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Analysis of short time-space range seismicity patterns in Italy

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Abstract
In our paper we analyze the data base obtained from the observations of the Italian Seismological Network from 1975 to 1994 by using a simple algorithm to determine the rate of occurrence of seismic events conditioned by the occurrence of previous events after a period of quiescence. The number of observed pairs of earthquakes depends on several parameters: the magnitude threshold of the two events, the spatial and temporal ranges of the quiescence period preceding the first (non aftershock) event, the time elapsed between the first and the second events and the spatial dimension of the alarm area. The Akaike information criterion was adopted to assess the optimal set of space-time parameters used in the definition of non-aftershock (events not related to a stronger previous one). In Central Italy, the rate of $M \geq 3.8$ earthquakes preceded by at least one $M \geq 3.3$ foreshock within 14.1 km and 2 days is 30%, while the rate of $M \geq 3.3$ earthquakes followed by a $M \geq 3.8$ mainshock in the same space time range is 7%. We observed that the probability that an earthquake of magnitude M_1 will be followed by an earthquake of magnitude M_2 (success rate) fits the law $\log \lambda = a+b\,(M_1-M_2)$ with b approximately equal to 1. By computing the success rate for given values of magnitude threshold of the first and the second events over a dense grid of spatial coordinates, we obtained maps of this feature over the investigated area. The results of this process document variations larger than a factor of five in the success rate over the Italian territory.

Key words *aftershocks – foreshocks – conditional probability – probability gain*

1. Introduction

It has been widely observed that any earthquake increases the probability that new earthquakes will occur closely in space and time (see, *e.g.*, Jones, 1984, 1985; Agnew and Jones, 1991; Console *et al.*, 1992, 1993; Console and Murru, 1996; Maeda, 1996; Ogata *et al.*, 1995, 1996). This circumstance may have a particular relevance studying the probability of occurrence of stronger earthquakes (mainshocks) following moderate earthquakes (foreshocks).

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To estimate the probability that a warning will be followed by an event (success rate or validity) and the probability that an event will be preceded by a warning (reliability), one needs a number of studied cases that should be relevant enough (Reasenberg and Matthews, 1988). The probability gain, defined by Aki (1981) as the ratio between the conditional rate and the average rate of occurrence, is another parameter that can be used to measure the effectiveness of a given prediction technique together with the validity or with the reliability. The probability gain is the factor by which the probability of occurrence of a given event increases under the issue of an alarm, compared to the average probability of occurrence of the same kind of events in the same space-time volume.

2. Existing methodologies for recognition of foreshocks

Console *et al.* (1993) analyzed the data of the Italian instrumental catalog covering the period 1975-1991, restricted to an area of 35 000 km^2 in Central Italy. This data set included 2671 events. By using a simple algorithm, the catalog was analyzed to obtain the frequency at which events of a given magnitude occur after a potential foreshock. Potential foreshocks (or «non-aftershocks») were defined as events of magnitude M_i exceeding a given threshold M_f that follow a period of quiescence of duration T_1 in a circular area of radius R_1. In this algorithm «quiescence» means total absence of events with magnitude exceeding M_f in the given period T_1. A potential foreshock becomes a real foreshock (or a success) if it is actually followed by a mainshock (event of magnitude exceeding M_i and greater than or equal to the mainshock threshold M_m) occurring within a given time-distance range (T_2, R_2) from the foreshock. The computer program was written in a way suitable for studying the influence of any of the six parameters (M_f, M_m, R_1, T_1, R_2, T_2) used in the definition of foreshocks on the validity and reliability of the predictions, in order to allow their optimization. This analysis led to the definition of the influence volume in which most of the mainshocks occur after their foreshocks (R_2 = 30 km, T_2 = 2 days).

In another algorithm proposed by Imoto (1993), all earthquakes with a magnitude larger than 2.0 are defined as potential foreshocks and an alarm is issued at every foreshock occurrence in a spherical volume of radius R_2. Aftershocks are not removed from the analysis. The alarm persists for T_2 days unless it is terminated earlier by the occurrence of an earthquake larger by ΔM than the foreshock. By means of this simple algorithm values from 0.5 to 0.9 and from 5 to 18, depending on the parameters used, were obtained respectively for the reliability and the probability gain on a data set obtained in the Kanto, Central Japan area. Imoto (1993) solved the problem of defining the best set of parameters R_2, T_2 and ΔM assuming that the mainshock occurrence may be modelled by two Poisson processes with different rates: the higher rate in the time-space volume defined by alarms, and the lower rate in the non-alarm time-space volume. The performance of the proposed algorithm is measured in terms of the likelihood ratio of the two Poisson rates model with respect to a uniform Poisson model. The criterion based on the difference in log likelihood between the two models, known in literature as the Akaike Information Criterion (Akaike, 1974; Sakamoto *et al.*, 1983), has also been used in this study. More details about this method can be found in Appendix.

3. Definition of the best set of parameters

In this study we have applied the method used by Imoto (1993) to the problem of optimizing the parameters R_1, T_1, R_2 and T_2 which appear in the algorithm introduced by Console *et al.* (1993) to issue alarms based on foreshocks. This test was carried out on the catalog of seismic events located by the Italian National Seismographic Network from 1975 to 1994 for two different areas: a zone located in Central Italy (the boundaries of which are defined in Console *et al.*, 1992) and the whole Italian territory (table I). Dealing with a larger data set has the advantage of increasing the statistical significance of the results, however, the possible inhomogeneous behaviour of different areas (described in section 5) cannot be observed. In order to obtain an optimal set of parameters it is necessary to fix the magnitude thresholds for the first and second events of each pair, which in our case are 3.3 and 3.8 respectively. The first magnitude threshold was chosen assuming that earthquakes with $M \geq 3.3$ have been completely detected by the Italian Seismological network for the entire period since 1975. This is probably true for Central Italy but the same may not hold for the rest of the Italian territory. The second magnitude threshold was chosen 0.5 magnitude units larger than the first, in order to have a number of observed cases large enough to obtain significant statistical results. In the following part of this section we will talk about foreshocks and mainshocks for the first and the second events of each couple respectively, following the terminology used by Console *et al.* (1993) and Imoto (1993). In fact, the algorithm used in this analysis does not take into account pairs of events in which the first has a magnitude larger than the second, so as to give information about the possibility of occurrence of stronger events after moderate earthquakes. However, as will be shown in the following section, the aim of this study is more general, including also mainshock-aftershocks interaction.

Table I. Features of the Italian and Central Italy catalogs from 1975 to 1994.

Data set	Total area S (km^2)	Number of events with $M \geq 2.5$
Italy	674 000	10 970
Central Italy	35 000	2 448

Table II. Range of variation for the parameters used in the optimization algorithm.

	Min	Max	N-step
R_1 (km)	10	1280	14
T_1 (days)	0.125	1024	12
R_2 (km)	10	56.6	10
T_2 (days)	0.125	2.828	9

Table III. The optimum values for R_1, T_1, R_2 and T_2 for Central Italy and the whole of Italy.

