Position and spatial orientation of magnetic bodies from archaeological magnetic surveys

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Abstract
For the last twenty years magnetic surveys have frequently been applied in archaeological prospection. Among the different processing techniques that can be applied to the magnetic data, we studied the possibility for delineating the position and the spatial orientation of shallow depth magnetic anomalous bodies. The approach namely cross-correlation filter (or matched filter) has been adopted. At first, theoretical magnetic anomalies of total magnetic field, of its components and the vertical gradient of these, due to three-dimensional bodies, oriented S-N, E-W and NW-SE, were calculated. These synthetic bodies were considered a sum of many elementary prisms. Field measurements were simulated adding a noise component with different signal-to-noise ratio on the theoretical anomalies. In order to improve signal-to-noise ratio and to locate and delineate the orientation of the anomalous bodies a bidimensional cross-correlation technique was applied. Different synthetic anomalies, due to bodies with different dimensions, depths, susceptibility contrasts and geomagnetic parameters were used, as operators, to apply the cross-correlation technique. The efficiency of the method was improved using a normalised cross-correlation of the field data.

Key words applied geophysics – archaeological prospection – magnetic method – cross-correlation technique – synthetic models

1. Introduction
Surface geophysical surveys have been applied to the problem of detecting and delineating shallow subsurface targets of archaeological interest (Aitken, 1974; Weymouth, 1986; Brizzolari et al., 1992a;b; Piro and Versino, 1996). Among these, magnetic prospection represents one of the most widely used techniques. However, geophysical methods used in conventional applications, for e.g., mining exploration, have always required some upgrading of data acquisition system, sensitivity and resolution as the investigated targets are usually of limited dimensions; that is, an aspect more pronounced in the case of archaeological investigations.

The experience of archaeological magnetic prospection has developed during recent years from qualitative help before or during excavation to accurate mapping and delimitation of sites. This improvement has been largely made possible by the development in the techniques of data acquisition, processing and interpretation continuously presented by many authors (Scollar et al., 1990; Brizzolari et al., 1992b, 1993; Beker, 1995; Tsokas and Hansen, 1995).
One of the main aims of processing is to transform the raw field data into a reasonable meaningful form and to enhance the maps to delineate buried structures whose surface expressions can be slightly detected or completely obscured. This is not only because of the disturbances due to inhomogeneous surfaces and any residual human activities but also in relation to the characteristics of the geoenvironmental condition, in which the archaeological sites are contained. Top-soil carries a much higher magnetic susceptibility than the rock or sub-soil from which it has been derived. As long as the rock surface (or the sub-soil surface) is smooth, the magnetic intensity across the site will be smooth too, and when man-made pits dug into the rock exist, that have silted up with top-soil, well-defined magnetic anomalies will be produced that can be detected given a sensitive enough magnetometer (Aitken, 1974).

Sources of perturbation of the Earth's magnetic field, in the case of archaeological prospection could be differentiated into two broad types: correlated and uncorrelated (Scollar et al., 1990), or as indicated by Brizzolari et al. (1993), into Gaussian type, stochastic or coherent types. The sources of these three types may include instrumental defects, orientation and spacing errors (Gaussian type), random irregularities of the surface or slight changes in the susceptibilities and earth current transients (stochastic type), minor superficial inhomogeneities or major deeper structures (coherent type).

The magnitude of the noise varies from quite small to considerable values, particularly for major irregularities in susceptibilities of the top soil. However, the noise effect has to be compared to the amplitude of the sought-after anomalies experienced on a site. In the case of archaeological investigation, a low signal-to-noise ratio may severely prejudice the identification of anomalous bodies.

The frequency band of the measured signal depends on the depth of burial of the features and on the susceptibility contrast between features and surroundings. High frequency anomalies, caused by the archaeological features, could be masked by local variations of the susceptibility distribution in the background (Scollar et al., 1990).

Preliminary processing includes filtering and convolution techniques as a trial to separate, as far as possible, the expected signals from any unwanted information contained in the field data. The simplest case is the well known low-pass, high-pass and band-pass filters; the latter techniques have proven helpful to eliminate some of the near-surface and deeper geological induced effects (Bhattacharya, 1965; Darby and Davies, 1967; Clement, 1973; Bath, 1974; Bhattacharya and Navolio, 1976; Scollar et al., 1990). However, application of filtering requires some knowledge about the frequency spectra of both signal and noise. As this information is not completely available from real data, it is therefore useful to consider simulated models for signal and imposed noise. Since the archaeological features we are looking for in the magnetic data have well defined orientation, it is intuitive to think of constructing directional filters to enhance or discriminate against a particular orientation of an asymmetric structure. Various trending features, if present, can be picked out, one by one, as different directional filters are applied.

