Mid-point electron density profiles from oblique ionograms

Xueqin Huang(*), Bodo W. Reinisch and Walter S. Kuklinski
University of Massachusetts Lowell, Center for Atmospheric Research, Lowell, MA 01854, U.S.A.

Abstract
A computationally efficient technique for the inversion of oblique ionograms into mid-point electron density profiles is described. The profile is given as the sum of quasi-parabolic functions suitable for ray tracing. The CPU time for a 486 desk top computer is 30 s.

Key words ionospheric radio – electron density profiles – oblique sounding

1. Introduction
Bistatic oblique ionograms can provide information on the mid-point vertical electron density profiles, that can serve as additional fix points for ionospheric mapping over large regions. The routine use of oblique sounding is handicapped by the non-availability of autoscaling techniques for Oblique Incidence (OI) ionograms. While reasonable progress has been made in the automatic processing of Vertical Incidence (VI) ionograms in terms of autoscaling and profile inversion, no operational procedures exist for the autoscaling of OI oblique incidence ionograms. An iterative approach is under development by the authors that scales the oblique traces in a variational process assuming a multi-parabolic mid-point profile \( N_e(h) \). One building block in this procedure is the calculation of \( N_e(h) \) from oblique ionogram traces; this inversion is described in this paper.

2. Models and assumptions
Operational real time applications require computationally efficient algorithms suitable for the PC environment. It is therefore necessary to make some simplifying assumptions.

1) The ionosphere is spherically stratified.
2) Each ionospheric layer has a quasi-parabolic electron density distribution.
3) The geomagnetic field can be neglected when using the ordinary traces (Krasheninnikov et al., 1994).

The electron density profile at the midpoint

(*) On leave from the Chinese Research Institute for Radiowave Propagation, Xinxiang, Henan, China.

Mailing address: Prof. Xueqin Huang, University of Massachusetts Lowell, Center for Atmospheric Research, 450 Alken Street, Lowell, MA 01854, U.S.A.; e-mail: XUEQINHUANG@POBOXES.COM
can then be written in the form (Croft and Hoogasian, 1968):

\[
N(r) = \begin{cases} 
N_m \left( 1 - \left( \frac{r - r_m}{y} \right)^3 \left( \frac{y}{R} \right)^2 \right), & \text{for } r_b < r < \frac{r_m r_b}{r_b - y} \\
0 & \text{elsewhere}
\end{cases}
\]  

(2.1)

where \( r \) is the geocentric radius, \( N_m \) is the maximum density at the layer peak \( r_m \), and \( r_b \) is the base radius. The semi-thickness of the layer is then given by \( y = r_m - r_b \). The index of refraction for the O-trace is given by

\[
\mu_0^2 = 1 - f_N^2 f^2
\]  

(2.2)

where \( f_N \) is the plasma frequency and \( f \) the sounding frequency.

Figure 1 shows a Composite Quasi-Parabolic (CQP) profile for a ionosphere with \( E \), \( F_1 \) and \( F_2 \) layers. The objective of the inversion process is to find \( N_m \) (or the maximum

![Fig. 1. Composite quasi-parabolic profile.](image)

plasma frequency \( f_m \), \( r_m \), and \( r_b \) for each layer from the oblique ionogram traces. The forward process of calculating the oblique ionogram for a distance \( D \) and a given mid-point electron density profile using eqs. (3.1a) and (3.1b) given below. For \( D = 2000 \) km and the \( f_N(h) \) profile of fig. 1, the resultant one-hop and two-hop oblique traces are shown in fig. 2. The inverse process is described in the next section.

3. Inversion procedure

A multi-variable minimization process is used to determine the CQP parameters from the oblique O-echo traces. It is assumed that trace data points \((f_s, P'_s)\) are identified as belonging to a low or high angle trace section of a specific layer. The \( E \) layer parameters are determined first using data points only from the \( E \) trace, then the \( F_1 \) parameters, then the \( F_2 \) parameters.