	Italy	Central Italy
R_1 (km)	452.5	28.3
T_1 (days)	0.25	16
R_2 (km)	11.9	14.1
T_2 (days)	2	2

The four parameters (R_1, T_1, R_2 and T_2) have been allowed to change within the ranges and with the number of steps given in table II. In order to span wide ranges of values without employing too many steps, we let the parameters increase at each step by a constant factor, which is 2 for T_1, $\sqrt{2}$ for R_1 and T_2, and $\sqrt[4]{2}$ for R_2. The lower limit for both R_1 and R_2 ranges was chosen as 10 km, because this is the tipical location error for the events reported in our catalog. For this reason, it was considered meaningless to investigate the interaction between events located at a distance shorter than this lower limit.

The optimization was carried out taking the set of the four parameters, which produces the maximum of the quality function defined by Imoto (1993) as the ratio between the likelihood of the two Poisson rates model and that of the uniform Poisson model, divided by the total number of mainshocks (target events).

The optimal values for R_1, T_1, R_2 and T_2 for Central Italy and the whole of Italy are given in table III.

Figure 1a-d shows three-dimensional plots of the validity, reliability, probability gain and quality factor in Central Italy *versus* R_1 and T_1, for the optimal R_2 (14.1 km) and T_2 (2 days) pair. The plots of the same quantities *versus* R_2 and T_2 for the optimal R_1 (28.3 km) and T_1 (16 days) pair are given in fig. 1e-h. Figures 2a-h show similar plots for the whole Italian seismicity. The plots of the validity *versus* R_1 and T_1 (figs. 1a and 2a) clearly show that the validity has its maximum values for large values of both of these parameters. This is in agreement with the intuitive concept that the probability of occurrence of a mainshock after a potential foreshock in a given place is higher if the area has been quiescent for a longer time. The difference in behaviour of the validity for the largest values of R_1 and T_1 between Central Italy and the whole of Italy can be explained in the following way: for $R_1 \geq 320$ km, in the case of Central Italy, all the explored area is included in the radius of possible interaction between events, so that there is a lower limit of the number of potential foreshocks, and a saturation of the validity. The same does not hold for the Italian territory, the size of which is of

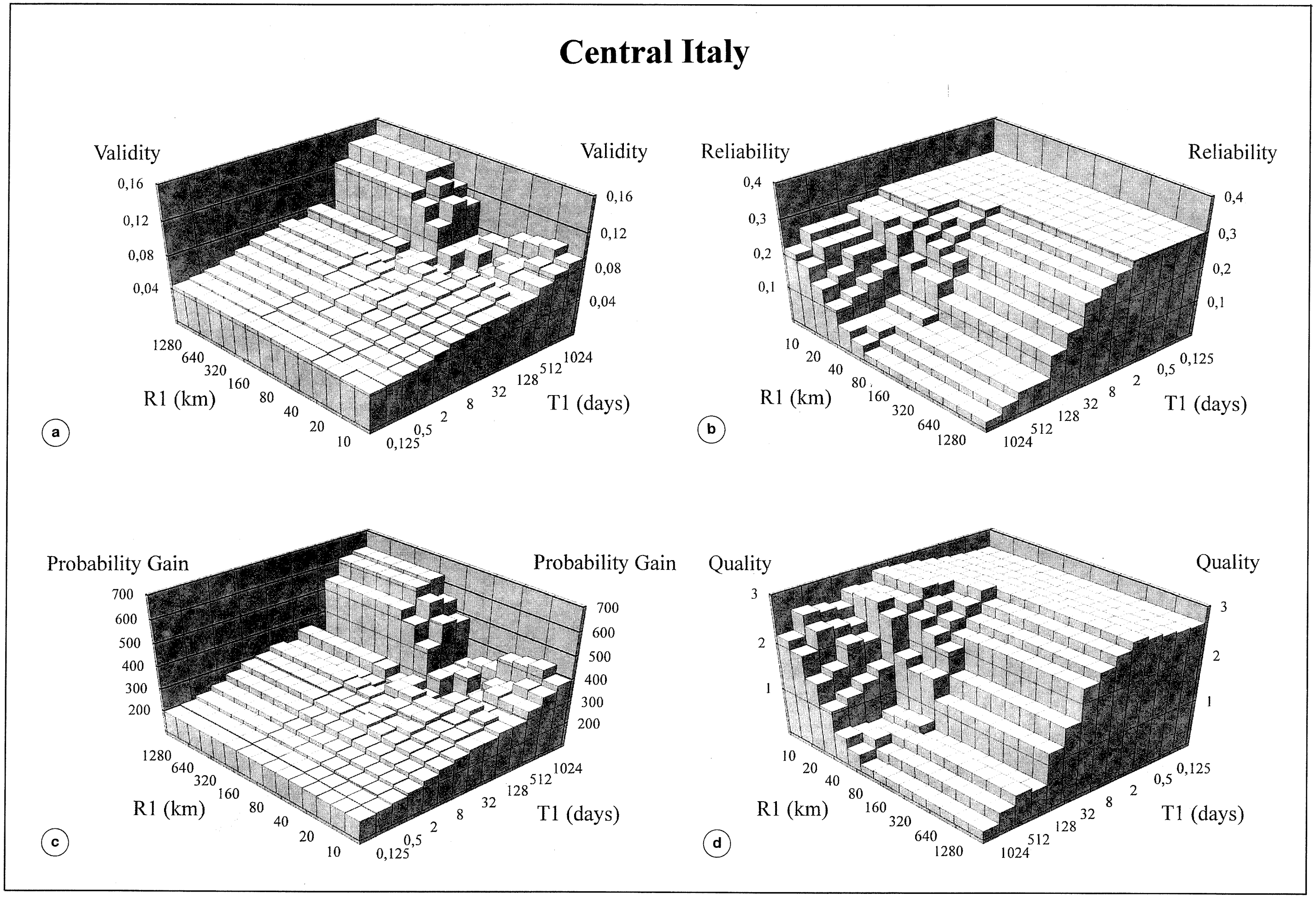


Fig. 1a-d. a) Validity; b) reliability; c) probability gain and d) quality factor for foreshocks of magnitude equal to or larger than 3.3 to be followed by mainshocks of magnitude equal to or larger than 3.8 within $T_2 = 2$ days and $R_2 = 14.1$ km for Central Italy, *versus* the quiescence time-space range T_1 and R_1.