If the absolute magnitudes of the susceptibility, magnetic viscosity and remanent magnetisation contrasts are assumed, the corresponding magnetic anomalies may be calculated in closed form for simple regular shapes. Some methods allow for differing susceptibilities in various parts of a structure and for remanent magnetisation with direction differing from that of the main field as well as for a magnetic moment of arbitrary strength (Scollar, 1969; Lington, 1972). In order to analyse, elaborate and interpret the magnetic anomalies, the directions of both the total and induced magnetisation vectors should also be estimated. But these parameters are often difficult to determine.

The present study is based on the application of the bidimensional cross-correlation technique (Brizzolari et al., 1993) to locate and delineate the possible spatial orientation of the archaeological structures.
2. Enhancement of S/N ratio

2.1. Defining the problem

The problem of recovering the anomalies masked by the noise is to choose and to apply suitable techniques to improve the signal-to-noise ratio.

Treibel and Robinson (1969) showed in detail that, if we have at least a rough estimation of the shape, dimensions and physical properties of the expected bodies, the best filter (operator) is the theoretical anomaly of the structure itself. This operator is « an absolute optimum » in the case of Gaussian noise; in case of coherent noise (autocorrelated) the best operator is still the theoretical anomaly which minimises the prevalent frequencies of the autocorrelated noise (Bernabini et al., 1988).

The application of this operator, in the space domain, consists in the cross-correlation of the raw field data with the calculated theoretical anomaly.

The cross-correlation function is a measure of similarity between sets of data. If the sought anomaly has a shape close to that of the theoretical one, the cross-correlation technique produces a signal with a shape similar to that of the autocorrelation of the theoretical anomaly. Whenever the two sets of input data are close, their cross-correlation will be usually positive and their value will be large. In the opposite case, some of the products will be positive while others are negative, and the sum tends to be smaller.

The magnetic surveys are performed along lines and the anomalies are usually selected from each profile. Thus the cross-correlation technique has generally been applied over single profiles using theoretical models calculated under the hypothesis that the profiles were centred with respect to the anomalous body (Brizzolari et al., 1993). The anomalous body also influences the contiguous profiles and therefore it is useful to employ these readings in the numerical elaboration of field data. For this reason the 2D cross-correlation technique should be applied using theoretical three-dimensional operators for the anomaly.

In the discrete form, the cross-correlation $C(x, y)$ can be written as:

$$C(x, y) = \sum_{k=-m_2}^{m_2} \sum_{s=-n_2}^{n_2} f(x + k; y + s) g(k + m_2 + 1; s + n_2 + 1)$$

where $m_2 = \frac{m_1 - 1}{2}$ and $n_2 = \frac{n_1 - 1}{2}$; with $m_1$ and $n_1$ the dimensions of the operator.

In the calculation, to correct for end effects, the field data are enlarged with samples equivalent to half the number of samples of the operator. The added samples are identified using a statistic distribution of the field data around the standard deviation. In this way the first value after cross-correlation corresponds to the position of the first value at the border.

In order to make the correlogram more meaningful it would be advantageous to normalise it in some way. Since the maximum output from the autocorrelation is normalized to unity when the data set is matched, the interpretation of the cross-correlograms is clearer by dividing the output cross-correlated values by the maximum autocorrelation values for each of the applied operators. If the normalised cross-correlated values should approach unity for one output, the best synthetic operator will fit the field data. From this information and our assumptions about the other source/field parameters, it seems possible to determine the location and to limit the depth-range of the investigated area.

2.2. Calculating synthetic anomalies

It is evident that above all it is necessary to calculate the synthetic magnetic anomalies for the three-dimensional anomalous bodies to be used as operators in the bi-dimensional cross-correlation, i.e. their dimensions, physical parameters and depths.

A series of studies have shown that both induced and remanent magnetization can be of importance. Of these two effects that of induced magnetization is always present.
All natural deposits, both rocks and soils, exhibit a magnetisation whose value seems to depend mainly on two factors: the total quantity of iron minerals, especially oxides, present in the deposit and the proportions between the different forms of these minerals. In particular, soil deposits are normally more magnetic than the rocks from which they are formed and this increase is even higher for many occupational deposits (Linnington, 1972).

Assuming a filling of uniform susceptibility without remanent magnetisation effect does not lead to significant error when constructing a synthetic model which is to be compared with the searched signal due to the anomalous body. The random fluctuations in the latter due to different causes will usually be more significant than that attributable to variable susceptibility in the fill (Scollar et al., 1990). Since the estimation of the remanent magnetization properties is often difficult, only the induced magnetisation effects have been considered in the present study.

