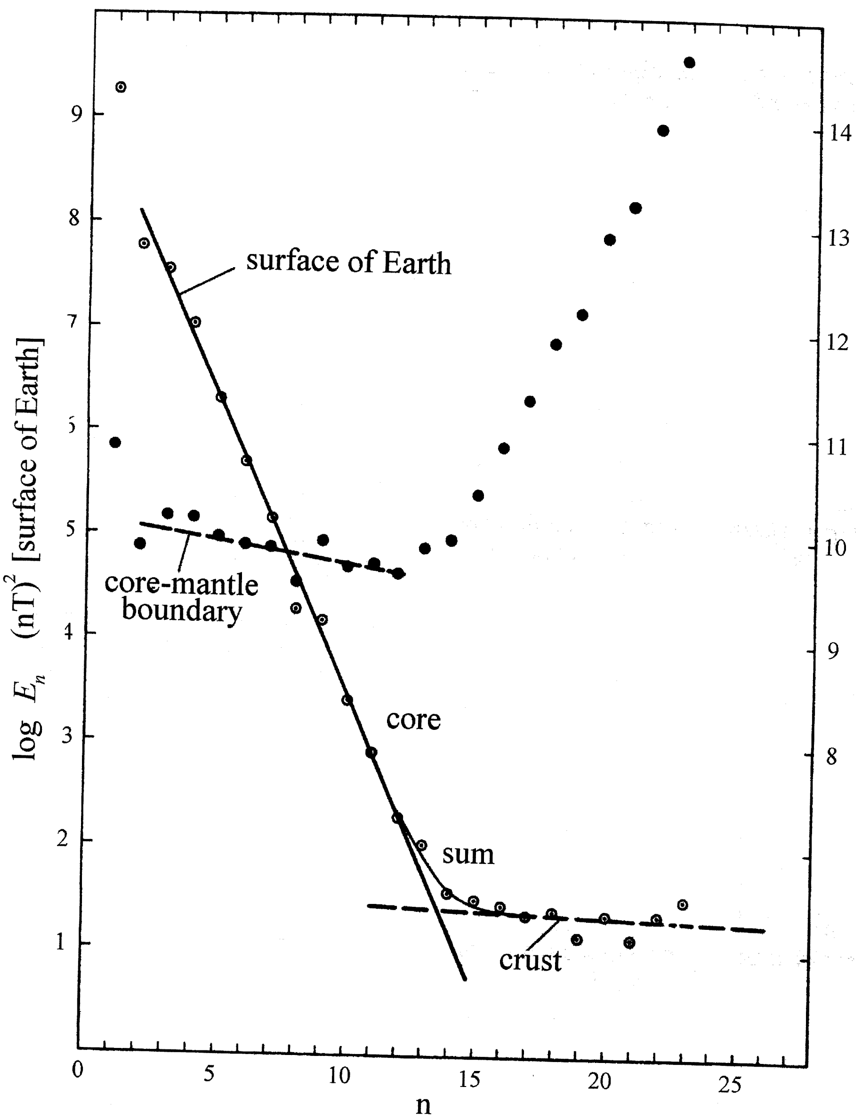

Fig. 3. This is a historical figure (here redrawn) by Langel and Estes (1982) showing for the first time the precise estimate of the 2 Lowes lines by means of MAGSAT records (epoch 1979.85). Those authors also computed and plotted in this same figure the tilt of such 2 lines, as they can be supposed to be inferred by a hypothetical observer located on the CMB, who is supposed to measure a field equal to the analytic continuation underground of the ground-measured field. This same argument, applied to the case of a hypothetical observer located at some pre-chosen, although varying, constant radial distance, leads to the definition of the «white spectrum» radius, being identified with the location of such an observer, who monitors one such Lowes' line as being perfectly horizontal. In this respect, we note that the core line, rotated as it appears to a hypothetical observer located on the CMB, still has a negative slope, reflecting the fact that, consistently with the «white spectrum» requirement, it becomes horizontal only at some depth greater than the CMB. See text.

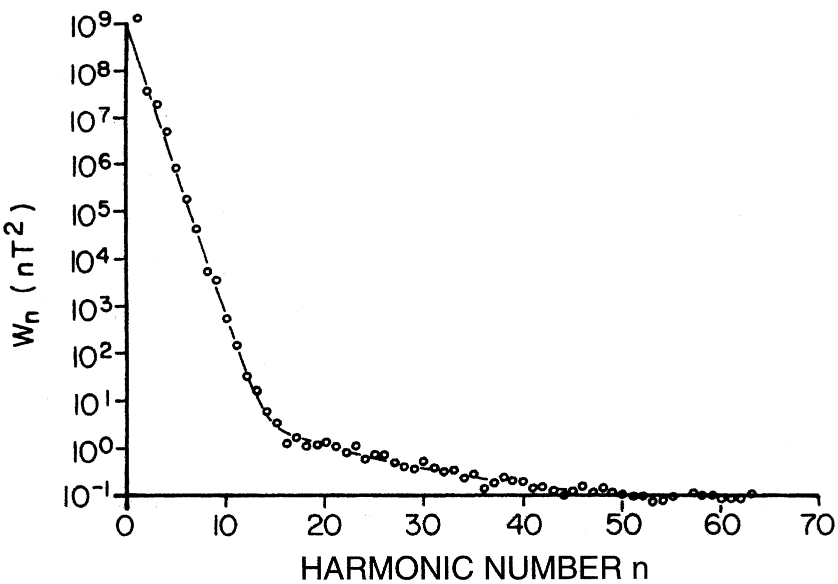

Fig. 4. «Power spectrum of geomagnetic field from the M07AV6 model at 420 km altitude (units are nT^2). Fitted curve is sum of core and crustal spectra plus a noise figure of 0.091 nT^2». The plotted curve is $E_n = 7.48 \cdot 10^8 (0.251)^n + 14.8 (0.876)^n + 0.091$. The 2 Lowes lines intersect for $n = 14.2$. Figure and captions after Cain *et al.* (1989).