The advantage of using the CQP model is that the ground distance \( D \) and the group path \( P' = c \cdot t_g \) (\( c \) = speed of light, \( t_g \) = group travel time) can be expressed by analytical functions.
The method of steepest decent is then used to find the optimal \( (r_m, r_b) \) values (in the least-squares sense) that best reproduce the measured oblique traces. The fitting process allows for partial overlapping of the parabolas.

To reduce the calculation time the search range for the take-off angle \( \beta \) is limited to only those values that are physically possible for the iteratively assumed profiles. In fig. 3, the functions \( D(\beta) \) (solid lines) and \( P'(\beta) \) (dotted lines) for a sounding frequency of 9 MHz are plotted for the profile given in fig. 1. The maximum take-off angles for \( E \) and \( F_1 \) layer propagation \( \beta_{maxE} \) and \( \beta_{maxF_1} \) (12.34° and

For a signal reflected in the \( F_2 \) layer:

\[
D = F_1 (f, \beta, f_0 E, r_{mE}, r_{bE}, f_0 F_1, r_{mF_1}, \\
r_{bF_1}, f_0 F_2, r_{mF_2}, r_{bF_2})
\] (3.1a)

\[
P' = F_2 (f, \beta, f_0 E, r_{mE}, r_{bE}, f_0 F_1, r_{mF_1}, \\
r_{bF_1}, f_0 F_2, r_{mF_2}, r_{bF_2})
\] (3.1b)

where \( f \) is the operating frequency, \( \beta \) the take-off angle of the ray, and \( f_0 E \) to \( r_{bF_2} \) are the model parameters. Starting with the \( E \) layer, the parameters \( f_0 E, r_{mE} \) and \( r_{bE} \) must be determined for each take-off angle during the homing process. The three-dimensional parameter surface \( S(f_0, r_m, r_b) \) can be reduced to a two-dimensional surface \( S(r_m, r_b) \) if the function frequency \( f_j \) has been scaled from the OI ionogram. In that case the critical frequency \( f_m \) of the parabolic layer can be calculated as function of \( f_j, r_m \) and \( r_b \)

\[
f_0 = f_0(f_j, r_m, r_b).
\] (3.2)
22.93° in this example) can be determined analytically. The homing process of finding the take-off angles for a given distance, for example 2000 km, can then be limited to values just above zero and just below β_{maxE} for low and high angle E layer rays, just above β_{maxF} and just below β_{maxE} for the F₁ layer rays, and just above β_{maxF} for the F₂ layer ray. This qualified homing procedure reduces computation time by a factor of 40 compared to a brute-force homing approach. A typical homing process for the F₂ layer parameters is illustrated in fig. 4, where the error contours |P' - P''_{observed}| are plotted versus r_{RF}, (horizontal axis) and y_{RF}, (vertical axis). The total CPU time for calculation of the mid-point CQP on a 486 PC is only about 30 s.

The CQP algorithm has been applied to the oblique ionogram in fig. 5 between Wallops Island and Millstone Hill (D = 629 km) using GPS synchronized DPs (Digisonde Portable Sounder; Reinisch et al., 1992; Haines, 1994). The resulting mid-point profile N_r(h) is shown in the lower part of fig. 6. The critical frequency, peak height and half width for each layer are listed on the right side. The upper plot in fig. 6 compares the observed ranges P'_{obs} (dots) with the calculated P' values (solid lines) for the profile N_r(h). The agreement is very good.

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Revr. ULCAR - MILLSTONE HILL, WESTFORD, MASSACHUSETTS
LAT. 42.6, LONG. 288.5
DIP 72.9 IH 1.4

Xmr: ULCAR - WALLOPS ISLAND, VA
LAT. 37.9, LONG. 255.5
DIP 69.9 IH 1.4

Lowell Digisonde Portable Sounder
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**Fig. 5.** Digital OI ionogram between Wallops Island and Millstone Hill (D = 629 km).
4. Summary

A computationally efficient algorithm has been developed that calculates a composite quasi-parabolic mid-point profile from oblique ionogram traces. The CPU time required to calculate the profile is typically half a minute on a 486 PC.

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REFERENCES