There are a number of methods given in geophysical literature which allow calculation of theoretical anomalies from objects of arbitrary shape. Only a few of these are of interest for archaeological structures. If higher resolution is needed, it may be advantageous to use Linington’s (1972) or Talwani’s (1965) method (Scollar et al., 1990).

In our paper, the theoretical magnetic anomalies for a three-dimensional model were computed using the relations proposed by Talwani (1965), which allow the calculation of the values of the three and total components of the magnetic field at any arbitrary point on the surface, generated by a body of arbitrary shape. The method assumes an uniform magnetisation $M$ of the body and the equations are given in relation to the parameters that characterise the body both from the geometric and magnetic points of view. Specific software has been developed for these calculations and includes the possibility of considering the contributions of several bodies.

We concentrated our attention to the depth-range 0.5-4 (m) for the anomalous body, because this range can represent the subsurface portion where more frequently the archaeological features are contained.

Taking into account the above considerations, many theoretical anomalies produced by varying alternative body dimensions (length, width and height), depths ($d = 0.5-4$, with step interval of 0.5 grid unit) and their susceptibility contrast, were calculated. These anomalies were utilized as operators to apply the cross-correlation technique.

As an example, fig.1a-e illustrates the total magnetic anomaly maps of simulated one vertical sided rectangular prism, with a SN extension, at different depths (grid unit): 0.8, 1, 1.5 and 2. The dimensions of the body, in grid unit, are: length = 5, width = 2, height = 1 and the uniform susceptibility contrast is $\Delta x = 10^{-3}$ SI. The geomagnetic parameters are: $F = 45,000$ nT, $I = 55^\circ$ and $D = 0^\circ$.

We added a noise component, with a S/N ratio of 1:2.5, to the original map of fig. 2a to simulate a field result.

Random noise is introduced, point by point, on the data using the following relation:

$$Z_n = Z_c + (RDN - 0.5) \times R \times K$$

where, $Z_n$ is the noise anomaly, $Z_c$ is the computed anomaly, $RDN$ is a random value ranging from 0-1, $R$ is the range of the synthetic anomaly’s values, and $K$ is a coefficient which changes the percentage of the noise on the anomalies.

The ratio of S/N ranges between 1:1.2 to 1:2.5. The latter value should not be considered unusual for field conditions. In such case, as illustrated in fig. 2b, the simulated feature has almost completely disappeared.

This map was successively cross-correlated with the theoretical anomalies of a SN prism, with different dimensions, depths and susceptibility contrast, calculated with the same grid interval of the simulated field map. The analysis of the different correlograms shows a good location of the searched for body centre, if there is a similarity between the shape of the signal of the anomalous body and the synthetic operator. On the contrary, it is not possible to delineate the spatial orientation of the body. In addition, the normalisation of the cross-correlated data with respect to the maximum value of the autocorrelation of the corresponding the-
Fig. 1a-e. Theoretical calculated anomalies (3D) of total magnetic field. Body dimensions (grid unit) length = 5, width = 2, height = 1, at different depths (grid units): 0.8, 1, 1.5 and 2. Earth’s magnetic field parameters: $F = 45,000$ nT, $I = 55^\circ$, $D = 0$ and with susceptibility contrast of $\Delta \chi = 1 \times 10^{-3}$ SI. a) Position in plant of the simulated body; b) $d = 0.8$; c) $d = 1$; d) $d = 1.5$ and e) $d = 2$. 

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Fig. 2a,b. a) Theoretical calculated anomaly of $\Delta F$. The body dimensions and the magnetic parameters are the same as in fig. 1a-e. Depth of the body $d = 1$ (grid unit). b) Noisy map containing the anomalous body, with S/N ratio 1:2.5.
oretical anomalies can limit the depth-range of the body.

In succession, the normalized correlograms, obtained using synthetic anomalies calculated at different depths, are presented and discussed.

The normalised correlograms, shown in fig. 3a-d, indicate that the location of the anomalous body comes through quite well, but not its orientation. Comparing the normalised results it is possible to observe that the values which approach unity have been obtained using synthetic operators calculated for a depth-range \( d = 0.8-1 \) grid unit.

Figure 4a illustrates the total magnetic anomaly map of simulated two vertical sided rectangular prisms, with a SN and N45°E extent, respectively, at a depth of 1 grid unit. The dimensions of the bodies are: length = 10, width = 2 and height = 1, with susceptibility contrast \( \Delta \chi = 10^{-3} \) SI. The geomagnetic parameters are the same as the previous calculations. We added a noise component to the original map with a S/N ratio of 1:2.5, fig. 4b.