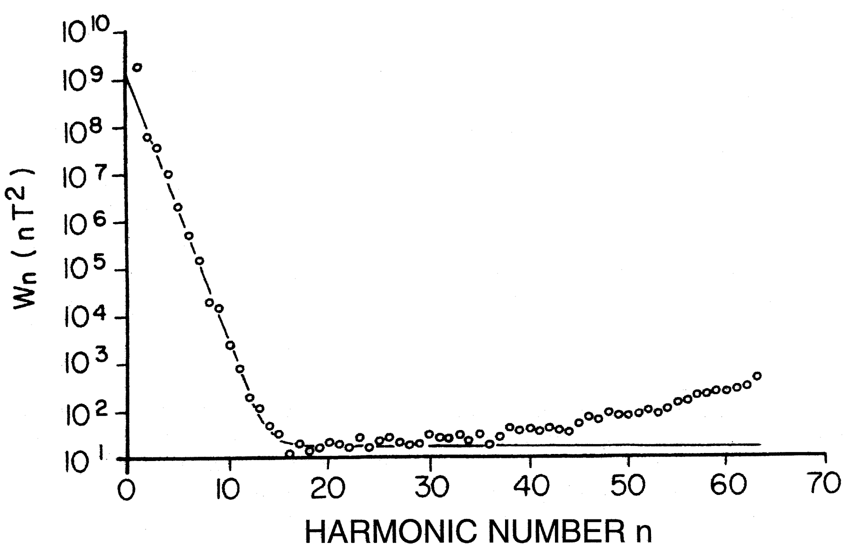

Fig. 5. «Power spectrum of geomagnetic field from the M07AV6 model at $r = 6371$ km (units are nT^2). Curve is that as in fig. 4 reduced to mean earth radius but without the noise term». The plotted curve is $E_n = 9.66 \cdot 10^8 (0.286)^n + 19.1 (0.996)^n$. The 2 Lowes lines intersect at $n = 14.2$. Figure and captions after Cain *et al.* (1989).

electric currents that flow in space can be neglected. The form (1.1) relies on the associated Legendre polynomials, first published by Legendre in 1785 and 1789 for his investigations on the figure of the planets. Such a choice results effective in the fact that it rapidly converges. This expresses the fitness of such a mere mathematical formalism to the actual physics of gravitation and of planetary magnetism, in the fact that, owing to physical reasons that can be either known or unknown, as a first order approximation there is a clear observational evidence that the figure of a planet is a flattened ellipsoid, while its magnetic field is dipolar.

I-2 – Also the Poisson equation (every component of $\boldsymbol{A}$ satisfies one such equation) allows for one unique solution (see *e.g.*, Durand, 1968, or Courant and Hilbert, 1953), although only provided that the distribution is known for the electric currents $\boldsymbol{j}$. In the case of the Earth we know $\boldsymbol{B}$, not $\boldsymbol{j}$. We want to estimate the $\boldsymbol{j}$ that flows in the Earth core, and that is responsible for the observed $\boldsymbol{B}$. This problem is expressed by an integral equation (*i.e.* by using the Laplace law for expressing the $\boldsymbol{B}$ produced by a known $\boldsymbol{j}$). This is called «inversion problem», that does not allow for a unique solution, as it is proven by Fukushima's (1972) theorem applied to the internal currents (rather than to the external circuits as in its original formulation).

I-3 – A harmonic function is analytic. Hence, let us also consider its formal analytic continuation below the Earth's surface.

I-4 – The uniqueness theorem of Laplace's equation can be extended to the following limit process. Consider a region V of space, contoured by a boundary S. Consider a sequence of boundary value functions f defined over S, such that they tend to a limit function f^*. Consider the sequence of the harmonic functions in V defined by means of every such f over S. Also such a sequence of harmonic functions has a limit, and it coincides with the harmonic function defined by f^*.

I-5 – Consider $\boldsymbol{B}$ outside the Earth's surface, and define a set of different $\boldsymbol{j}$-loops inside the Earth, such that every such $\boldsymbol{j}$-loop gives, alone, a correct description of $\boldsymbol{B}$ at the Earth's surface. Let the Earth radius $\mathsf{a} \to 0$ while the aforementioned $\boldsymbol{j}$-loops contract and always satisfy the same condition above, *i.e.* they represent for $r \geq \mathsf{a}$ the analytic continuation of $\boldsymbol{B}$. Note that, for such a purpose, it is sufficient to require that every such $\boldsymbol{j}$-loop is such that its $\boldsymbol{B}$, in the limit $\mathsf{a} \to 0$, converges to a limit that coincides with the analytic continuation inside the Earth's surface of the $\boldsymbol{B}$ that is observed outside it.

I-6 – Refer to (1.1), and, according to the aforementioned procedure and warnings, consider its analytic continuation inside the Earth's surface. A convenient and rigorous mathematical representation is in terms of a spherical shell (*ss*) of radius $\mathsf{a} \to 0$, with some suitable $\boldsymbol{j}$ distribution over it (Chapman and Bartels, 1940). According to (1.1), $\boldsymbol{B}$ can be computed only for $r \geq \mathsf{a}$, while for $r < \mathsf{a}$ one should use a different formula. Also the total energy of such a $\boldsymbol{j}$ system can be computed, it is a continuous function and it is finite, although it diverges for $\mathsf{a} \to 0$ (actually, it diverges before getting $\mathsf{a} \sim 0$; see below).

I-7 – The smaller $\mathsf{a} \to 0$, the larger $\boldsymbol{B}$ for $r = \mathsf{a}$, and such an increase *versus* decreasing a is comparatively much larger when one considers terms of higher degree n in (1.1), due to their comparatively larger power of $1/r$.

I-8 – Refer to (1.1) and consider every addendum alone. Whenever r gets larger, the relative role of the terms of higher order n becomes less important. The limit for $r \to \infty$ implies that the observed field results purely dipolar, *i.e.* it contains only the terms with $n = 1$. This is particularly convenient, because, owing to physical reasons as mentioned above, (1.1) rapidly converges. Such a fact is the logical justification for several papers in the literature that attempted at dealing with the origin of the geomagnetic field in terms of some suitable distribution of dipoles inside the Earth. Such papers were sometimes criticised as mere mathematical and nonphysical descriptions. Instead, in principle, they can provide a physical indication of the sites where the prime sources are located. For brevity, such papers cannot be reviewed here, although they definitely cannot be simply ruled out as nonsense.

I-9 – All such facts are just a matter of mathematics, and the only physics they contain derives from neglecting air-Earth currents, while all remaining arguments simply rely on the formal mathematical theory of the potential, that was extensively exploited by Gauss. But, when making any practical application to geomagnetism, a few basic additional physical drawbacks must be considered. In fact, the Earth interior is not an empty space, and this is normally accounted for by means of the (relative) electric constant ϵ and of the (relative) magnetic permeability μ, and ϵ depends on the medium, while it is usually assumed $\mu = 1$ because a randomly chosen sample of soil has a feeble ferromagnetic behaviour, determined by an eventual small percent of ferromagnetic component that overwhelms the effect of its either para- or diamagnetic components. But, when the Curie point is attained, it is $\mu = 1$. Neither does it appear feasible to introduce some correction for such assumptions, that therefore remain unavoidable and substantial physical drawbacks for every analytic continuation below the Earth's surface of the geomagnetic field.