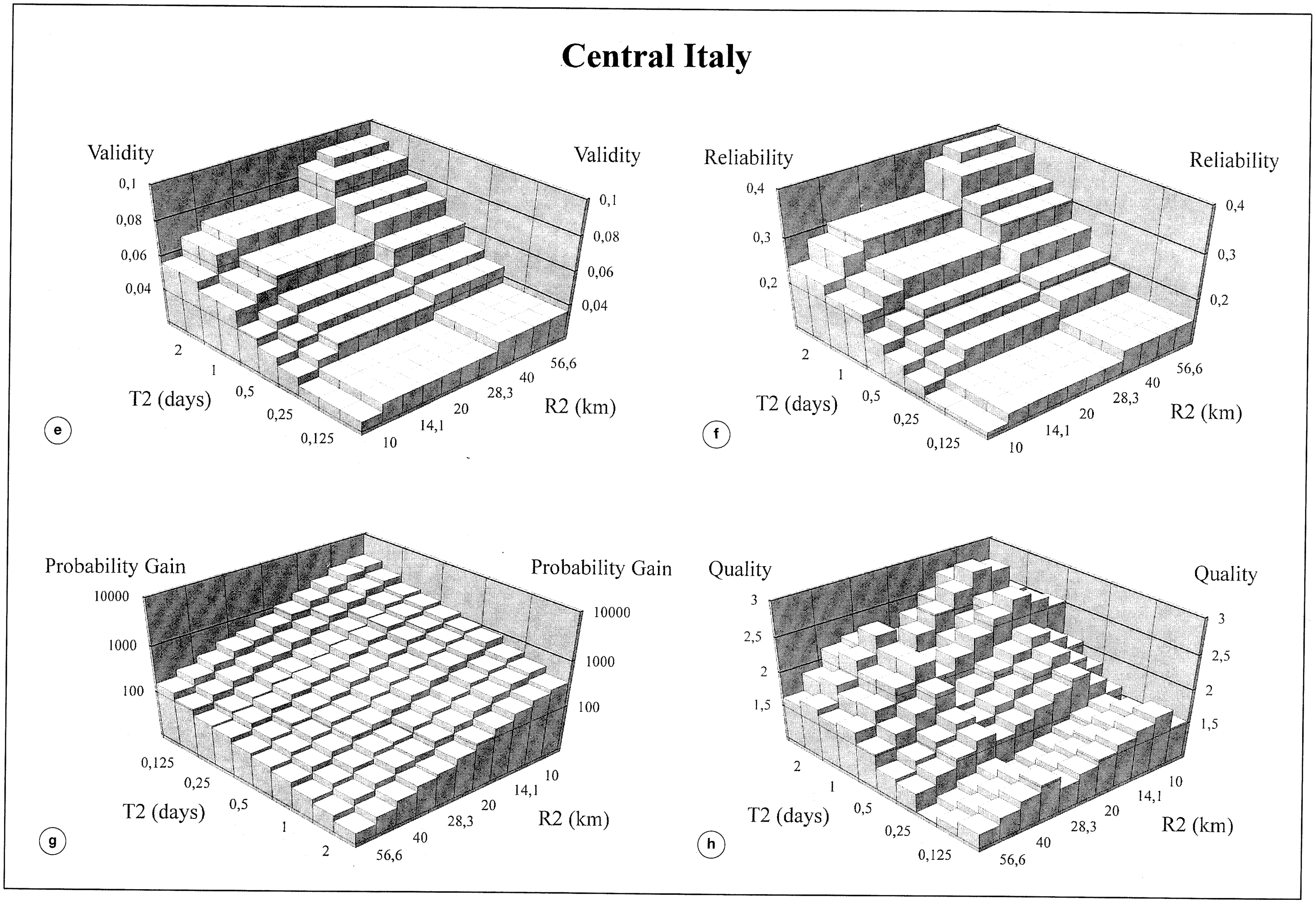


Fig. 1e-h. e) Validity; f) reliability; g) probability gain and h) quality factor for foreshocks of magnitude equal to or larger than 3.3 preceded by a quiescence time-space range ($T_1 = 16$ days and $R_1 = 16$ km) to be followed by mainshocks of magnitude equal to or larger than 3.8 for Central Italy, *versus* the alarm time-space range R_2 and T_2.

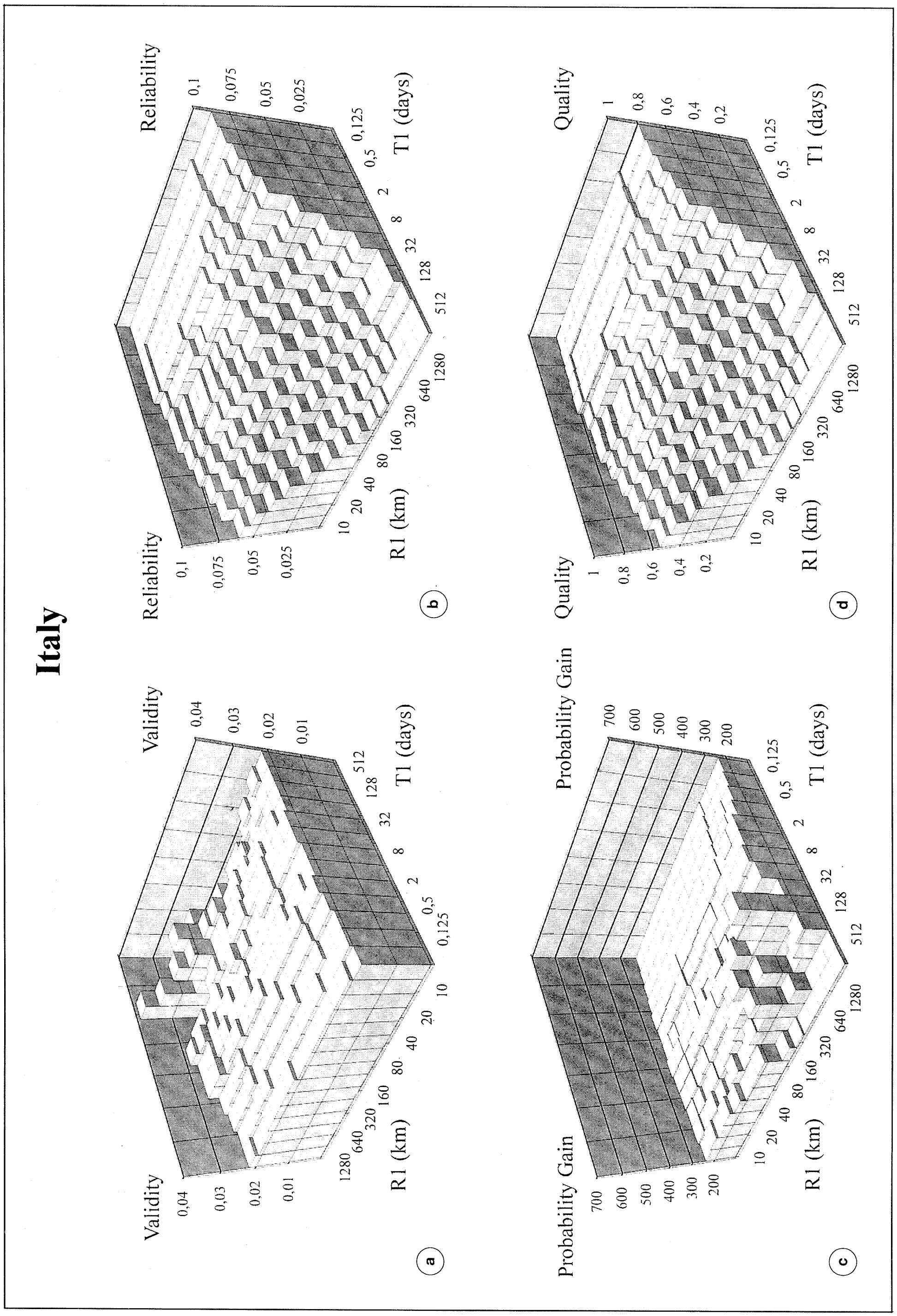


Fig. 2a-d. a) Validity; b) reliability; c) probability gain and d) quality factor for foreshocks of magnitude equal to or larger than 3.3 to be followed by mainshocks of magnitude equal to or larger than 3.8 within $T_2 = 2$ days and $R_2 = 11.9$ km for Italy, *versus* the quiescence time-space range T_1 and R_1.