The simulated field map was successively cross-correlated, first with the theoretical anomaly of a SN prism, followed by the anomaly of the N45°E prism, one at a time. The correlograms, normalised with the maximum of the autocorrelation of the corresponding synthetic anomaly, shown in the fig. 5a,b, indicate that the position of the single elongated anomalous body comes through, one by one, but it is not possible to locate simultaneously, using the same theoretical anomaly, the position of two bodies and their orientation.

In our case we considered the magnetic anomaly as an integrated effect of the anomalies due to many elementary cube-shaped bodies, with dimensions \( 1 \times 1 \times 1 \) (grid unit) placed side by side, to form the vertical sided rectangular prisms. Therefore, instead of using a synthetic model generated by a body having presumed dimensions close to the one searched after, we performed a number of \( N \times N \) cross-correlations with the anomaly due to a body with dimensions: \( 1 \times 1 \times 1 \) (grid-unit). Also in this case many theoretical anomalies were calculated, varying either the depth \( d = 0.5-4 \), with step interval of 0.5 grid unit) or the susceptibility contrast. In succession, the normalised correlograms obtained using synthetic anomalies calculated with the same geomagnetic parameters and susceptibility contrast of the previous calculations, at different depths \( (d: 0.8, 1, 1.2, 1.5 \text{ and } 2 \text{ grid unit}) \) are presented. The small operator, at \( d = 1 \) (grid unit), works out quite well and the position and the orientation of the anomalous prisms are clearly outlined in fig. 5c. This figure shows that some isolated values approaching unity, are also present where there are no sources \( (x, 2 \text{ – } y, 2; x, 22 \text{ – } y, 22) \). This fact can be due, probably, to the chosen number of samples of the synthetic operator, which matches a signal strongly corrupted by noise. Taking into account that the aim of the study is to identify the orientation of the main bodies, only the isolines characterized by a clear spatial correlation should be considered.

As mentioned above, by dividing the output cross-correlated values by the maximum autocorrelation values of each theoretical anomalies being applied (fig. 6a-d), it is possible to exert a good control on the interpretation. When the normalised cross-correlated values approach unity for an output the best theoretical value fits the field data. Considering the maps shown in fig. 6a-d, only the result for the body at depth: 1 (grid unit), after normalisation, gives a value equal to unity; while the model at depth of 0.8 and 1.2 (grid unit) give a result near unity. The other results are far from unity.

We experimented this procedure of isolating and delineating the orientation of anomalous bodies by cross-correlating a noisy effect of bodies elongated in various directions with the synthetic anomaly due to body with dimensions \( 1 \times 1 \times 1 \) (grid unit). All tests made with varying depth, susceptibility contrast, thickness and horizontal size (this last ranging from 0.5 to 2, with step interval of 0.5 grid unit) allow similar results to those above.

Another example is the simulated total magnetic map for a square frame of four connected vertical sided rectangular prisms, at depth of 1 grid unit, fig. 7a. The dimensions of the prisms are: length = 10, width = 2, height = 1 and susceptibility contrast \( \Delta \chi = 10^{-3} \) SI.
Fig. 3a-d. Normalized cross-correlation of simulated field data of fig. 2b, using the synthetic anomalies of fig. 1a-e for different depths. a) $d = 0.8$; b) $d = 1$; c) $d = 1.5$; d) $d = 2$. 

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Fig. 4a,b. a) Theoretical anomalies of $\Delta F$ due to two bodies. The magnetic parameters and susceptibility contrast are the same as in fig. 1a-c. Body dimensions and depth (grid unit): length = 10; width = 2; height = 1; depth = 1. Directions: SN and N45°E. b) Noisy map containing two anomalous bodies; S/N ratio 1:2.5.
Fig. 5a-c. Normalized cross-correlation of simulated field data using alternatively the synthetic anomaly of SN elongated body (a) and the anomaly of N45°E body (b). c) Results using the anomaly of body with dimensions $1 \times 1 \times 1$ at depth $d = 1$ (grid units). For the other parameters see the fig. 1a-e. Each data set has been normalized with the maximum value of the autocorrelation of the corresponding theoretical anomaly.
Fig. 6a-e. Normalized cross-correlation of simulated field data of fig. 4b using the synthetic anomalies of a body with unit dimensions at different depths. a) \( d = 0.8 \); b) \( d = 1 \); c) \( d = 1.2 \); d) \( d = 1.5 \) and e) \( d = 2 \). Each data set has been normalized with the maximum value of the autocorrelation of the corresponding theoretical anomaly.
Fig. 7a,b. a) Theoretical anomalies of $\Delta F$ due to four connected bodies, forming a square frame. Bodie's dimensions (grid unit) length = 10, width = 2, height = 1 and depth $d = 1$. For the other parameters see fig. 1a-e. b) Noisy map containing the four anomalous bodies; S/N ratio 1:2.5.
The simulated field map with S/N ratio of 1:2.5 is shown in fig. 7b. We applied the same SN prism (1 x 1 x 1 grid units), where its magnetic anomaly was cross-correlated with the noisy map. The position and the orientation of the bodies are delineated in the normalised correlogram of fig. 8.