II – Meyer *et al.* (1983, 1985) and Harrison *et al.* (1986) attempted at evaluating a model for the crustal field, by dividing the Earth surface into small elementary areas just like *tesserae* of a mosaic, then by evaluating the crustal field of every such *tessera* by means of satellite data, finally by interpolating a SHE over such tesseral data. The field of every *tessera* was estimated by likening it to an idealized layered model, and by considering its average geologic features etc., then by defining an equivalent dipole representing such an average field: such a dipole was then used for mathematical interpolation. Meyer *et al.* (1983) showed that the final result well agrees with the $n > 14$ branch of Lowes' law. The trend is practically a horizontal line, that Meyer *et al.* (*ibid.*) call «white spectrum», a concept and a word already used by Lowes (1974). See below for discussion. The convergence of the energy of the geomagnetic field is discussed here below, where it is shown that it is physically related to the evaluation of the average depth of the lithosphere (more exactly to the average depth of the «geomagnetic definition» of lithosphere). Therefore, the terms with higher n need not to be truncated: whenever actually available, they ought to be investigated by searching whether they have some regular pattern or not, although, for large n, the errors can severely bias the result (concerning their fractal analysis, that is not here discussed, refer *e.g.*, to de Santis and Barraclough, 1996, 1997; Barraclough and de Santis, 1997, and references therein). Moreover, owing to the argument explained here below and based on an energy balance, both lines in Lowes' plot must be decreasing for increasing n, in order to ensure the necessary condition for convergence of (2.1a) for $N \to \infty$. Hence, the apparently horizontal line that was found by Meyer *et al.* (1983) ought to be actually considered either a result of their error-bars rather than of physics, or, and much more likely from a less underestimating viewpoint, an actual physical result denoting the role of the permanent magnetization of the crust (as per Section 4).

III – The next logical step is in performing the analytic continuation of the field, downward from the Earth's surface, and in considering the form displayed by Lowes' and Nevanlinna's laws, as they can be envisaged by a hypothetical observer located on a spherical layer concentric with the Earth and having a given radius $r < \mathrm{a}$. One can thus suppose *e.g.* to choose $r = R_{\mathrm{CMB}}$ (*i.e.* the radius of the core mantle boundary). Such a procedure was applied by Lowes (1974), by Langel and Estes (1982), and by Meyer *et al.* (1983): the 2 lines of Lowes' plot appear thus rotated counter-clockwise (fig. 3). When r is decreased, the line for $n > 14$ largely tilts and finally gets a steep trend by which [log E_n] increases *versus* n. This is of no interest (due to the divergence of the energy, see below). In contrast, the Lowes line for $n < 14$ becomes progressively closer to horizontal. This suggests the possibility that the field associated with the Lowes line for $n < 14$ should be originated on an approximately spherical shell (*ss*) inside the Earth, having some suitable radius R_{ss} to be determined just in such a way that a hypothetical observer located at $r = R_{ss}$ should observe a transformed line of Lowes' law that is exactly horizontal, *i.e.* such as to simulate a «white spectrum». This argument (*i.e.* choosing r in such a way that the $n < 14$ line of Lowes' law becomes horizontal) was first applied by Lowes (1974). Later on, Meyer *et al.* (1983) applied it to six different models (five ones evaluated by J.C. Cain and one by R.A. Langel, but all of them by means of a mutually non-fully-independent experimental database): the result is substantially stable, showing that the origin of this part of the magnetic field is likely to be located on the *ss* having radius $R_{ss} \sim (R_{CMB}-147) \pm 50$ km.

Reconsider all this from the viewpoint of the mathematics mentioned at item **I** above. By this, realise that the counter-clockwise tilt of Lowes' lines appears to be just a consequence of mere mathematics. In this same respect, also the «white spectrum» argument could be just one particular example, or case history, of a counter-clockwise rotation. Hence, essentially this should be only a mathematical property, although the intrinsic fact, that a Lowes' line eventually becomes horizontal, can have some physical implication. This, however has to be proven in terms of some rationale, that has to be suitably exploited by means of some physical interpretation of observations. One key for such an attempt is in terms of energy balance (see below).

Harrison *et al.* (1986) emphasised that there is no strict logical reason in favour of such a «white spectrum» requirement. That is, they stressed that such a derivation of ~ 147 ± 50 km ought to be considered a reasonable guess, an indicative speculation, not as a strict logical requirement (on the other hand, it appears unlikely that such a result is only a mere coincidence). It will be here shown that such a «white spectrum» argument coincides with the search for a necessary mathematical condition for convergence for $N \to \infty$ of the SHE (2.1a). Hence, according to such an inference, the apparent radius of the CMB, that is derived from such an algorithm, has to be re-interpreted as being the lower bound for its physical (*i.e.* energetically constrained) value (see below).

In any case, Meyer *et al.* (1983) concluded that the geomagnetic field seems to be divided into three different parts: i) a relatively stable dipole «originating probably in the deeper core or in the core as a whole»; ii) a part originated in «a relatively thin surface layer of the core» (this part includes a dipole component and, in addition to it, also the entire geomagnetic secular variation; this is a substantial new and original result by Meyer *et al.*, 1983); and iii) the field originated within the crust. Their final statement is: «although this may in effect concern merely the description of the core field, the results about the core field energy spectrum must certainly be considered for any true theory of the core field generation.»

IV – Nevanlinna (1987) applied the «white spectrum» argument to the aforementioned $n = 1, 2$ line in the Lowes plot, he evaluated the corresponding radius using the IGRF 1980.0, and found a value $R_{ss} \sim 1160$ km that is quite close to the radius (~ 1200 km) of the commonly accepted R_{ICB} (*i.e.* the radius of the inner core boundary ICB, also called Lehmann discontinuity). Actually, such a ~1160 km value results slightly inside the ICB, as should be expected, analogously to the argument about the re-interpretation of the ~ 147 km value by Meyer *et al.* (1983) (see below). Drawing a line through only 2 points implies that the tilt of such a line is eventually the result of some coincidence, as we lack any third point to test it. However, such a possible interpretation of Nevanlinna's line is strengthened by the fact that the «white spectrum» argument, applied by analogy from the 2 Lowes lines, gives a radius that is close to the ICB, much like the Lowes lines give radii that are close to the CMB and ALB.

