Short Note

Space-time combined correlation integral and earthquake interactions

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Abstract
Scale invariant properties of seismicity argue for the presence of complex triggering mechanisms. We propose a new method, based on the space-time combined generalization of the correlation integral, that leads to a self-consistent visualization and analysis of both spatial and temporal correlations. The analysis was applied on global medium-high seismicity. Results show that earthquakes do interact even on long distances and are correlated in time within defined spatial ranges varying over elapsed time. On that base we redefine the aftershock concept.

Key words *seismicity - correlation integral - fractal dimension - clustering*

1. Introduction

Seismicity appears to be scale invariant in many of its aspects. Several papers (Kagan, 1994; Bak *et al.*, 2002; Parson, 2002; Marsan and Bean, 2003; Corral, 2004) investigate spatial and temporal correlations of epicentres, involving for example the concepts of Omori law and fractal dimension. We think that the complex phenomenon of seismicity calls for an approach capable of analysing spatial localisation and time occurrence in a combined way and without subjective *a priori* choices. In this paper we introduce a new method of analysis that leads to a self-consistent analysis and visualization of both spatial and temporal correlations based on the definition of correlation integral (Grassberger and Procaccia, 1983).

2. Method

We define the space-time combined correlation integral as

$$C_c(r, \tau) = \frac{2}{N(N-1)} \cdot \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \left(\Theta\left(r - \|\mathbf{x}_i - \mathbf{x}_j\|\right) \cdot \Theta\left(\tau - \|t_i - t_j\|\right) \right)$$

where Θ is the Heaviside step function ($\Theta(x)=0$) if $x \leq 0$ and $\Theta(x)=1$ if $x>0$) and the

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sum counts all pairs whose spatial distance $\|\boldsymbol{x}_i - \boldsymbol{x}_j\| \leq r$ and whose time interval $\|t_i - t_j\| \leq \tau$. When applied over all possible values of τ or r, the well-known correlation integral (Grassberger and Procaccia, 1983) is returned. It results that $C_c(r, \tau)$ is the generalisation of the correlation integral for a phenomenon that explicates in diverse dimensions with not comparable measurement units. When applied on seismicity $C_c(r, \tau)$ takes into account the distribution of all time intervals and epicentral inter-distances between all pairs of events, irrespective of the relationship between the main event and any aftershock.

From the space-time combined correlation integral we define the time correlation dimension and the space correlation dimension for sets of events within space-time distances r and τ, respectively as

$$D_t(r,\tau) = \frac{\partial \log C_c(r,\tau)}{\partial \log \tau}$$

and

$$D_s(r,\tau) = \frac{\partial \log C_c(r,\tau)}{\partial \log r}.$$

If $C_c(r, \tau)$ was a pure power-law in both variables, then D_t and D_s would correspond to the temporal and spatial fractal dimensions, respectively. More generally, the behaviour of D_t and D_s as a function of r and τ will characterise the clustering features of earthquakes in space and in time. This method has been applied to global seismicity to study the space-temporal correlation between earthquakes all over the world. Data come from the catalogue of the National Earthquake Information Center, USGS, in the time period between 1973 and 2002, with magnitudes m_b greater than 5. This catalogue selection was conditioned by completeness criteria and it presents medium to high magnitude distribution.

3. Results

The space-time combined correlation integral C_c for global seismicity is represented in fig. 1 with black contour lines. The local slopes of this surface in the direction parallel to the time axis is the time correlation dimension D_t, plotted in colours as a function of space and time. The colour coding of each pixel quantifies the time correlations existing between events occurring within a given distance and time interval. $D_t \cong 1$ corresponds to the random occur-