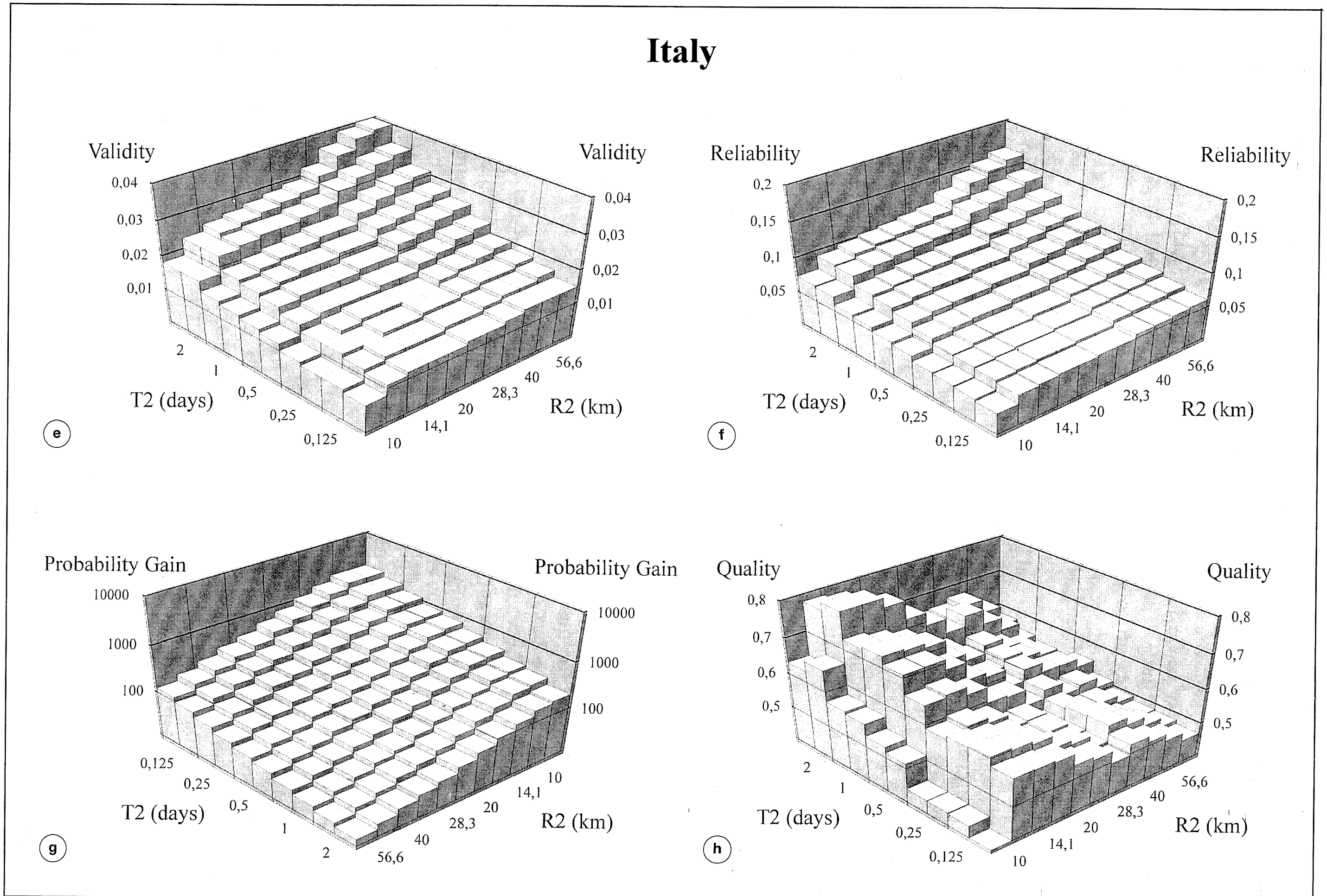


Fig. 2e-h. e) Validity; f) reliability; g) probability gain and h) quality factor for foreshocks of magnitude equal to or larger than 3.3 preceded by a quiescence time-space range ($T_1 = 0.25$ days and $R_1 = 452.5$ km) to be followed by mainshocks of magnitude equal to or larger than 3.8 for Italy, *versus* the alarm time-space range R_2 and T_2.

the order of 1000 km. In this case the random seismic activity spread on the investigated area is high enough to inhibit the identification of independent events defined as potential foreshocks, except for the earthquakes of large magnitude, which are unlikely to be followed by other events of larger magnitude. The number of successes and the validity in this case drop to zero. These circumstances suggest that the choice of higher limits for R_1 is of no use in our investigation.

For both data sets examined in this study, the behaviour of the reliability shows a monotonical trend to decrease increasing both R_1 and T_1 (figs. 1b and 2b). This is obviously related to the fact that large R_1 and T_1 values imply a smaller number of potential foreshocks, that means smaller values of reliability, taking into account that the number of mainshocks is kept constant.

All the comments made for the plots of the validity *versus* R_1 and T_1 also apply to those of the probability gain (figs. 1c and 2c) because the latter is proportional to the former, the number of mainshocks being fixed. Values as large as 200, or larger, are found for the probability gain in Italy for a target volume defined by R_2 = 14.1 km and T_2 = 2 days.

With regard to behaviour of the quality factor, figs. 1d and 2d clearly show a monotonical decrease *versus* both R_1 and T_1. The values obtained for the quality, which are larger than zero, indicating that the model containing two different rates has to be preferred to that characterized by a constant rate, are nearly flat at their maximum value for $T_1 \leq 2$ days. This can be explained considering that seismic activity exhibits its most significant non-random behaviour in relation to short-term interaction, such as foreshocks-mainshock and mainshock-aftershocks series, seismic swarms and other kinds of induced seismicity. This circumstance indicates that the choice of lower limits for T_1 is not relevant in our investigation.

The 3D plots for the validity and the reliability (figs. 1e-f and 2e-f) show, as expected, a steady increase *versus* both R_2 and T_2. For the optimal values R_2 = 14.1 km and T_2 = 2 days, we found that, in Central Italy, 30% of $M \geq 3.8$ earthquakes are preceded by at least one $M \geq 3.3$ foreshock, and 7% of $M \geq 3.3$ earthquakes are followed by $M \geq 3.8$ mainshocks. The corresponding values for Italy are 9% and 2.3% respectively, for the optimal values R_2 = 11.9 km and T_2 = 2 days.

The probability gain, which is inversely proportional to the total target volume, has a very sharp increase towards smaller values of R_2 and T_2 (figs. 1g and 2g). Values higher than 10^3 are obtained for R_2 = 14.1 km and T_2 = 0.25 days. The plots of the quality factor *versus* R_2 and T_2 (figs. 1h and 2h) show a maximum within the range of variation adopted for these two parameters.