V – For the sake of completeness, one brief mention ought to be made (with no presumption of giving full justice to them) of the two very important papers by Fanselau and Kautzleben (1958) and Fanselau and Mundt (1965) (briefly reported also by Bucha *et al.*, 1983, p. 220-226, based on a contribution by Wolfgang Mundt; they had no subsequent development, private communication by Wolfgang Mundt), that were real rigorous precursors of the Lowes plot and analysis, and of the «white spectrum» argument, although applied to

$$\left\{ \frac{E_n}{(n+1)(2n+1)} \right\}^{1/2} \tag{3.1}$$

instead of to E_n like Lowes' and Meyer's. They computed one value of R_{ss} by equating every couple of terms (3.1) with different degrees n and i. Their values are plotted in fig. 6. They

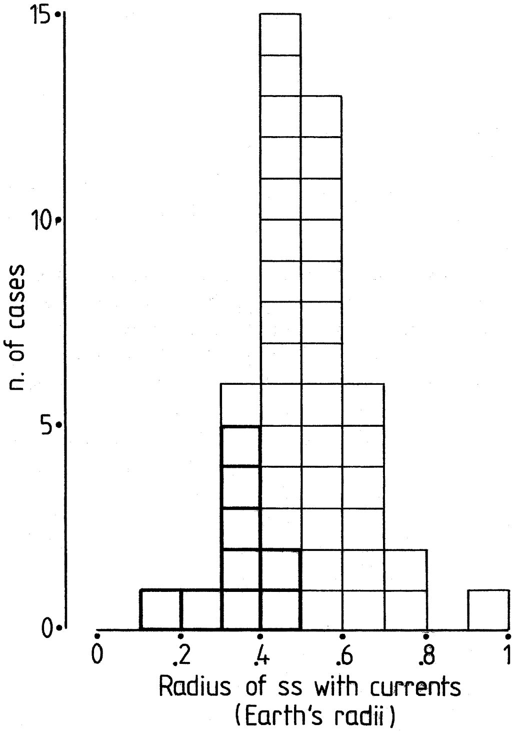

Fig. 6. «White spectrum» radii R_{ss} listed by Fanselau and Kautzleben (1958), computed by means of the «Potsdam field» (epoch 1955). They do not call «white spectrum» such a procedure, but, in any case, they evaluated the tilt of the (first) Lowes line by means of every couple of terms (3.1) corresponding to different degrees n, i = 1, 2, ..., 10. The present figure is drawn by denoting by a thicker line the values computed when either one n, i is equal to 1 (in which case, we know that it partially refers to Nevanlinna's line, not to the first Lowes line). This is the very first historical estimate of the lower bound for R_{CMB}, derived from geomagnetic evidence alone.

concluded as follows: i) the average for all couples of degrees n, i = 1, 2, ..., 10 gives $R_{ss} \sim 0.50$ a or a depth of ~ 3200 km; ii) the average restricted to the couples n, i = 2, 3, ..., 10 (this amounts to excluding the dipole term as a non-fitting part, as in fig. 1) gives $R_{ss} \sim 0.54$ a or a depth of ~ 2900 km; and iii) the average restricted to i, i + 1 = 1, 2, ..., 9 gives $R_{ss} \sim 0.52$ a or a depth of ~ 3100 km. They concluded (*ibid.*, pag. 52; translation by the authors):

«Therefore, the average spherical surface over which the average values are equal, of the terms of the geomagnetic field that have a different order, apparently corresponds to the core boundary».
«According to other arguments (Fanselau and Lucke, 1956), it is probable that the cause for the main part of the geomagnetic field is to be searched for within some very shallow layer on the outer boundary of the core, and for sure it is possible to infer that the properties that are here considered of the terms of the potential are to be originated in terms of the geometrical structure of the system of the electric currents that flow inside such a layer».

They also emphasized that the pure dipole terms appear to be originated at some greater depth. Their results suffered, however, from some relevant bias related to the insufficient quality of their available SHE, and such far-looking and absolutely remarkable papers were almost unnoticed by the international scientific community, maybe also due to their very innovative and unconventional approach.

VI – Owing to the importance of the next item, it is important to report in detail a comment by Backus *et al.* (1996):

«Figure 3 shows a plot of the spectrum at the earth's surface taken from a fit to the MAGSAT data up to n = 23 given by Langel and Estes (1982). The usual interpretation of the spectrum is that $\boldsymbol{H}$(a) comes from the core if $1 \le n \le 12$, from the crust if $16 \le n$, and from both if n = 13, 14, 15. The argument for this interpretation is as follows: ...»

They claim that

$$<< \boldsymbol{H}^2(r) >> = \left(\frac{\mathrm{a}}{r}\right)^4 \sum_{n+1}^{\infty} E_n(\mathrm{a}) \left(\frac{\mathrm{a}}{r}\right)^{2n} \tag{3.2}$$

The series on the right of (3.2) converges as long as all the sources are located inside the sphere $S(r)$ of radius r. Hence, every individual term in the series is certainly bounded. Differently stated, there is a constant such that

$$E_n(\mathrm{a}) \left(\frac{\mathrm{a}}{r}\right)^{2n} \le \text{const} \quad \text{or} \quad E_n(\mathrm{a}) \le \text{const} \left(\frac{r}{\mathrm{a}}\right)^{2n} \tag{3.3a,b}$$

«If we could actually observe the E_n (a) for all n, we could use (3.3b) to argue that if the sources are inside $S(r)$, there is a constant such that

$$\ln E_n(\mathrm{a}) + 2n \ln \frac{\mathrm{a}}{r} \leq \text{const} \qquad (3.4)$$

for all n. Therefore, if we choose an r so small that the left side of (3.4) $\to \infty$ as $n \to \infty$, we can be sure that there are sources outside $S(r)$.

This is the only rigorous conclusion contained in (3.3b). The converse is false. It is perfectly possible to have (3.4) valid for all n, and yet have sources outside $S(r)$. For example, a uniformly magnetized sphere could have any radius < a, and (3.3a,b) would be valid for all $r > 0$ because E_n (a) = 0 if $n > 1$.

And, of course, we cannot observe E_n (a) for all n. Figure 3, in fact, goes only to n = 23. Nevertheless, the linear dependence of ln E_n (a) on n in that figure may be more than a coincidence. The simplest interpretation of fig. 3 is that if $S(r)$ is the smallest sphere containing all the sources, then for that value of r, the $E_n(r) = \ll H_n^2(r) \gg$ are not very dependent on n (the «white noise source hypothesis»). If we take $E_n(r)$ = const then we have

$$E_n(\mathrm{a}) = \left(\frac{r}{\mathrm{a}}\right)^4 E_n(r) \left(\frac{r}{\mathrm{a}}\right)^{2n} \qquad (3.5)$$

In fig. 3, the curve drawn for the surface of the earth can be written

$$E_n(\mathrm{a}) = \left(\frac{r_1}{\mathrm{a}}\right)^4 E_n^{(1)}(r_1) \left(\frac{r_1}{\mathrm{a}}\right)^{2n} + \left(\frac{r_2}{\mathrm{a}}\right)^4 E_n^{(2)}(r_2) \left(\frac{r_2}{\mathrm{a}}\right)^{2n} \qquad (3.6)$$

This suggests that there are two statistically independent sources, one at radius r_1 and one at radius r_2. For $n \leq 12$, the source inside $S(r_1)$ dominates the data, while for $n \geq 15$ the source, which extends up to $S(r_2)$, dominates the data. Using the parameters of best fit from fig. 3, we obtain

$$r_1 = 3310 \text{ km}$$

$$\sqrt{\ll H_n^{(1)}(r_1) \gg} = 136\ \mu\text{T}$$

$$r_2 = 6288 \text{ km } (= 6371 - 83)$$

$$\sqrt{\ll H_n^{(2)}(r_2) \gg} = 6.3\ \mu\text{T}$$

The seismically observed core radius is 3486 km. Clearly we cannot use magnetic data to measure this radius accurately, but r_1 is close enough to 3486 km to suggest that the core contributes to the H_n with $1 \leq n \leq 12$.

