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Tripotential data processing for HES interpretation

Pietro Cosentino and Dario Luzio
Istituto di Geofisica Mineraria, Università di Palermo, Italy

Abstract
In this paper some methods are proposed and compared to correct the experimental measurements for preliminary processing of tripotential data which are acquired for HES prospecting. However, the use of those methods should be based upon an accurate analysis of all experimental data. Such an analysis ought to involve: 1) an estimate of the averaged measurement errors with their variance and distribution in both the space and the three apparent-resistivities domains; 2) the choice of a resistivity model capable of describing the actual volume under study. The differences among the three values of apparent resistivity measured on a point are generally influenced both by the resistivity distribution below ground as well as by the eventual measurement errors. The proper choice of the method of correction which may be useful to merge the resistivity values and minimize the measurement errors is also linked to the separation of modelling effects. Consequently, the model chosen should be selected in relation to the above mentioned analyses. It thus becomes useful to know the general relations among the three apparent resistivity values for some simple structures, *e.g.* the two-layer model with a slowly changing first layer thickness. This theoretical model is presented and discussed using two new «composed» apparent resistivities, namely ρ^{μ} and ρ^{τ}, which seem to be useful tools in HES interpretation. The behaviour of the calculated responses can be useful also for a fast data inversion.

Key words *geoelectrical prospecting – tripotential method – data processing – two layer model*

1. Introduction

The tripotential method, proposed by Carpenter (1955) and developed by Carpenter and Habberjam (1956), is based on the detection of a triad of resistivity values for each selected measurement position. These three measurements (namely ρ^{α}, ρ^{β} and ρ^{γ}) should be carried out using a linear four-electrode arrangement (fig. 1), alternatively using them as current (namely A and B) and potential (namely M and N) electrodes. In this paper only the array having equidistant electrodes with ρ spacing is examined, as this kind of array is the most widely used for Horizontal Electrical Soundings (HES).

Each array shown in fig. 1 gives a value of resistance R (defined as the ratio between the absolute values of the measured potential and the inserted current, $\Delta V/I$) as well as an associated value of apparent resistivity. The following relations can be written

$$R^{\alpha} = \Delta V^{\alpha} / I^{\alpha} \tag{1.1}$$

$$R^{\beta} = \Delta V^{\beta} / I^{\beta} \tag{1.2}$$

$$R^{\gamma} = \Delta V^{\gamma} / I^{\gamma} \tag{1.3}$$

and

$$\rho^{\alpha} = R^{\alpha} \cdot 2 \cdot \pi \cdot p \tag{1.4}$$

$$\rho^{\beta} = R^{\beta} \cdot 6 \cdot \pi \cdot p \tag{1.5}$$

$$\rho^{\gamma} = R^{\gamma} \cdot 3 \cdot \pi \cdot p \tag{1.6}$$

The measured resistances, as well as the apparent resistivities are not independent of each

*	p	*	p	*	p	*	
A		M		N		B	α
A		B		N		M	β
A		M		B		N	γ

Fig. 1. Sketch of the three possible distinguished arrays, with p spacing, having A and M, as well as B and N, electrically concordant.

other, since the following theoretical relations are in effect:

$$R^{\alpha} - R^{\beta} - R^{\gamma} = 0 \tag{1.7}$$

$$3\rho^{\alpha} - \rho^{\beta} - 2\rho^{\gamma} = 0 \tag{1.8}$$

Consequently an experimental tripotential measurement (without errors) should give a triad of values of apparent resistivity satisfying (1.8), which would correspond to a single point of the 3-D cartesian space of the three apparent resistivities $[\rho^{\alpha}, \rho^{\beta}, \rho^{\gamma}]$ pertaining to the plane which is described by (1.8) and contains the straight line $\rho^{\alpha} = \rho^{\beta} = \rho^{\gamma}$.

The various experimental data are generally affected by errors both with regard to the electrical (currents and potentials) and geometrical (*e.g.*, the relative positions of the four electrodes) measurements. The propagation of the errors involved in the electrical measurements can produce a default in (1.7) and (1.8), giving

$$R^{\alpha} - R^{\beta} - R^{\gamma} = E \tag{1.9}$$

and

$$3\rho^{\alpha} - \rho^{\beta} - 2\rho^{\gamma} = \varepsilon\,, \tag{1.10}$$

where E and ε, respectively are the degree of incompatibility in resistance and resistivity with respect to the observed values.