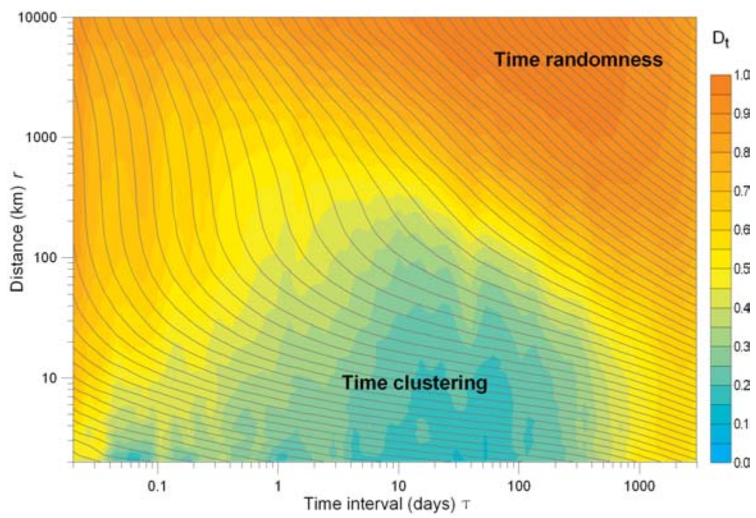

Fig. 1. Space-time combined correlation integral $C_c(r, \tau)$ (dark contour lines) and time correlation dimension D_t (coloured shaded contour) for the catalogue of global seismicity.

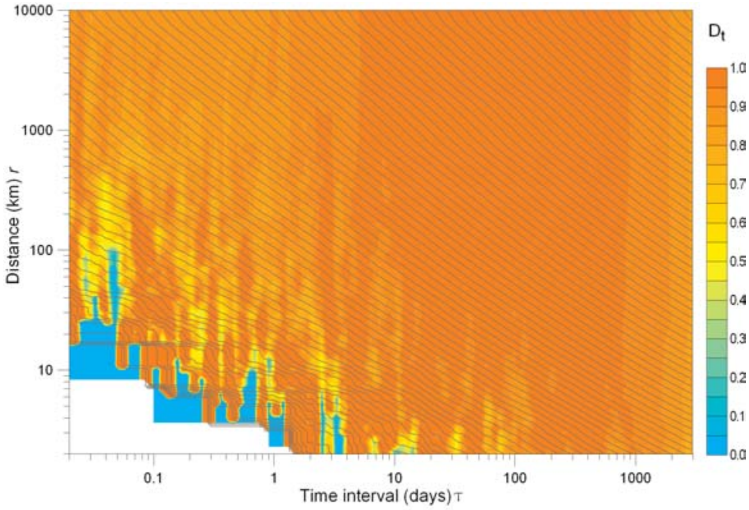

Fig. 2. Same as fig. 1 for the reshuffled catalogue.

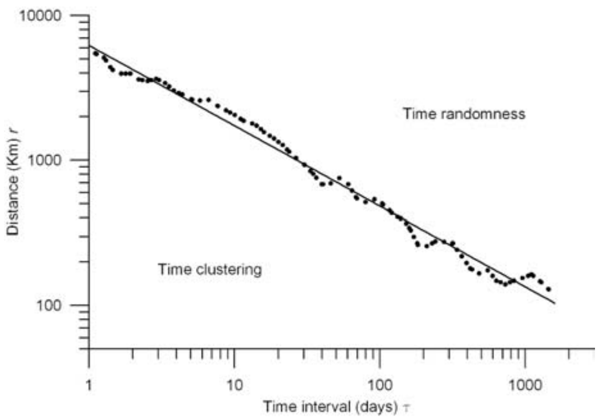

Fig. 3. The limit separating time clustering from time randomness (fixed to D_t=0.8) as a function of distance is shown. Points are fitted by the line of equation $\log r = -0.55 \log \tau + 3.8$.

rence of events, while a lesser value of D_t indicates time clustering. In fig. 1 two main domains appear: one at shorter inter-distances with low D_t representing time clustering; the other with $D_t \cong 1$ indicating a random time occurrence of events. The patterns observed in fig. 1 significantly support the hypothesis that earthquakes are correlated inside some space-temporal ranges. In order to check this hypothesis we applied the same analysis to the global catalogue after a reshuffling procedure. Reshuffling consists in mixing the time occurrence of each event keeping fixed its epicentre coordinates. The characteristic of this procedure is that of maintaining the separate statistical properties of data. The results show (fig. 2) that all patterns vanish, evidencing constant high values of D_t at all distances and time intervals.