A glance at fig. 3 shows that the straight-line fit above n = 15 is not too good. The curve by Cain *et al.* (1989) [*i.e.* fig. 4] is better determined because it has more data, but it appears to be swamped by noises of measurement (satellite position and orientation errors, instrument errors) above about n = 45. For $45 \leq n \leq 66$, $E_n(r_3)$ is clearly independent of n if r_3 = a + *satellite altitude* (~ 6371 km + 400 km). In any case, one would be ill advised to use these data to put the depth to the crustal magnetic sources at 10 km. However, the depth is clearly very shallow, so above $n \geq 15$ it is probably the crust that produces H_n».

Backus *et al.* (1996) do not even mention Nevanlinna's law, perhaps because it is drawn only through 2 points (n = 1, 2). Nevertheless, their association of the seismically determined radius R_{CMB} with the «white spectrum» radius R_{ss} is noteworthy. Concerning the crustal field, they claim (in contrast we our proposed interpretation) that the spectrum for $45 \leq n \leq 66$ reflects different kinds of errors.

They also discuss in some detail how one can attempt to separate the contribution of the core field from the crustal field. In addition to the aforementioned investigation by Cain *et al.* (1989), they also mention Meyer *et al.* (1983), who found that r_3 is the radius of the Earth and $\sqrt{\ll H_n^{(3)2} \gg}$ = 3 nT. They also assume that $S(r)$ it is

$$\ll \boldsymbol{H}^2(r) \gg = \sum_{n=1}^{\infty} E_n^{(3)}(r) = \sum_{n=1}^{\infty} E_n^{(3)}(r_3) \left(\frac{r_3}{r}\right)^{2n+4} \qquad (3.7)$$

and by supposing that $E_n^3(r_3)$ is constant with respect to n, they sum up such a series

$$<<\boldsymbol{H}^2(r)>> = E_n^{(3)}(r_3)\left(\frac{r_3}{r}\right)^6\left[1-\left(\frac{r_3}{r}\right)^2\right]^{-1} \tag{3.8}$$

This vaguely recalls about a similar computation carried out by the authors (see below), although in terms of some substantially different rationale. It has to be emphasized that such a series diverges for $r \le r_3$, *i.e.* it must be $r > r_3$. They also claim that $<<\boldsymbol{H}^2(\mathrm{a})>>$ is very sensitive to the very small difference between r_3 and a, so that such values «should not be taken too seriously». They quote the following estimates

	for r = a	for r = a + 400 km	by
$\sqrt{<<\boldsymbol{H}^2>>}$	78 nT	12 nT	(1)
$\sqrt{<<\boldsymbol{H}^2>>}$	13 nT	15 nT	(2)

(1) Cain *et al.* (1989)
(2) Langel *et al.* (1982).

They also discuss how one can attempt to recognize for $n \le 12$ the crustal contribution compared with the core field. They recall Langel *et al.* (1982) who used ~ 6 months data of MAGSAT measurements at an average altitude of ~ 350 km, and computed the SHE up to $n = 13$. Then they computed the resulting predicted field at the site of every ground-based observatory: the difference between the observed and the predicted field at every such observatory ought to be basically independent of time for $n \ge 15$. Such station corrections resulted typically of the order of ~ 100 nT with outliers as large as ~ 4000 nT (Scott Base and South Pole). They also recall about Shure *et al.* (1985) who attempted to minimize the r.m.s. value of the radial field H_r on the CMB, while still fitting the satellite data to ~ 10 nT r.m.s. They found serious deviations compared with the standard fit by means of truncated SHE applied to satellite data. Finally, Backus *et al.* (1996) review in some detail, and give also valuable references on, an estimate of the role of the truncation of a SHE. For such a purpose, they include one additional arbitrary weight within every addendum of the SHE. In the case of an infinite series, it can be shown that the optimum choice of such a weight function is a Dirac δ-function. In the case of a truncated SHE, the best fit is, rather, some function that has some finite width, to be eventually expressed in terms of Tchebichef polynomials. For example, for $n = 10$ they find a «circle of confusion» on the CMB of at least ~ 20° radius.

VII – Gregori (1993, 1994, 1996) and Gregori and Dong (1996) re-interpreted, in terms of an energy threshold, both the «white spectrum» argument and the aforementioned Backus *et al.* (1996) interpretation. In fact, the Lowes-Nevanlinna plot contains 3 straight lines, that are presumed to be associated with 3 distinct sources, whatever their location and origin. The warning has to be suitably considered that the Nevanlinna line is defined only by means of 2 points. However, the fitness of such a working hypothesis about Nevanlinna's line is to be tested *a posteriori* after comparing the inferences obtained by its analogy with the Lowes lines. In any case, either one of such 2 or 3 sources is supposed to be composed of electric currents, as the Curie point is such that only the crustal sources can be permanently magnetised bodies, and, on the other hand, the electrical conductivity σ within the crust is excessively low for allowing telluric currents of adequate intensity to flow. Instead, it is well known that the magnetisation of the crust is definitely insufficient for justifying the observed geomagnetic field.

Let any kind of whatever dynamo generate some electric currents anywhere inside the Earth, and in terms of any given, even unknown, mechanism. Owing to Hamilton's principle, every such current must expand in space as much as possible, *i.e.* as far as it experiences a local drop of σ, where it shall propagate outward like fringe currents, and rapidly decay by Joule's heating. The Hamilton principle is classic and well known since the last century. It is an alternative and rigorous way of giving an axiomatic definition of classical physics (instead of Newton's principles plus Maxwell's laws). Its elementary version specifies that a loop with an electric current flowing inside it attempts at expanding in space as much as possible (*e.g.*, Bruhat, 1963). Another related and equivalent result (by Fermi and Chandrasekhar) is that a plasma cannot be self-contained by its own magnetic field (*e.g.*, Rossi and Olbert, 1970).

Owing to several independent inferences by other geophysical parameters (seismic waves, and e.m. induction within deep Earth), we know that there are physical discontinuities across the ICB, across the CMB and at the base of the lithosphere, *i.e.* across the ALB. Namely, we know that there are step-like decreases of σ *versus* r occurring across both ALB (by ~ 1 order of magnitude) and CMB (by ~ 4 orders of magnitude), while we know nothing about the trend of σ across the ICB. As a provisional working hypothesis, let us suppose that one such negative step of σ *versus* r also occurs across the ICB.