Relations (1.9) and (1.10) appear to be useful to check the experimental data. Considerable discrepancies between the ε values and the probable error of measurement are in fact due to trivial errors during the acquisition process: they are often readily recognized and removed. On the contrary minor errors, due to the instrumental accuracy and the acquisition procedures, can often be minimized using a suitable correction routine in which each triad of experimental resistivity data must be shifted from the experimental point to the plane $3\rho^{\alpha} - \rho^{\beta} - 2\rho^{\gamma} = 0$, here after the $\varepsilon = 0$ plane.

2. Reduction to the plane $\varepsilon = 0$

A few correction techniques were discussed in a previous paper (Cosentino *et al.*, 1992). In principle those techniques can be divided into three groups:

1) techniques in which a probability distribution on the $\varepsilon = 0$ plane is associated with each experimental point;

2) techniques of shifting of the actual ex-

perimental points to the $\varepsilon = 0$ plane, on the basis of error theory;

3) techniques of composed reduction, in which the above criteria are integrated through some constraints which arise from the general or particular characteristics of the model which is selected to describe the actual investigated volume. Generally these constraints are expressed by relations – equalities or inequalities – among functions of the three measured resistivities.

With regard to point (2), we will show later that the criteria used to choose the directions of the shifting procedure can also be suggested by the analysis of the expected errors on each measured resistivity (ρ^α, ρ^β and ρ^γ).

In principle the projection of an experimental point to the $\varepsilon = 0$ plane using ℓ_α, ℓ_β, ℓ_γ direction parameters is carried out using the following correction formulas:

$$\rho_c^\alpha = \rho^\alpha - \varepsilon \cdot \ell_\alpha / (3\ell_\alpha - \ell_\beta - 2\ell_\gamma) \tag{2.1}$$

$$\rho_c^\beta = \rho^\beta - \varepsilon \cdot \ell_\beta / (3\ell_\alpha - \ell_\beta - 2\ell_\gamma) \tag{2.2}$$

$$\rho_c^\gamma = \rho^\gamma - \varepsilon \cdot \ell_\gamma / (3\ell_\alpha - \ell_\beta - 2\ell_\gamma) \tag{2.3}$$

In particular it is possible to calculate for each measurement point a quantity – hereafter ρ^μ – proportional to the mean apparent resistivity. This is equivalent to a normal projection of the experimental point on the $\rho^\alpha = \rho^\beta = \rho^\gamma$ straightline (Cosentino *et al.*, 1992). The physical meaning of ρ^μ will be discussed below.

Another useful correction can be obtained by projecting each experimental point normally to the $\varepsilon = 0$ plane. It can be performed using the following formulas:

$$\rho_c^\alpha = \rho^\alpha - 3 \cdot \varepsilon / 14 \tag{2.4}$$

$$\rho_c^\beta = \rho^\beta + \varepsilon / 14 \tag{2.5}$$

$$\rho_c^\gamma = \rho^\gamma + \varepsilon / 7 \tag{2.6}$$

This procedure corresponds to the minimum possible correction of the data which, nevertheless, insures a partial reduction of the errors. Error theory suggests choosing the directional parameters of (2.1), (2.2) and (2.3) in such a manner that the corrections are proportional to the probable errors on the three measured apparent resistivities. It is possible to evaluate such errors starting from the probable errors on the measured ΔV, I and the geometric factor K, using

$$\sigma_\rho = \rho \sqrt{\left(\frac{\sigma_K}{K}\right)^2 + \left(\frac{\sigma_R}{R}\right)^2} \tag{2.7}$$

where σ_ρ, σ_K and σ_R are, respectively, the standard deviations of the resistivity measurements, the geometric factor and the measured resistences. The values of σ_ρ can be used for a correction of the various measurements based on the analysis of the errors bearing on each measurement.

It should be noted that, as (1.7) is independent from the geometry of the array, the errors on the estimate of K shift the resistivity points along directions which are parallel to the $\varepsilon = 0$ plane.