The analysis of the normalised correlograms for the case of simulated maps (figs. 3a-d, 5a-e, 6a-e and 8) shows that using a synthetic anomaly due to an elementary cube-shaped alloed us to locate the position and spatial orientation of all bodies.

3. Elaboration of field data

A survey example is shown in fig. 9a, illustrating the elaborated gradiometric measurements carried out to detect filled graves and a portion of ancient road excavated in the tufaceous outcrop at «Acqua Acetosa» archaeological site near Rome (Brizzolari et al., 1993). In this zone a differential magnetic survey was performed in a rectangular area with sides of 30 m and 16 m. The measurements of total magnetic field were collected along S-N profiles with 1 m sampling interval using a proton
Fig. 9a,b. Acqua Acetosa Archaeological site (Laurentina, Roma - Italy). a) Contour map of the gradient of total magnetic field. b) Normalized cross-correlation using synthetic anomaly of the gradient of total magnetic field, due to a body with dimensions: $1 \times 1 \times 1$ m, depth $d = 0.8$ m and susceptibility contrast $\Delta \chi = -2 \times 10^{-4}$. Main magnetic field parameters: $F = 44.128$ nT, $I = 55^\circ$, $D = 0$. The dotted lines A and B indicate the position of two excavated roads. C indicates the position of excavated filled grave (tomb).
precession magnetometer G856 (Geometrics) in gradiometric configuration, with two sensors at different heights: 0.5 and 1.5 m. In the previous paper (Brizzolari et al., 1993) the data set was elaborated using bidimensional filtering techniques (high-pass filter), to eliminate a trend which masked the magnetic anomalies.

Taking into account that geologically the investigated area is characterized by a series of lithoid pozzolans, the searched for archaeological structures are probably filled with sediments which present a susceptibility value less than average. Therefore, for the calculation of theoretical anomaly, a negative value of susceptibility contrast was applied.

Using the above described approach, a synthetic anomaly of the gradient of the total magnetic field due to a body with dimensions $1 \times 1 \times 1$ m, susceptibility contrast $\Delta \chi = -2 \times 10^{-4}$, at depth $d = 0.8$ m was calculated. The adopted geomagnetic parameters were: $F = 44.128$ nT, $I = 55^\circ$, $D = 0$. The values of normalised cross-correlograms are shown in fig. 9b.

The map illustrates the presence of four anomalous zones with different values of normalised cross-correlation, which can be arranged in two groups.

In the first we have the zones with the following co-ordinates: $x = 18$, $y = 13$ and $x = 29$, $y = 2$. These zones present the highest values of normalised cross-correlation. The extent of these area is similar to the dimensions of the known filled graves, which are present in the necropolis. The second group, situated in the interval: $x = 5; 15$ and $y = 1; 12$, present values between 0.3 and 0.8. Taking into account the spatial orientation of this zone and the archaeological information about the direction of the hypothesized road, these two zones can be related to the presence of the structure boarding the Acqua Acetosa Necropolis.

To verify the validity of this interpretation, and to confirm the presence of the hypothesized archaeological structures, excavations were made to identify the road remains, indicated with A and B in fig. 9a,b, and the position C of a filled grave (tomb).

4. Conclusions

The analysis of all calculated correlograms shows that to delimit the position and orientation of the sources it is not useful to use the theoretical anomaly due to a body with large dimensions, because in this way only the body barycentre can be detected. On the contrary, a set of theoretical anomalies due to an elementary cube-shaped body (each one sized $1 \times 1 \times 1$), the unit meaning the grid unit, works very much better. Our tests show that the technique is relatively not dependent on the other source parameters, i.e. depth, thickness and susceptibility.

Moreover, the normalisation of the cross-correlated data with respect to the maximum value of the autocorrelation of the different theoretical anomalies can help to limit the depth-range of the searched for body.

The improvement of this processing technique is a model characterised by a variable susceptibility to study the case for remanent magnetisation with different direction.

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