To delineate a limit of the clustering resulting from fig. 1, all the points with D_t=0.8 were plotted on a separate figure (fig. 3). It is interesting to see that, within the temporal ranges shown, the clustering boundary can well be approximated by a straight line on this log-log plot, thus indicating a power-law behaviour. Using the least squares fitting we obtained $\log r = -0.55 \log \tau + 3.8$. This relation places a strong constraint on time relations among events, evidencing how distance plays a dynamic role. In particular, the relation can be read as defining a temporal correlation

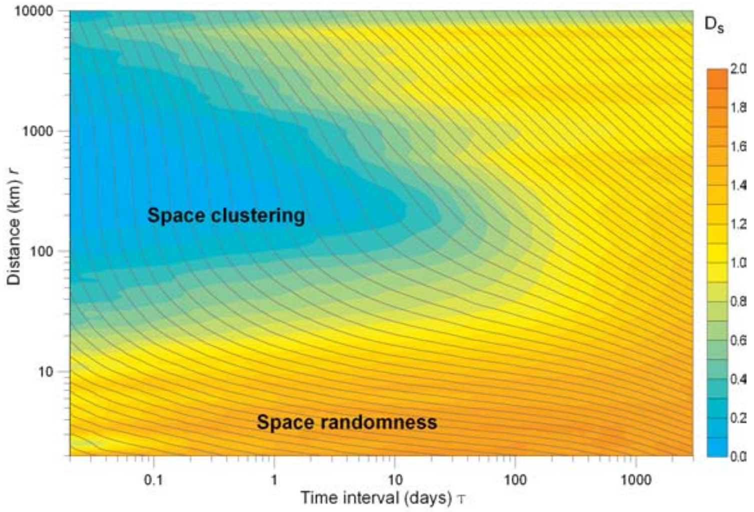

Fig. 4. Space-time combined correlation integral $C_c(r,\tau)$ (dark contour lines) and space correlation dimension D_s (coloured shaded contour) for the catalogue of global seismicity.

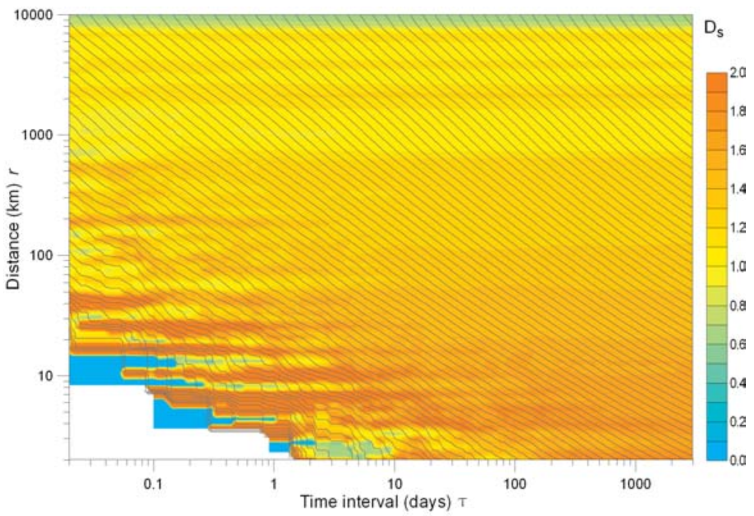

Fig. 5. Same as fig. 4 for the reshuffled catalogue.

reaching long distances that quickly shrinks over time following a power-law. For relatively short spatial ranges (around 100 km) events are time clustered and correlated for long time intervals (around 3 years). Over longer distances time correlation lasts for a short period (less than 30 days for 1000 km).

The local slopes of the surface of the space-time combined correlation integral, in the direction parallel to the space axis, correspond to the space correlation dimension D_s, plotted in colours in fig. 4 as a function of space and time. The colour coding of each pixel quantifies the space clustering existing between events occurring within a given distance and time interval. $D_s \cong 2$ identifies a random distribution of earthquakes, $D_s \cong 1$ indicates that epicentres tend to dispose along lines and $D_s < 1$ corresponds to space clustering. Even in this plot different domains are easily recognised. At short distances, a high space correlation dimension domain is clearly separated from space clustering $(0 < D_s < 1)$ that is present at greater distances: both conditions last for inter-time up to 100 days. The disappearance of clustering with time leaves room to a general $D_s \cong 1$, interpreted as the activity of seismicity on plate boundaries. Even in this case we tested the goodness of the results applying the same analysis on the reshuffled catalogue. The resulting plot in fig. 5 shows that the single statistical properties of data are not sufficient to produce the clustering domains appearing in the combined approach of fig. 4, but a real connection between space and time is needed.