If the term under the root in (2.7) is quite independent from the values of R, (2.1), (2.2) and (2.3) reduce to the corrections proposed by Habberjam (1979)

$$\rho_c^\alpha = \rho^\alpha - \varepsilon \cdot \rho^\alpha / (3\rho^\alpha + \rho^\beta + 2\rho^\gamma) \tag{2.8}$$

$$\rho_c^\beta = \rho^\beta + \varepsilon \cdot \rho^\beta / (3\rho^\alpha + \rho^\beta + 2\rho^\gamma) \tag{2.9}$$

$$\rho_c^\gamma = \rho^\gamma + \varepsilon \cdot \rho^\gamma / (3\rho^\alpha + \rho^\beta + 2\rho^\gamma) \tag{2.10}$$

The composed reduction techniques seem to introduce a new tool which can improve both the correction techniques as well as the selection of the model type. As a matter of fact these techniques are characterized by an analysis of the experimental data both in the space and resistivity domains. These analyses should give different information on the model to be used in the final step; such information can also be used in the preliminary step of reduction of the data to the $\varepsilon = 0$ plane.

Thus, the experimental data, without correction or with a preliminary rough correction, should be examined on the $\varepsilon = 0$ plane and compared with the general behaviour of the resistivity data calculated for each particular model.

3. The composed apparent resistivities ρ^{μ} and ρ^{τ}

In order to describe the point distribution in the $\varepsilon = 0$ plane it is very useful to use a rotated reference frame constructed in such a way that two of its axes are on the $\varepsilon = 0$ plane, while the third one is ρ^{ε}, and, thus is equal to zero if the measurements are correct. The two axes ρ^{μ}, ρ^{τ} on the $\varepsilon = 0$ plane can be chosen with directional parameters respectively $\ell_{\alpha} = 1$, $\ell_{\beta} = 1$, $\ell_{\gamma} = 1$ and $\ell_{\alpha} = 1$, $\ell_{\beta} = -5$, $\ell_{\gamma} = 4$ with respect to the old frame.

The new composed resistivities can be expressed as a function of the old ones, as follows:

$$\rho^{\mu} = \sqrt{3} \cdot (\rho^{\alpha} + \rho^{\beta} + \rho^{\gamma}) / 3 \tag{3.1}$$

$$\rho^{\varepsilon} = \sqrt{14} \cdot (3\rho^{\alpha} - \rho^{\beta} - 2\rho^{\gamma}) / 14 \tag{3.2}$$

$$\rho^{\tau} = \sqrt{42} \cdot (\rho^{\alpha} - 5\rho^{\beta} + 4\rho^{\gamma}) / 14 \tag{3.3}$$

It can be useful to stress that in the new rotated frame only the two composed resistivities ρ^{μ} and ρ^{τ} are connected with the resistivity distribution in the earth, while the third apparent resistivity, namely ρ^{ε}, is only connected with all the measurement errors carried out during the acquisition of the triad $[\rho^{\alpha}, \rho^{\beta}, \rho^{\gamma}]$. So, in principle, once the corrections are carried out, one can use only the two following composed resistivities to generate a complete tripotential information:

$$\rho^{\mu} = \sqrt{3} \cdot (4\rho^{\alpha} - \rho^{\gamma}) / 3 \tag{3.4}$$

$$\rho^{\tau} = \sqrt{42} \cdot (\rho^{\gamma} - \rho^{\alpha}) \tag{3.5}$$

In particular, if the measurements are carried out on a homogeneous and isotropic earth whose resistivity is ρ_o, and no measurement error is done, the composed resistivities should respectively result $\rho^{\mu} = \sqrt{3} \cdot \rho_o$, $\rho^{\tau} = 0$ and $\rho^{\varepsilon} = 0$.

It is easy to deduce the qualitative behaviour of the data distribution in the resistivity domain when the data pertain to a simple ground model (see, *e.g.*, Cosentino *et al.*, 1992). It can be very useful to known their quantitative behaviour in many theoretical cases: such behaviour is calculated below for a two-layer model characterized by low-gradient variability of the thickness of its first layer.

4. Two-layer model with variable depth of the boundary

The normal two-layer model can be characterized by a slow variability of the depth of the boundary in the investigated area. In this case the distribution of the set of the experimental data in the resistivity domain can be approximately calculated by collecting the responses of many two-layer models having a plane and parallel interface at different depths, for the range of depths considered. Thus the single responses can be calculated using two-layer monodimensional models.