It is important to stress that, in principle, in the case of an ideal step-like variation of σ, every such layer of currents has the ideal thickness of one atom. In fact, such electric currents imply no motion of atoms or nuclei, rather only of conduction electrons in solids and/or of free electrons in an ionised medium. Electrons must propagate outward as much as possible, at a speed comparable to that of light, until they find some difficulty preventing their further expansion. Therefore, in the case of an ideal step-like variation of σ, one should expect that such electrons go as far as the ultimate edge of the step. In natural reality, the physical thickness of such a layer depends, rather, on the finite gradient of σ *versus* r, *i.e.* on the fact that the step is not ideal. Even in this case, however, the thickness of such a current layer must be as thin as possible, just due to Hamilton's principle. Moreover, such an almost vanishing ideal thickness of any such current layer is also implicitly mentioned in the reported quotation from Fanselau and Kautzleben (1958). That is, in any case, when stating that some fraction of the field is generated in some given way within either the core or the mantle, the dynamo mechanism can be effective within some large physical volume, but its generated currents must always flow on some very thin layer, the figure of which is determined by the local radial gradient of σ.

Therefore, owing to such an entire argument, it is concluded that we must expect 3 very thin layers, that can be described, as a first order approximation, in terms of 3 *ss*' of electric currents flowing on the ICB, CMB, and ALB, respectively. The acronym ALB was proposed by Gregori (1993) to denote that, although the transition between lithosphere and asthenosphere is eventually not very sharp, Hamilton's principle *per se* requires that such electric currents must be expected to flow within some very thin layer, right across the region where σ rapidly drops outward: the ALB is such a sharp layer, the morphology of which depends on the radial gradient of σ. That is, it is an electrically defined parameter, and it ought not to be confused with any seismic definition of asthenosphere, or other.

Consistently with the aforementioned argument by Backus *et al.* (1996), let us suppose that every one of such 3 lines in the Lowes-Nevanlinna plot are representative of 3 actual lines extending between $1 \leq n < \infty$, although, owing to their physical overlapping, we can observe only one segment of every one of them. Henceforward, let us assume that they are originated by 3 different suitable systems of $\boldsymbol{j}$-loops, located somewhere inside the Earth. By using the same wording that is common for time spectra, in the present case of a space spectrum we can state that the effects of such 3 sources can be observed at the Earth surface only within 3 different frequency bands (respectively, either for $n = 1, 2$, or for $n = 3, ..., 13$, or for $n = 14, ...$). Every such source, in general, is expected to contribute also within other frequency bands, although it is completely overwhelmed by the contribution given by either one of the other sources. *A priori*, we do not know the reason for such a property, that has to be investigated by means of a different rationale (to be discussed elsewhere). In this respect, it ought to be stressed that such a screening, in the space-frequency domain, is completely different from the well known Faraday screening in the time-frequency domain, caused by the high σ of the Earth interior, by which *e.g.*, a signal from the core cannot reach the Earth's surface unless its (time) frequency is adequately low, etc.

A well known tool for modelling the geomagnetic field of internal origin (Chapman and Bartels, 1940) is in terms of a supposedly *ss* of some arbitrary and unknown radius R_{ss}, with a suitable distribution of electric currents $\boldsymbol{j}$ flowing on it. Therefore, let us apply such an algorithm, separately to every one of such 3 lines of the Lowes-Nevanlinna plot. Let every such *ss* have its own radius $R^{(k)}$, with k = 0,1,2, corresponding, respectively, to either n = 1, 2, or n = 3, ..., 13, or $n \geq 14$. For every such $ss^{(k)}$ we can therefore also compute (the detailed algebra is not here given) the formal associated total energy $E^{(k)}(R^{(k)})$ that is needed for constructing such a system of electric currents: it is a function of the unknown, and arbitrarily chosen, $R^{(k)}$. Let us plot $E^{(k)}(R^{(k)})$ *versus* $R^{(k)}$, and thus find out that it has an asymptote, that can be formally and rigorously shown (details are not here given) to coincide with the aforementioned «white spectrum» radius (fig. 7a-c).