4. Magnitude relationship for couples of events

We have applied another algorithm to study the magnitude relationship between subsequent shocks. According to Reasenberg and Jones (1989), the occurrence rate, λ, of events with magnitude M_2 or larger within the time T_2 after another event (non-aftershock) not related to a stronger previous one of magnitude M_1, may be expressed as

$$\lambda(T_2, M_2) = 10^{a+b(M_1-M_2)} \tag{4.1}$$

where a and b are parameters depending on R_2, T_2 and on geological features of the area. Equation (4.1) implicitly assumes that the rate of occurrence of subsequent events does not depend separately on M_1 and M_2 but only on their difference. This implies, for instance, that the probability of a foreshock of magnitude 3.3 being followed by a mainshock of magnitude 4.3 is the same as a foreshock of magnitude 5.3 being followed by a mainshock of magnitude 6.3. In the following analysis we have fixed both the time-space dimensions of the quiescence volume and those of the alarm volume as in table III. These values are taken from the average of the optimal parameters obtained from the analysis of the previous section.

In this study we do not fix our attention only on small events (foreshocks) followed by larger ones (mainshocks) but also on large events (mainshocks) followed by smaller ones (aftershocks). Doing so, we assume that both the foreshock-mainshock and the mainshock-aftershock pairs are examples of the same physical process described by eq. (4.1). This assumption has been discussed elsewhere (Jones *et al.*, 1997). The magnitude of the first shock (M_1) and the magnitude of the second shock (M_2) have been allowed to vary over a range of 0.5 magnitude units. We obtain 5 non overlapping cases shifting each range of magnitude from 3.3 to 5.3 in steps of 0.5. For all these 25 cases, we may compute both the number of successes and the conditional rate (validity). Table IV gives the numbers of successes obtained for the catalog of Central Italy.

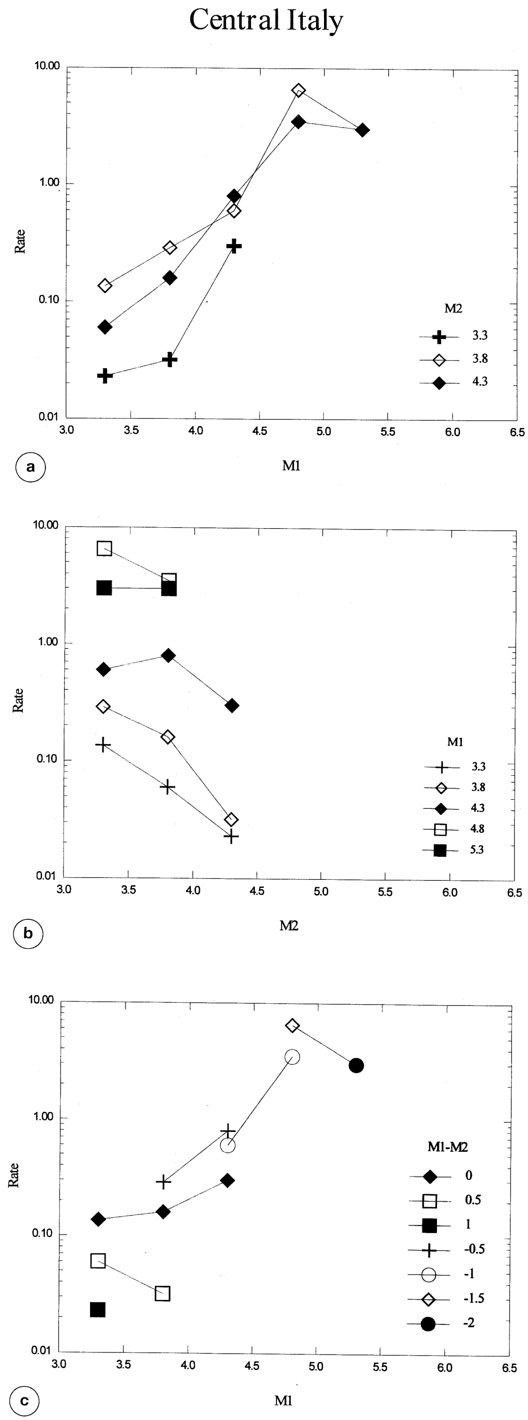
This table was obtained counting the number of cases in which an event is followed by another within T_2 days and R_2 km radius, respectively, with the condition that the first event of a pair occurs after a quiescent period of T_1 days within R_1 km radius. This means that we exclude pairs composed of aftershocks of the same earthquake. Also, pairs of events the second of which is an aftershock of an event different from the first one, are excluded. As a consequence, the search for subsequent events for a given first event is terminated by the occurrence of another event of magnitude larger than the first one. In this case, a correction is made for the computation of the validity and of the other parameters under study, so as to compensate the reduction of the target time duration. Cases in which the conditions are fulfilled by more than one subsequent event are counted as separate successes, so the condi-

Table IV. Number of earthquake pairs within 2 days and 14.1 km for Central Italy (1975-1994).

Magnitude of the first event	Magnitude of the second event				
	$5.3 \le M < 5.8$	$4.8 \le M < 5.3$	$4.3 \le M < 4.8$	$3.8 \le M < 4.3$	$3.3 \le M < 3.8$
$5.3 \le M < 5.8$	0	0	0	6	5
$4.8 \le M < 5.3$	0	0	0	7	12
$4.3 \le M < 4.8$	0	0	3	6	6
$3.8 \le M < 4.3$	0	0	1	5	7
$3.3 \le M < 3.8$	0	0	3	8	17

Table V. Number of earthquake pairs within 2 days and 11.9 km for Italy (1975-1994).

Magnitude of the first event	Magnitude of the second event						
	$6.3 \le M < 6.8$	$5.8 \le M < 6.3$	$5.3 \le M < 5.8$	$4.8 \le M < 5.3$	$4.3 \le M < 4.8$	$3.8 \le M < 4.3$	$3.3 \le M < 3.8$
$6.3 \le M < 6.8$	0	0	0	1	4	8	11
$5.8 \le M < 6.3$	0	1	0	3	6	14	20
$5.3 \le M < 5.8$	0	1	1	0	3	15	16
$4.8 \le M < 5.3$	0	0	0	1	4	15	34
$4.3 \le M < 4.8$	1	1	2	1	3	26	51
$3.8 \le M < 4.3$	0	1	2	1	10	58	100
$3.3 \le M < 3.8$	0	2	0	0	8	50	157