Let ρ_1 be the resistivity of the upper layer and ρ_2 that of the lower one. The reflection coefficient k is given by

$$k = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} \tag{4.1}$$

If δ is the ratio between the thickness h of the upper layer and the p spacing the image method directly gives:

$$\frac{\rho^{\alpha}}{\rho_1} = 1 + 4 \sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{1 + 4\delta^2 n^2}} + \frac{1}{\sqrt{4 + 4\delta^2 n^2}} \right] \tag{4.2}$$

$$\frac{\rho^{\beta}}{\rho_1} = 1 + 6 \sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{9 + 4\delta^2 n^2}} + \frac{1}{\sqrt{1 + 4\delta^2 n^2}} - \frac{2}{\sqrt{4 + 4\delta^2 n^2}} \right] \tag{4.3}$$

$$\frac{\rho^{\gamma}}{\rho_1} = 1 + 3\sum_{n=1}^{\infty} k^n \left[\frac{1}{\sqrt{1+4\delta^2 n^2}} - \frac{1}{\sqrt{9+4\delta^2 n^2}}\right] \tag{4.4}$$

which obviously satisfy (1.8).

On the $\varepsilon = 0$ plane the distribution of the experimental points that can be measured on a two-layer model having a slowly variable thickness of the upper layer will be approximately given by

$$\frac{\rho^{\mu}}{\rho_1} = \sqrt{3} + \frac{\sqrt{3}}{3}\sum_{n=1}^{\infty} k^n \left[\frac{13}{\sqrt{1+4\delta^2 n^2}} + \frac{3}{\sqrt{9+4\delta^2 n^2}} - \frac{16}{\sqrt{4+4\delta^2 n^2}}\right] \tag{4.5}$$

$$\frac{\rho^{\tau}}{\rho_1} = \sqrt{\frac{14}{3}} \cdot \sum_{n=1}^{\infty} k^n \left[\frac{4}{\sqrt{4+4\delta^2 n^2}} - \frac{1}{\sqrt{1+4\delta^2 n^2}} - \frac{3}{\sqrt{9+4\delta^2 n^2}}\right] \tag{4.6}$$

The relative behaviour of the ratios between the two composed resistivities and the upper layer resistivity on the $\varepsilon = 0$ plane is presented respectively in figs. 2 and 3, for positive k values (that is for conductive overburden) and negative k (that is for resistive overburden). Finally in figs. 4, 5 and 6 such calculated values are projected upon the three coordinate planes of the resistivity space and are logarithmically represented. The general trend of the three sets of curves seem to be similar, even though the relative distances from the main diagonal are

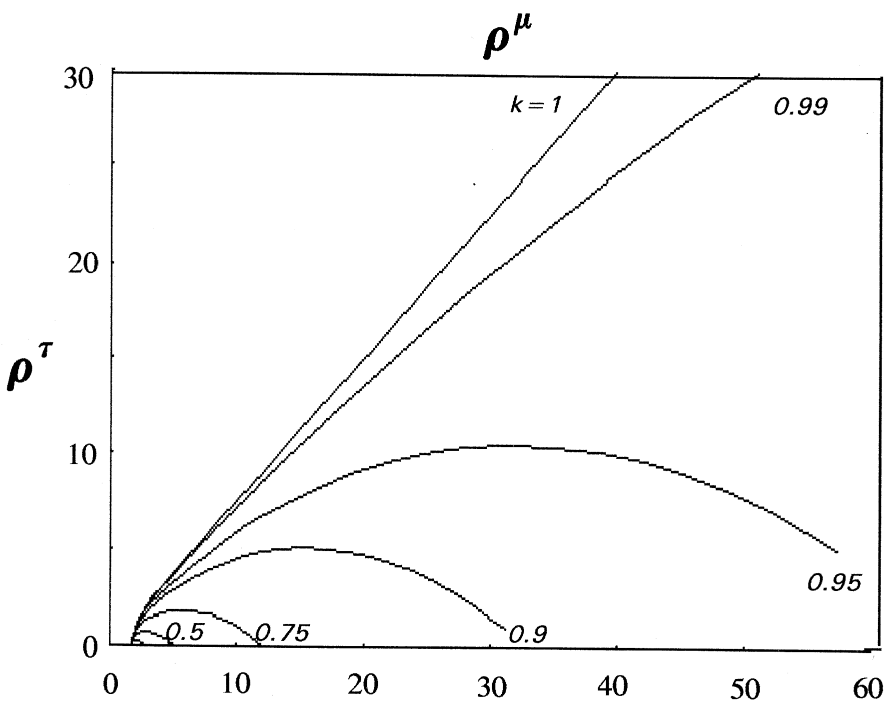

Fig. 2. Behaviour of the composed apparent resistivities for all the δ-values and for selected positive k-values.