Localisation errors certainly plays an important role at short spatial ranges, generating high values of D_s (fig. 4), but it appears that the area with high space correlation dimension is evolving with time calling for the presence of a physical process. In particular, plotting in fig. 6 the points with $D_s = 1$, chosen as the limit separating random behaviour from space clustering, it appear that in the shown ranges they follow a straight line. Fitting with least squares we obtained the relation $\log r = 0.1 \log \tau + 1.2$. The separation line defines an area around each epicentre, slowly growing in time, within which seismic events are randomly distributed.

3. Discussion

Figures 1 and 4 show that earthquakes are connected with each other in a non trivial way and that a dynamic interaction appears when space and time are analysed together with the combined correlation integral. The results reveal a statistical property of the global seismicity of medium-high magnitude, but interpreting them as an average behaviour of events after the occurrence of each earthquake of magnitude greater than 5, a possible scenario appears. The term ‘aftershock’ can be redefined on the basis of our findings: aftershocks are all earthquakes con-

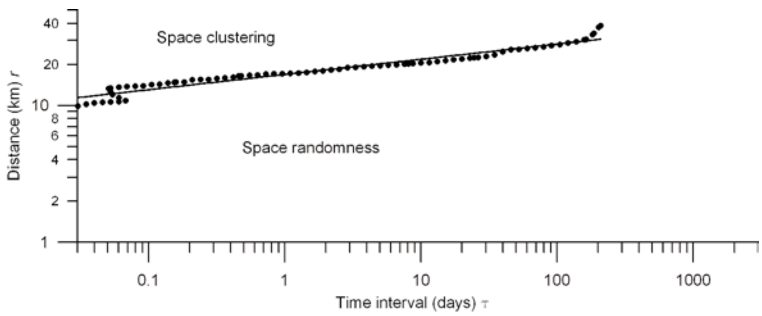

Fig. 6. The limit separating spatial clustering from spatial randomness (fixed to $D_s = 1.0$) as a function of time is shown. Points are fitted by the line of equation $\log r = 0.1 \log \tau + 1.2$.

nected to one reference event preceding them, as revealed by their temporal correlation (low D_t). In this sense all earthquakes occurred at a distance from reference event less than the radius r, defined by the relation $\log r = -0.55 \log \tau + 3.8$. (where τ is the elapsed time, fig. 3), are aftershocks of that event. This aftershock region reaches long distances from the reference 'main' event, but it quickly shrinks over time. If compared to an homogeneous time distribution, this area of influence can be interpreted as a region of modified probability of earthquake time occurrence. The epicentres of these connected events tend to cluster in space, apart from the near field (an area of radius 10-20 km), where earthquakes are randomly placed. Even this near field aftershock region has a dynamic boundary, increasing slowly in size according to the equation $\log r = 0.1 \log \tau + 1.2$ (fig. 6). This result is in agreement with other authors (Tajima and Kanamori, 1985; Marsan *et al.*, 2000; Helmstetter, 2003; Huc and Main, 2003) who found a migration of aftershocks, defined with classic methods, away from a main shock. This migration is described in terms of a law $\bar{d}(t) \sim t^H$, where $\bar{d}(t)$ is the mean distance between main event and aftershocks occurring after time t, with an exponent $H < 0.5$ corresponding to a sub-diffusive process.

4. Conclusions

In summary, we have introduced a new statistical tool, the combined space-time correlation integral, which allows us to perform a simultaneous and self-consistent investigation of the correlation properties of earthquakes. This tool leads to the discovery, visualization and deep analysis of the complex interrelationships existing between the spatial distribution of epicenters and their occurrence in time. The analysis performed on the worldwide seismicity catalogue and the corresponding reshuffled catalogue, strongly suggests that earthquakes of medium-high magnitude do interact with each other. This result led to a new definition of aftershocks, as all earthquakes with non-random occurrence with respect to the reference 'main' event, without considering their magnitude. Finally the analysis revealed how the aftershock region modifies over elapsed time.

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