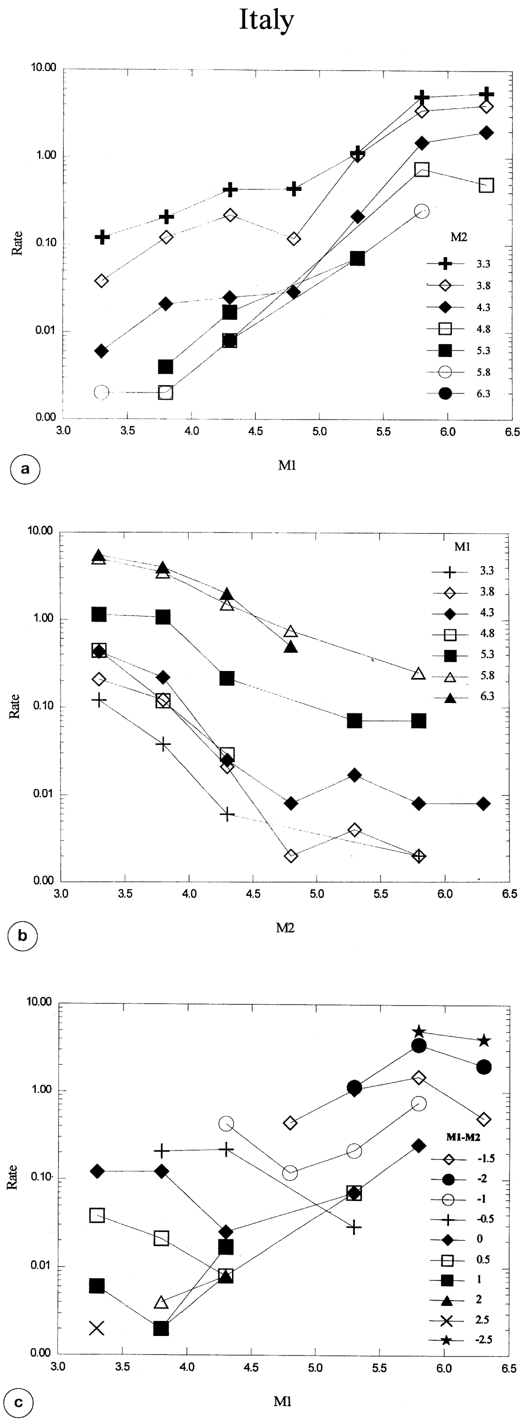
tional rate may be, in general, larger than 1.0. The main diagonal of the tables contains data referring to pairs of events in the same magnitude range. The cases below the diagonal represent the interaction between events the first of which (mainshock) has a magnitude larger than the second one (aftershock), while in the cases above the first event (foreshock) has a magnitude smaller than the second one (mainshock).

Table V has the same meaning as table IV but it was obtained from the catalog of the whole Italian territory, using the parameters given in table III. In this second case, the maximum magnitude considered is 6.8.

Figures 3a-c and 4a-c show, for Central Italy and the whole of Italy respectively, the conditional rate of occurrence, plotted in logarithmic scale in three ways: a) *versus* the magnitude M_1 of the first event for different values of the subsequent magnitude; b) *versus* the magnitude M_2 of the subsequent event for different values of the magnitude of the first event; c) *versus* the magnitude M_1 of the first events for different values of the difference $M_1 - M_2$.

These sets of data allow a test of the scaling law modeled by formula (4.1). In fact, if the data were perfectly fitted by formula (4.1), the lines of plots 3a and 4a would be straight lines with angular coefficient b, those of plots 3b and 4b would be straight lines with angular coefficient $-b$, and those of plots 3c and 4c would be horizontal lines. Any discrepancy between the theoretical formula and the real data may be ascribed either to incompleteness of

Fig. 3a-c. a) Occurrence rate of subsequent events within 2 days and 14.1 km *versus* the magnitude of the first event for different magnitudes of the second event. b) Occurrence rate of subsequent events within 2 days and 14.1 km *versus* the magnitude of the second event for different magnitudes of the first event. c) Occurrence rate of subsequent events within 2 days and 14.1 km *versus* the magnitude of the first event, for various values of their magnitude difference.



the catalog or to a real deviation from the model. A least-squares fit of all non-zero conditional rates of occurrence for Central Italy gives:

$$\log \lambda = -(0.52 \pm 0.08) + (0.84 \pm 0.08)(M_1 - M_2)$$

with an rms = 0.256. Fitting the data for Italian catalog, we obtain:

$$\log \lambda = -(0.89 \pm 0.06) + (0.70 \pm 0.04)(M_1 - M_2)$$

with an rms = 0.336. For practical uses of these results it must be recalled that here the occurrence rate of subsequent shocks is defined within a space-time window of 14.1 km and 2 days for Central Italy and 11.9 km and 2 days for Italy.

5. Geographical distribution of foreshock rate

We applied a visualization technique to the algorithm for the analysis of foreshocks, with the aim of investigating variations in space for the success rate, the reliability and the probability gain in the Italian seismicity.

In this analysis (in accordance to the results of section 3) we have fixed the following parameters:

1) time (T_1 = 0.25 days) and radius (R_1 = 452.5 km) of the quiescence area;

2) time-space separation between the potential foreshock and the mainshock (T_2 = 2 days and R_2 = 11.9 km);

Fig. 4a-c. a) Occurrence rate of subsequent events within 2 days and 11.9 km *versus* the magnitude of the first event for different magnitudes of the second event. b) Occurrence rate of subsequent events within 2 days and 11.9 km *versus* the magnitude of the second event for different magnitudes of the first event. c) Occurrence rate of subsequent events within 2 days and 11.9 km *versus* the magnitude of the first event, for various values of their magnitude difference.

3) minimum magnitude for foreshocks $M_f = 3.3$;

4) minimum magnitude for mainshocks $M_m = 3.8$.

The analysis is carried out on circular areas, each of which is centered on a rectangular grid of points equally spaced on both the x and y axes. In order to deal with uniform sets of data in our statistical analysis, we consider areas of varying sizes with a constant number of earthquakes, rather than equal-sized areas with a varying number of earthquakes. Therefore, the radius of the circles is a function of space and inversely proportional to the local density of earthquakes. Having fixed the number of earthquakes N_i to be included in each circular area, we limit the maximum values of the radii (200 km), excluding from the analysis those points for which the maximum was exceeded.

Each point of the grid gives a sub-catalog, and to each of them the procedure for estimating the validity of the predictions based on foreshocks (Console *et al.*, 1993) is applied.

To visualize the variations in the degree of validity, we assign to each validity value a

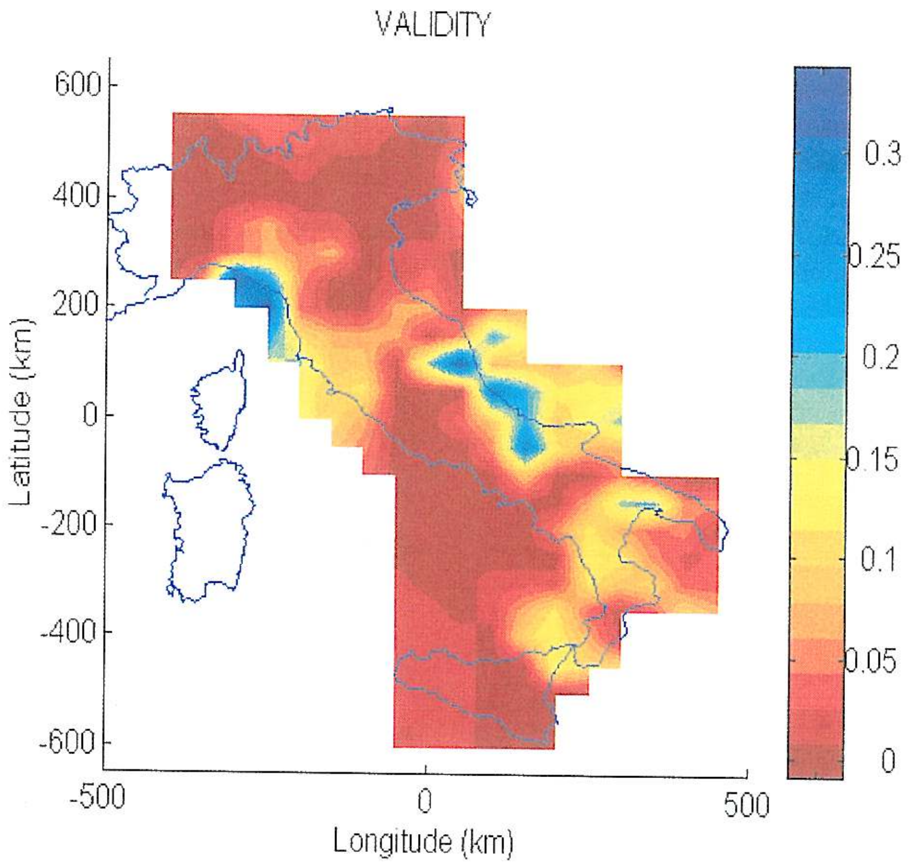


Fig. 5. Map of the occurrence rate for foreshocks of magnitude equal to or larger than 3.3 to be followed by mainshocks of magnitude equal to or larger than 3.8 within T_2 = 2 days and R_2 = 11.9 km, over the Italian territory.