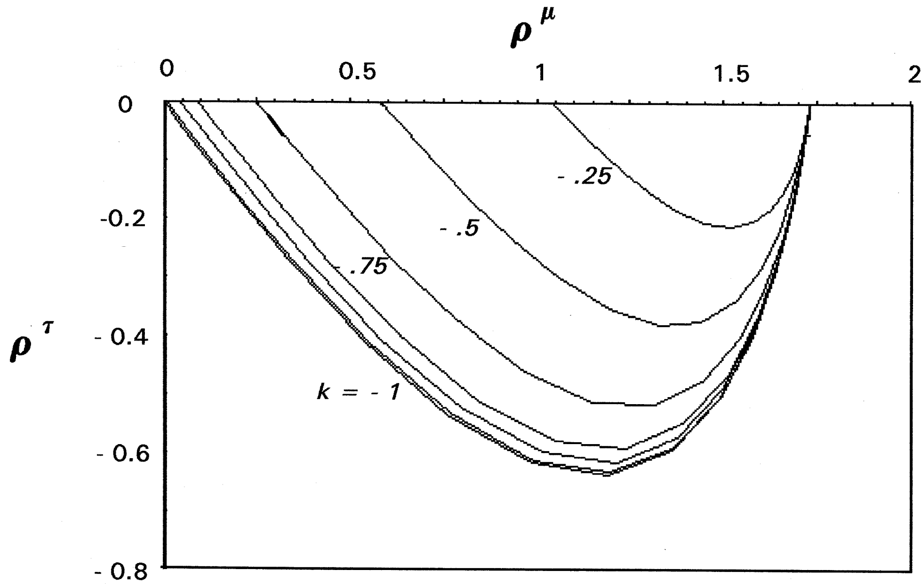

Fig. 3. Behaviour of the composed apparent resistivities for all the δ-values and for selected negative k-values.

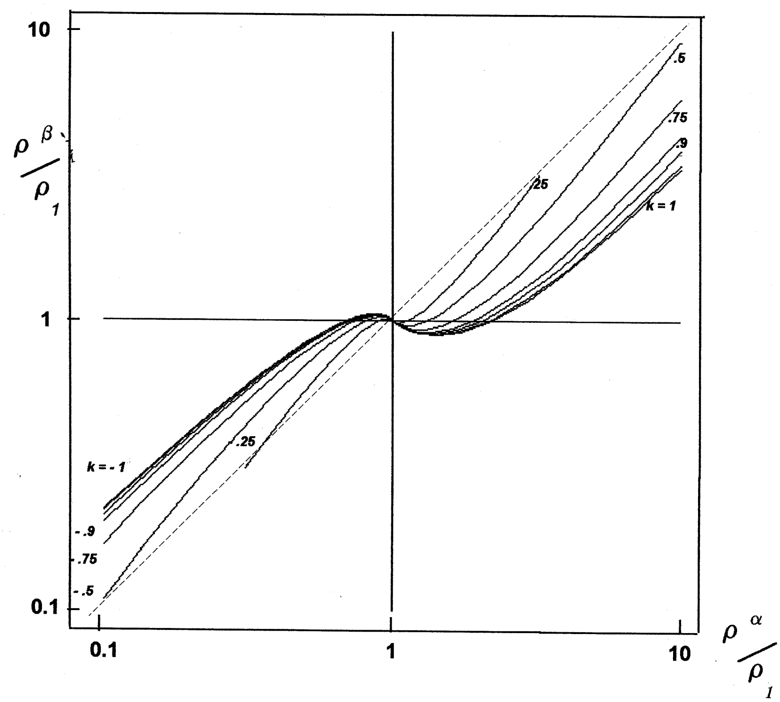

Fig. 4. Behaviour of the apparent resistivities ρ^{α} and ρ^{β} for all the δ-values and for selected k-values.

