Some considerations on the hypocentre location problem

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ABSTRACT

Some aspects of the problem of accuracy in the hypocentre location are discussed, and some difficulties are shown to arise within the conventional approach. The statistical basis of the commonly employed location procedure is examined, in orded to clarify under which assumptions it can be considered as correct; in particular an example is given where the validity of the probabilistic statement connected to confidence regions is evaluated via Montecarlo simulation.

The hypocentre location problem is next re-examined regarding it as a measurement problem: in this scheme, special attention is paid to the analysis of the features of the measurement instrument, and to the problem of the possibility of repeating the trials.

The analysis suggests that the attention should be shifted on to joint treatment of several events, including a different probabilistic interpreta-

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tion of confidence regions, in order to overcome the inadequacies which have been pointed out.

Accordingly, the joint focal and crustal parameters determination is interpreted as a change of perspective in seismological measurements.

RIASSUNTO

Gli Autori considerano alcuni aspetti del problema della accuratezza nella localizzazione ipocentrale evidenziando alcune difficoltà inerenti l'approccio convenzionale.

Vengono analizzate le ipotesi statistiche sulle quali si basano le procedure più comunemente usate per la localizzazione, al fine di valutare sotto quali condizioni queste possono ritenersi corrette; in particolare si dà un esempio di valutazione, attraverso una simulazione di tipo Montecarlo, della validità degli enunciati probabilistici connessi alle regioni di confidenza.

Successivamente il problema della localizzazione ipocentrale viene riesaminato e visto come un problema di misura: nell'ambito di tale impostazione una particolare attenzione è dedicata all'analisi delle caratteristiche dello strumento di misura e alla possibilità di ripetere le misurazioni.

L'analisi suggerisce, per superare le inadeguatezze messe in luce, l'opportunità di spostare l'attenzione verso una trattazione congiunta di più eventi che comprenda una diversa interpretazione probabilistica delle regioni di confidenza.

Conseguentemente la determinazione congiunta dei parametri focali e crostali viene interpretata come un cambiamento di prospettiva nelle misure sismologiche.

INTRODUCTION

The terms "accuracy" and "error" in the hypocentre location procedure are related to a scheme in which hypocentre location is considered as a measurement procedure (Freedman, 1968).

Such scheme is convenient to allow the application of methodologies and quantities derived from the measurement theory: it is a matter of fact, indeed, that the measurements of seismic parameters are performed by means of instruments, and thus reading errors and other typical measurement concepts are to be considered. On the other hand it is also clear that seismological measurements do not actually fit all measurement traditional hypotheses, first of all the possibility of repeating trials. This fact, as we will see, is to be carefully considered in order to avoid misunderstandings or incorrect interpretations. The quantities to be measured are commonly the focal parameters, that is time-space coordinates of a point: the traditional assumption of a point-source, to represent the limited portion of earth where seismic energy is released, has been the basis for the development of technology (networks, arrays), of mathematical techniques, and, at the same time, of a certain number of applications which gave origin to "users", which "need" in a certain sense point-sources; this is mainly the case of attenuation curves, maps of seismic risk according to various definitions, epicentre maps at scale such that source dimensions play no ultimate role.

In some applications, however, such as seismotectonics maps, and in general maps and elaborations aiming at a link between geological structures and statistically computed quantities, point-sources are proving inadequate.

Moreover, it is just in such applications that the customary use of quantities, such as the "accuracy" and the "error", evaluated whithin the conventional approach to the point-source determination problem, turns out to be difficult or misleading.

The aim of this paper is essentially of proposing some reflexions on the problem of the accurancy in the hypocentre location, by trying to re-examine the entire procedure under the point of view of a measurement problem, rather than introducing a new method. Some aspects of the problem will be first discussed, in order to clarify under which conditions and in which way the accuracy can be correctly evaluated by some commonly used quantities.