colour, and plot these as a function of latitude and longitude. Figure 5 shows an example (for the time period 1975-1994) for N_i = 200, M_f = 3.3, M_m = 3.8 over the whole area of study. Blue represents a high success rate, red a low success rate. This figure shows how foreshocks behave differently in different seismogenic zones.

Figures 6 and 7 show maps, obtained making use of the same catalog and the same set of parameters as in fig. 5, of the reliability and probability gain respectively for foreshocks on the Italian territory.

6. Conclusions

A computer routine to identify foreshocks and to estimate their validity and probability gain, in space, time and magnitude, has been applied to the data of the Italian earthquake catalog (1975-1994).

The result of our analysis demonstrates various features of precursory seismic activity in the Italian territory:

– The success rate and the probability gain of foreshocks have a maximum for large values of the size R_1 and the duration T_1 of the

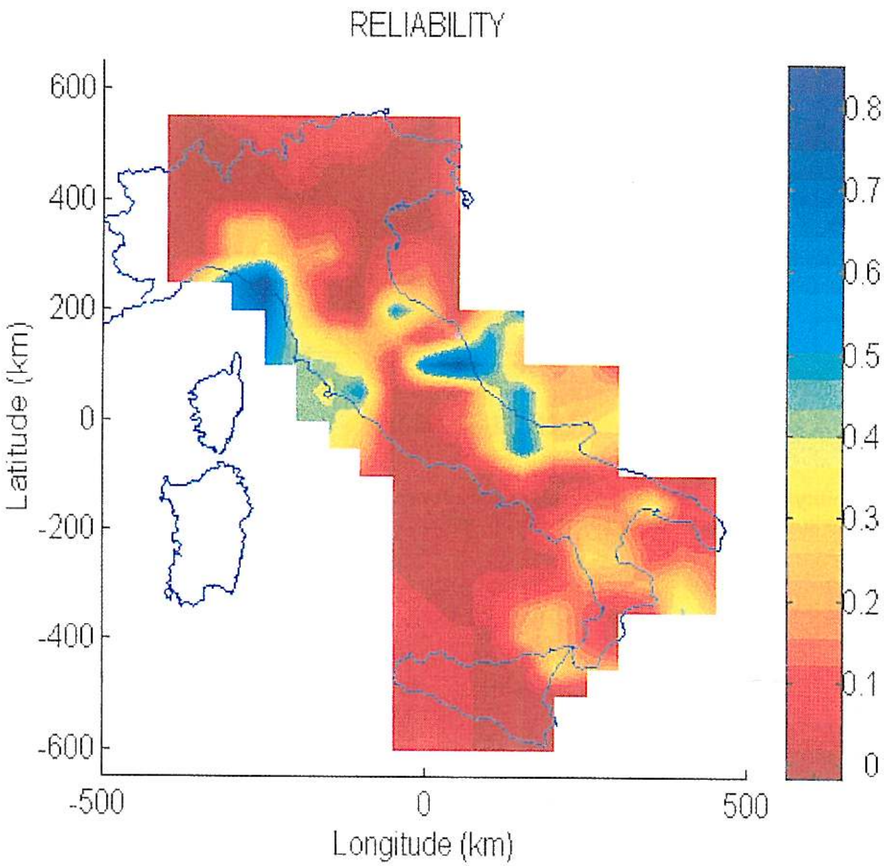


Fig. 6. Map of the reliability values of mainshocks of magnitude equal to or larger than 3.8 to be preceded by foreshocks of magnitude equal to or larger than 3.3, within T_2 = 2 days and R_2 = 11.9 km, over the Italian territory.

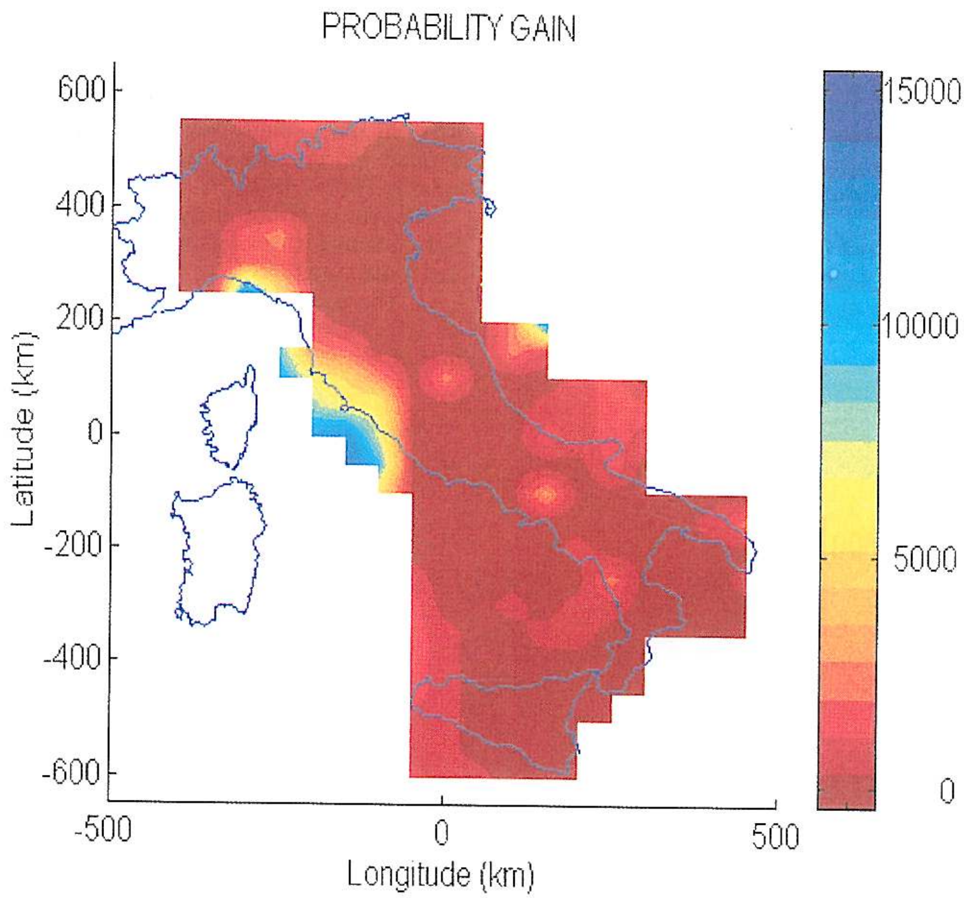


Fig. 7. Probability gain of precursors defined as foreshocks of magnitude equal to or larger than 3.3 for mainshocks of magnitude equal to or larger than 3.8 within T_2 = 2 days and R_2 = 11.9 km, over the Italian territory.

quiescent area. The maximum of the validity and the quality factor is found for small values of R_1 and T_1 (clustered seismicity).