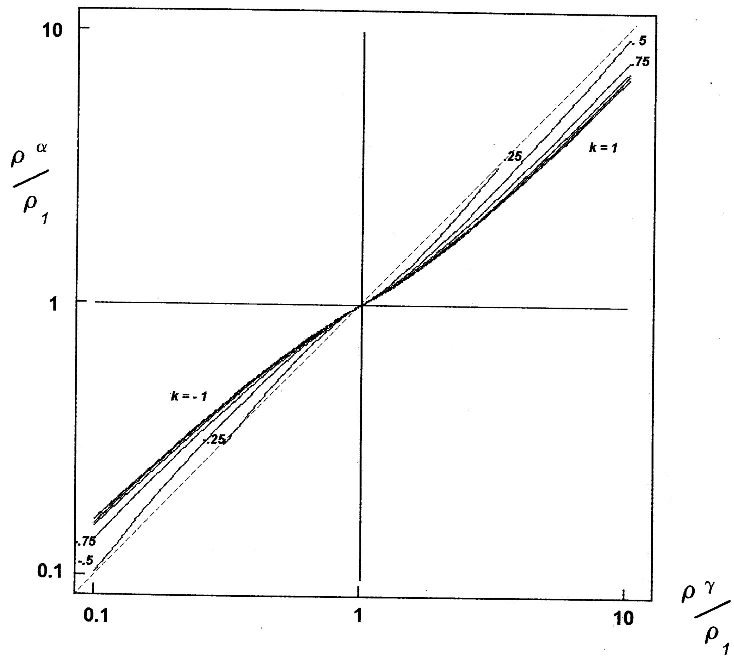

Fig. 5. Behaviour of the apparent resistivities ρ^{α} and ρ^{γ} for all the δ-values and for selected k-values.

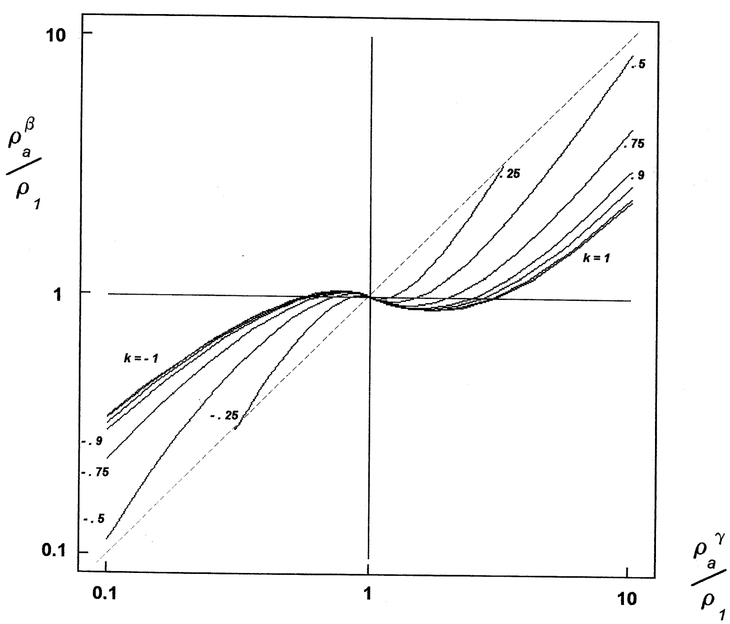

Fig. 6. Behaviour of the apparent resistivities ρ^{β} and ρ^{γ} for all the δ-values and for selected k-values.

significantly different due to the different dihedral angles among the $\varepsilon = 0$ plane and the three coordinate ones.

5. Conclusions

It can be concluded that it is useful to study the distribution of the experimental data in the apparent resistivity domain, and, in particular, in the plane of the composed apparent resistivities, once the necessary corrections of the data are carried out.

As a matter of fact such distributions can reveal some general characteristics of the type of model which should be used, so that it is possible to optimize the correction methodology. In particular, in the two-layer model having a «slowly» variable thickness of the first layer, the experimental points, in the $\rho^{\mu} - \rho^{\tau}$ plane should be approximately distributed along section curves similar to those shown in figs. 2 and 3. In such model, as can be expected, a tripotential measurement can theoretically give both the local thickness of the first layer as well as the resistivity of the second, if the resistivity of the first layer is known.

It is important to note that often, in practice, the actual resistivity distributions should be represented by models more complex than the studied one.

If two or more sectors of the investigated area are characterized by different models, the whole set of data should be differentiated in various different sub-sets: otherwise the distribution on the $\varepsilon = 0$ plane would be quite irregular and scatted due to «geological» and measuring noise.

Consequently it may be important to discriminate the possibility of finding different components in the whole data set, preferably after a suitable low-pass filtering: such approach can represent a standard preliminary methodology, which may also be useful in order to optimize the correction procedure.

Finally it should be noted that it is often necessary to filter the various maps in the space domain (for instance, a low-pass filtering in order to reduce the noise component due to random errors and to low-wavelength resistivity anomalies). In such cases it is difficult to evaluate the optimal characteristics of the filter by analyzing only the filter effects in the space domain: the optimization can be performed also analyzing the data in the resistivity domain, especially if those are compared with the distributions given by the chosen resistivity models.

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