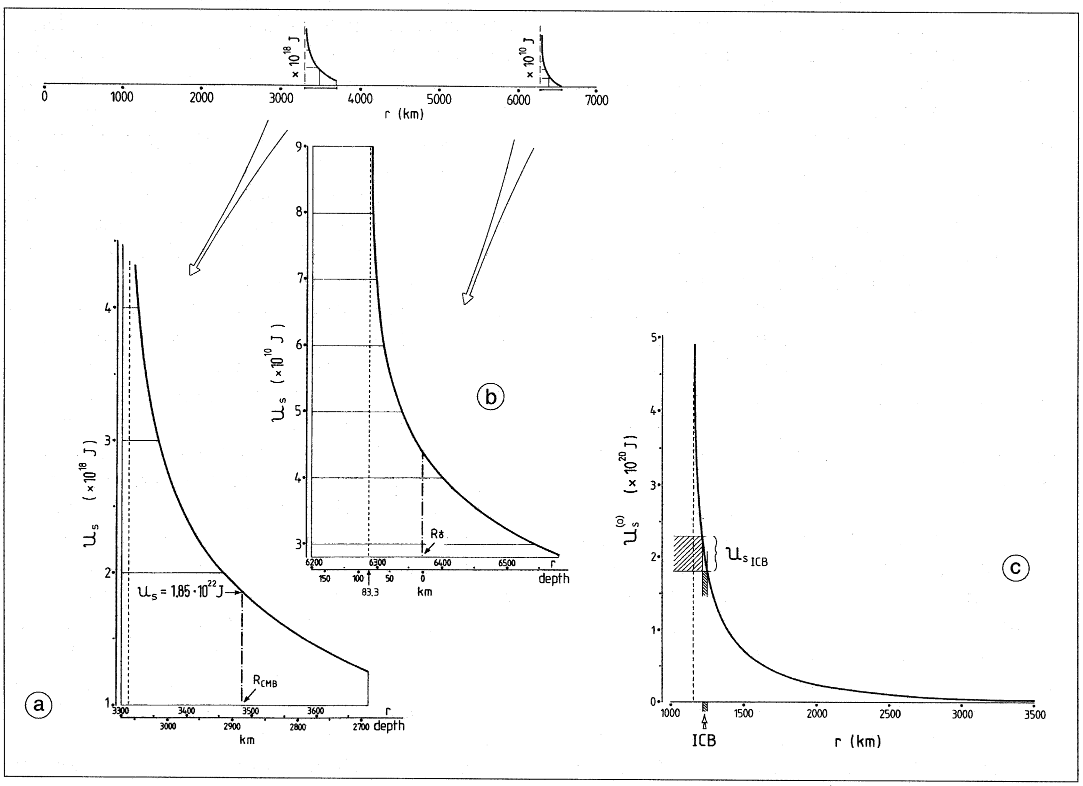

Fig. 7a-c. The total energy $\mathcal{U}_s^{(k)}$ is here plotted of the system of electric currents, that flow over either one of the 3 spherical shells (*ss*), being supposedly responsible for the field that is associated with either one of the 2 Lowes' lines (k = 1, 2; figs. 7a,b), or with Nevanlinna's line (k = 0; fig. 7c). $\mathcal{U}_s^{(k)}$ is proportional to $E^{(k)}$ $(R^{(k)})$ and the plot is *versus* the arbitrary chosen radius $R^{(k)}$ of its respective k-th $ss^{(k)}$. Every such $\mathcal{U}_s^{(k)}$ has an asymptote for $R^{(k)} = \mathrm{a}\sqrt{Q^{(k)}}$, where a is the Earth radius, and $Q^{(k)}$ is the tilt of the k-th line in the Lowes-Nevanlinna plot. Such $R^{(k)}$ (k = 0, 1, 2) can be rigorously and formally shown to coincide, just by a matter of definition, with the corresponding «white spectrum» radius (see text). A supposed correspondence of every such $R^{(k)}$ (k = 0, 1, 2) with the radius of some seismically determined discontinuity within the deep Earth, let us call it $R^{(k)}_{\mathrm{seism}}$, requires that it must result $R^{(k)}_{\mathrm{seism}} > R^{(k)}$ (for every k = 0, 1, 2). This, in fact, is always observed, and an obvious inference is that $R^{(k)}$ (k = 0, 1, 2) can be considered a geomagnetically derived lower bound for the seismically derived radius of the ICB, CMB, and ALB, respectively. See text.

Differently stated, the «white spectrum» radius, that according to the aforementioned discussion appears to be a mathematical inference, is the asymptotic lower bound of every such $R^{(k)}$, a lower bound that can never be attained in terms of a *ss*, provided that we want to keep finite the total energy of the system of the electric currents that flow over it. The aforementioned ~ 147 km estimate by Meyer *et al.* (1983) is not a thickness of such a current layer, rather it is the physical difference between the actual value of R_{CMB} and its asymptotic lower bound $R^{(1)}$, that will never be attained, as long as the *ss* is supplied only by some finite energy.

Such 3 asymptotes, for either one $ss^{(k)}$ ($k = 0$, 1, 2), are, as a matter of an observational inference, be it either a coincidence or not, just inside (as it must be expected, not outside) the ICB, CMB and ALB, respectively. Therefore, it is concluded that it appears very likely that they are the geomagnetic signatures, respectively, of such 3 σ discontinuities that are either known (for CMB and ALB), or speculated (for ICB) to exist inside the Earth. Differently stated, this can be considered the geomagnetic definition of either one of such 3 boundaries, the knowledge of which was previously attained in terms of geophysical quantities other than the geomagnetic field.

It is also possible to give the formal and rigorous mathematical proof (in terms of some algebra not here given; but, the conclusion is also intuitive) that such Lowes' and Nevanlinna's lines must have a negative tilt, in order to ensure the convergence of the series (2.1a). That is a necessary condition for $E^{(k)}(R^{(k)})$ to be actually evaluated and it is finite, is that the tilt of its respective Lowes' or Nevanlinna's line be negative. Moreover, the amount of such a tilt can be shown to specify, in a univocal way, the value of $R^{(k)}$, *i.e.*

$$R^{(k)} = a\sqrt{Q^{(k)}} \tag{3.9}$$

Moreover, notice that the intensity of the field (*i.e.* an eventual displacement along the ordinate axis of any one Lowes' or Nevanlinna's line, provided that it remains parallel to itself) is proportional to the intensity of the currents that flow on its corresponding $ss^{(k)}$, while it does not affect $R^{(k)}$, that rather depends only, and strictly only, on the tilt. Such a property can be shown to be basically the same as the fact by which a SHE of the geomagnetic field that is based on experimental records only of declination and inclination and not of intensity, can be computed apart from an arbitrary multiplication constant. This fact is quite important, as the tilt of Nevanlinna's and Lowes' lines can thus be significantly evaluated also in such a case, unlike their actual position along the ordinate axis.

For the sake of completeness, it ought to be stressed that all such inferences are completely independent of any assumption on the mechanism that is supposed to sustain the dynamo. The arguments of the present paper simply take for granted that the 3 expected step-like drops of the electrical conductivity $\sigma(r)$ *versus* r seem to be in close correspondence with the 3 Lowes-Nevanlinna lines, and with the 3 seismic discontinuities (*i.e.* the ICB, CMB, ALB, respectively). One may even be skeptical about the significance of Nevanlinna's line, due to its definition only in terms of 2 points. The close analogy, however, of its inferences compared to Lowes' lines appears to be a consistent support for its physical significance. The remarkable comparative difference of the orders of magnitude of their respective $E^{(k)}(R^{(k)})$ according to fig. 7a-c envisages that the efficiency of the dynamo strongly depends on r. According to Gregori (1993), the prime dynamo ought to generate currents only on the ICB and CMB, while the ALB currents ought to result from a leakage of currents that outspread from the CMB. But such additional details are the concern of the physics by which the dynamo is operated and they have no direct relation to the present paper.

All such statements are in close agreement with the review and discussion by Backus *et al.* (1996) who gave the best previous picture of the present state of the art, consistent with the aforementioned results by all previous authors.

4. Discussion and conclusions

Take for granted that either one of the 3 lines in the Lowes-Nevanlinna plot are the direct geomagnetic signature of ICB, CMB, and ALB, respectively. Let us consider the precision of their respective determination. The ICB is the best determined such surface, as it depends only on the terms with degree $n = 1,2$, that can be computed even by a comparatively poor observational database. Instead, the CMB relies on the terms with $n = 3, ..., 13$, that need some suitable good observations, adequately distributed all over the globe. Hence, the CMB geomagnetic signature is certainly more biased by instrumental errors than ICB's. Owing to the same reason, the most biased result ought to be concerned with the ALB signature. In fact, the two examples mentioned in Section 2 (*i.e.* the Lowes, and the Langel and Estes determinations of figs. 1 and 3) appear to be, the best such plots that can be found in the literature, unlike figs. 4 and 5, that ought to be expected much more accurate than fig. 3, but that, in contrast, originated the extensive discussion by Cain *et al.* (1989) mentioned in Section 2.

A fundamental logical warning, however, is that the argument (mentioned in Section 3) based on Hamilton's principle applies to every system of electric currents, while it does not work for any permanent magnetised source. Within much of the literature it is customary to consider the terms with low n as being originated by the dynamo deep into the Earth, and those ones of higher n as being originated in the crust. Such a feeling, however, is also generally reported to be excessively oversimplifying, in the fact that, in principle, the deep dynamo can also originate high n terms, while the crust can also have some regular hidden trend, resulting in a significant and non-vanishing low n field.