– The probability that a mainshock will occur after a foreshock decreases in time (T_2) and distance (R_2) from the potential precursory event. The decrease is rapid soon after and at short distance from the foreshock and becomes slower at larger time-distance ranges.

– The number of potential foreshocks followed by a mainshock (as well as that of a mainshock followed by aftershocks) of a given magnitude is exponentially proportional to the magnitude of the former, with a «*b*» coefficient approximately equal to 0.9.

– The rate of occurrence of foreshock-mainshock (as well as of mainshock-aftershock) pairs is fairly constant for a given magnitude difference between the events of the pair.

– Foreshock activity is a characteristic of some areas rather than of others, with more than a factor of five variation in the rate of occurrence from site to site.

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REFERENCES

AGNEW, D.C. and L.M. JONES (1991): Prediction probabilities for foreshocks, *J. Geophys. Res.*, **96**, 11959-11971.

AKAIKE, H. (1974): A new look at the statistical model identification, *IEEE Trans. Automat. Control*, **AC-19**, 716-722.

AKI, K. (1981): A probabilistic synthesis of precursory phenomena, in *Earthquake Prediction, an International Review, Maurice Ewing Series* **4**, edited by D.W. SIMPSON and P.G. RICHARDS, *Am. Geophys. Union*, Washington, D.C., 566-574.

CONSOLE, R. and M. MURRU (1996): Probability gain due to foreshocks following quiescences tested by synthetic catalogs, *Bull. Seism. Soc. Am.*, **86** (3), 911-913.

CONSOLE, R., M. MURRU and B. ALESSANDRINI (1992): Foreshock statistic in the Italian seismicity, in *Proceedings of the XXIII General Assembly of the European Seismological Commission*, Prague, Czechoslovakia, 7-12 September 1992, **1**, 126-130.

CONSOLE, R., M. MURRU and B. ALESSANDRINI (1993): Foreshock statistics and their possible relationship to earthquake prediction in the Italian region, *Bull. Seism. Soc. Am.*, **83**, 1248-1263.

IMOTO, M. (1993): Foreshock occurrence in Kanto, Central Japan, and its performance as a precursor, in *Proceedings of Conference LXII, Eigth Joint Meeting of the U.S. – Japan Conference on Natural Resources (UJNR), USGS open-file No. 93-542, Menlo Park, California, 16-21 November 1992*, 199-213.

JONES, L.M. (1984): Foreshocks (1966-1980) in the San Andreas system, California, *Bull. Seism. Soc. Am.*, **74**, 1361-1380.

JONES, L.M. (1985): Foreshocks and time-dependent earthquake assessment in Southern California, *Bull. Seism. Soc. Am.*, **75**, 1669-1680.

JONES, L.M., R. CONSOLE, F. DI LUCCIO and M. MURRU (1997): Are foreshocks mainshocks whose aftershocks happen to be big? *Bull. Seism. Soc. Am.* (submitted).

MAEDA, K. (1996): The use of foreshock in probabilistic prediction along the Japan and Kuril trenches, *Bull. Seism. Soc. Am.*, **86** (in press).

OGATA, Y., T. UTSU and K. KATSURA (1995): Statistical features of foreshocks in comparison with other earthquake clusters, *Geophys. J. Int.*, **121**, 233-254.

OGATA, Y., T. UTSU and K. KATSURA (1996): Statistical discrimination of foreshocks from other earthquake clusters, *Geophys. J. Int.*, **127**, 17-30.

REASENBERG, P.M. and L.M. JONES (1989): Earthquake hazard after a mainshock in California, *Science*, **243**, 1173-1176.

REASENBERG, P.M. and M. V. MATTHEWS (1988): Precursory seismic quiescence: a Preliminary Assessment of the Hypothesis, *Pageoph*, **126**, 373-406.

SAKAMOTO, Y., M. ISHIGURO and G. KITAGAWA (1983): Akaike information criterion statistics (D. Reidel, Dordrecht), pp. 290.

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Appendix

The Akaike Information Criterion is known in literature as useful for comparing the validity of two statistical models. In this case, we apply it to assess whether the observed number of subsequent events is better modelled by a Poisson model with a constant rate or by a model characterized by two rates.

If N_S is the number of successes and N_M is the total number of target events, the reliability is defined as:

$$R = \frac{N_S}{N_M}$$

and the probability gain:

$$G = \frac{N_S}{V_A} \cdot \frac{V_M}{N_M} = R\frac{V_M}{V_A}$$

where V_A is the total volume of alarm area and V_M is the total space-time volume in study.

The occurrence rate of the events to be predicted (target events) may be modeled in two ways:

1) by a uniform Poisson rate model:

$$\lambda_M = \frac{N_M}{V_M}$$

2) by two Poisson processes with different rates:

$$\lambda_S = \frac{N_S}{V_A}$$

$$\lambda_{NA} = \frac{N_{NA}}{V_{NA}}$$

with N_{NA} number of missed alarms ($N_{NA} = N_M - N_S$) and V_{NA} the corresponding volume.

In the first case, the likelihood function is defined as:

$$L_1 = \mathrm{Prob}\,(N_S,\, \lambda_M,\, V_A) \cdot \mathrm{Prob}\,(N_{NA},\, \lambda_M,\, V_{NA}) = \frac{e^{-N_M}}{N_S!\,N_{NA}!}\,\lambda_M^{N_S}\,V_A^{N_S}\,\lambda_M^{N_{NA}}\,V_{NA}^{N_{NA}} \tag{1a}$$

in the second case:

$$L_2 = \mathrm{Prob}\,(N_S,\, \lambda_S,\, V_A) \cdot \mathrm{Prob}\,(N_{NA},\, \lambda_{NA},\, V_{NA}) = \frac{e^{-N_M}}{N_S!\,N_{NA}!}\,\lambda_S^{N_S}\,V_A^{N_S}\,\lambda_{NA}^{N_{NA}}\,V_{NA}^{N_{NA}}. \tag{2a}$$

The log-difference between (1) and (2)

$$\ln \frac{L_2}{L_1} = N_M \ln G + N_M\,(1-R) \ln\left(\frac{1-R}{G-R}\right)$$

is connected with the difference in AIC (Akaike Information Criterion) (Akaike, 1974; Sakamoto *et al.*, 1983):

$$d\mathrm{AIC} = \mathrm{AIC}(1) - \mathrm{AIC}(2) = -2 \cdot \ln \frac{L_1}{L_2} + 2 \cdot (n_1 - n_2)$$

where n_1 and n_2 are the number of free parameters for the first and second models (1) and (2) respectively.

Defining the quality as the ratio between dAIC and the total number of target events, the best values for R_1, T_1, R_2, T_2 are obtained corresponding to the maximum value of this parameter.