Figure 1 is accurate, because those authors did a very clever job. In fact, they even extended their expansion up to $n = 500$, which appears impressive. In this way, they synthesized a signal that displays the correct trend (*i.e.* a negative tilt, with the appropriate value that ought to be expected according to the estimated expected average depth of the asthenosphere all over the globe, *i.e.* ~ 80-100 km). That is, the clever selection of an adequate number of recording points, all along one great circle of the Earth, apparently provided a physically significant result, suitable for separating the high n component of the observed field, component that is expected to be associated with the electric currents that ought to flow on the ALB. Instead, anyone of the other SHE models published in the literature, based on huge sets of records from different sources, sometimes is apparently less successful in getting rid of the irregular bias by the crustal field. In fact, their final result for the second Lowes line often does not give the expected negative tilt.

Figure 3 appears to be very accurate, as it relies on a database that is entirely collected by MAGSAT at several hundred kilometres height. In fact, every crustal source of the geomagnetic field gives a contribution that can be expressed in terms of a SHE algorithm analogous to the one used for defining (1.1), and by using a coordinate frame centred on every such permanent magnetised sample. Its respective dipolar and non-dipolar components, however, decrease *versus* distance with a different power of the radial distance, as is formally expressed by (1.1). In any case, their contribution to the field observed at satellite altitudes is certainly reduced, by some substantial amount, caused by such a geometrical damping factor. Therefore, the accurate tilt of the second Lowes law in fig. 3 just denotes the fact that the database was collected at high altitude, so that the bias by the permanent sources in the crust completely faded off due to such a geometrical damping.

In contrast, when Cain *et al.* (1989) attempted to reinterpret the same MAGSAT data in terms of a much more accurate formal analysis and gave figs. 4 and 5, they found an apparently worse result (notwithstanding they used a wealthier database, because they added to MAGSAT's also its respective simultaneously collected ground-based, ship- and air-borne database). Such an apparently paradoxical inference was found because the non-satellite data contain the bias originated by the permanent sources of the crustal field, that was not damped off by the effect of the geometrical distance.

Summarising, the ultimate conclusion is that the aforementioned clever analysis by Cain *et al.* (1989; *i.e.* fig. 5) is useful in the fact that it actually computes the entire joint field originated altogether by the electric currents that flow on the ICB, CMB, and ALB, and by the permanent magnetised sources in the crust. In contrast, the SHE model derived by means of the MAGSAT data alone (*i.e.* fig. 3) is only concerned with the field of the electric currents of the ICB, CMB, and ALB, while the contribution by the permanent crustal sources is negligible. Whenever you use some suitable very dense network of recording sites, such as for the Alldredge *et al.* (1963) great circle of the Earth, so that you even afford to go up to $n = 500$ or so, in such a case the bias by the crustal sources is averaged off and statistically filtered away. Otherwise, compute the formal difference between fig. 5 and fig. 3, and the result appears to be the best possible estimate of the expected field originated only by the permanent magnetised sources in the crust.

The separation of the field produced only by electric currents flowing on either one of the 3 surfaces ICB, CMB, or ALB can be accomplished by considering the extrapolation of either one of their respective Lowes-Nevalinna's line in fig. 3, by which we know that

$$E = \sum_{n=1}^{\infty} E_n = \sum_{n=1}^{\infty} P^{(k)} [Q^{(k)}]^n \quad (k = 0,1,2) \quad (4.1)$$

where the constants $P^{(k)}$ and $Q^{(k)}$ are known, and by which we can give a precise estimate also for the terms of degrees n that lie outside the range of observability in fig. 3 of every given such Lowes-Nevanlinna's line. However, by such an algorithm we cannot estimate the expected values for every g_n^m and h_n^m, rather only their corresponding contribution to the average energy density over every *ss* of a given and arbitrary radius. For example, we can just guess

$$\left|g_n^{(k)m}\right| \sim \left|h_n^{(k)m}\right| \sim \sqrt{\frac{1}{2(n+1)} P^{(k)} [Q^{(k)}]^n}$$

$$(k = 0,1,2) \qquad (4.2)$$

and choose an arbitrary sign for every g_n^m and h_n^m. By this, we just guess one possible, although basically arbitrary, model for the k-th field, that, however, is physically constrained by the correct energy content. On the other hand, considering that, in general, the orders of magnitude of such energy contents are significantly different compared with each other, sometimes such an arbitrariness has no relevant consequences for practical purposes.

Cain *et al.* (1989) attempted to get rid of such an effect, that is here interpreted in terms of the crustal field, and they referred to a speculated noise, just like, in general, is always done by anybody when dealing with a signal that does not fit into any simple regular trend or «logical» expectation, and in such a case one generally speculates about random occurrence. As a matter of fact, such a kind of noise is a bias that can be originated either by instrumental errors, or by some regularity (unknown to us) that ought to exist within the crustal sources: but, such an eventual regularity, whenever it actually exists, is to be evidenced and focused by the specific procedure here proposed.

Such an entire data handling here proposed appears to be, to the authors' knowledge, the best possible way of providing with a physical separation of the field originated either on the ICB, or on the CMB, or on the ALB, or by the permanent magnetised sources of the crust. It ought to be emphasised that the method is physical, rather than mathematical, because in terms of mere mathematics such an inversion problem is strictly indeterminate (see also the discussion on Fukushima's theorem in Gregori, 1999). Such an entire concern only deals with the planetary scale. But, *mutatis mutandis* (*), this same argument also applies to much smaller spatial scales, an application, however, that needs for two additional distinctions.

On one side, the crustal field can be promptly separated even on a small area, and on the basis of a good magnetic map of any kind, provided that one has a good model (obtained from satellite records) that represents altogether

(*) This means «provided that all items were changed that had to be changed». The *Concise Oxford Dictionary of Current English* (VI ed., 1976) defines it «with due alteration of details (in comparing cases)». Such a concise and sharp, common construction of Latin syntax is called ablative absolute.

the sum of all fields observed in that same area, and that are originated on the ICB, CMB, and ALB.

On the other hand, the aforementioned proposed separation of the fields that are originated on either ICB, or CMB, or ALB, relies (on the planetary scale) on Hamilton's principle. Let us suppose that we use satellite records referring only to one specific and restricted area of the globe, and that we carry out the formal computation of a SHE only by means of such an observational subset. Such a mathematical application can be accomplished *e.g.*, either by a standard SH analysis, or by using suitable modified algorithms, specifically envisaged for data sets collected over a spherical cap (*e.g.*, Haines, 1985; de Santis and Torta, 1997; and references therein). In either case, let us suppose that we draw the corresponding Lowes-Nevanlinna plot, by means of such computed Gauss' elements of terrestrial magnetism. Suppose that we repeat this same procedure over different regions of the world. An open question is whether the corresponding Lowes and Nevanlinna lines are the same, or not, for the same epoch and for different regions, and whether their eventual difference is physically significant or not. Such an approach could, perhaps, give evidence of the deviation of the deep Earth's structure with respect to spherical symmetry, a problem, however, that can be tackled also from other viewpoints and that is being actively investigated (in progress).

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