Accordingly, it will be analyzed what is the "measurement instrument", which are its features, and what we may consider as the "object", that is the quantity to be measured. We will try to show how some difficulties in using the results of these measurements arise from the scheme of the approach, rather than from lack of technological precision: we want to stretch that these difficulties exist even in case that the structural model could be considered as "exactly known", a condition which is in general far to be fulfilled. Finally, we will indicate how they may be probably overcome changing the leading thread of the approach, though no definite alternative will be proposed at this stage.

Hypocentre location as a nonlinear regression problem

The classical procedure for hypocentre location, viewed as a measurement problem, can be summarized as follows:

a) a point-source is to be located in time and space;

b) an instrument is used, typically a network, which consists of a set of homogeneous tools providing a set of homogeneous observations (onset times of P-waves, S-waves and so on);

c) a model is given, or some experimental travel-time table, to characterize the portion of the earth crossed by the seismic waves;

d) an algorithm is chosen which, if successful, gives normally the four coordinates of a point in the time-space domain;

e) the algorithm seeks the minimum of a suitable function of the unknown coordinates. A stop condition which the minimum should satisfy must be established for the algorithm;

f) some quantities are finally computed, which should represent the uncertainty in the location.

Clearly the location performed according to the above procedure, and its accuracy, will depend on all steps a) - f) and possibly on other factors. In order to set up a proper function to be minimized and to evaluate the accuracy in the location, a statistical scheme has to be established for the location problem.

Geiger, 1910 first considered the following nonlinear regression scheme:

$$t_i = t_0 + f_i(\mathbf{x}_0) + e_i \qquad i = 1, \ldots, m \qquad [1]$$

where

t_i = observed P-waves first arrival time at the i-th station
t_0 = origin time
$\mathbf{x}_0$ = vector of the spatial coordinates of the hypocentre
f_i = travel time function for the i-th station
e_i = nonsystematic error at the i-th station

The errors e_i are usually considered as independent normally distributed random variables with mean zero and known variances s_i^2. Then the parameters t_0, $\mathbf{x}_0$ can be estimated via the Maximum Likelihood Principle (see for example Mood et al., 1972)

$$\underset{t,\mathbf{x}}{\mathrm{Max}} \prod_{i=1}^{m} \exp \left\{ - \frac{1}{2} \frac{(t_i - t - f_i(\mathbf{x}))^2}{s_i^2} \right\}$$

which is equivalent to the classical least squares criterion with weighted residuals

$$\min_{t,\mathbf{x}} \sum_{i=1}^{m} \frac{(t_i - t - f_i(\mathbf{x}))^2}{s_i^2} \qquad [2]$$

In the following $\hat{t}_0$, $\hat{\mathbf{x}}_0$ will indicate the estimates of t_0, $\mathbf{x}_0$ obtained by [2].

In Freedman, 1968, Buland, 1976, the limits of the above statistical assumptions for the errors e_i, and hence of the interference about t_0, $\mathbf{x}_0$ given by 2, are discussed. Buland also stresses the fact that serious difficulties aise in the actual computation of $\hat{t}_0$, $\hat{\mathbf{x}}_0$, especially when the seismic ecent is recorded by a small localized network; algorithms which do not suitably tackle the numerical instability of the problem may fail to converge to $\hat{t}_0$, $\hat{\mathbf{x}}_0$ or even diverge.

In the framework of optimization theory several highly efficient algorithms have been developed to overcome the above difficulties. For a recent survey on the subject we refer to Dennis and Moré, 1977.

As far as the problem of giving a measure of the accuracy of the estimates, confidence regions provide a suitable tool through a probabilistic statement of the type: "The probability that the true parameters lie in the region A is...", where A is random region in the parameters space.

When the residuals are nonlinear functions of the parameters, as in the case of [1], a linearization procedure for the construction of confidence regions is based on the following assumptions (Bard, 1974):

1) Let the regression model be of the form

$$\mathbf{y} = \mathbf{g}(\mathbf{w}) + \mathbf{e}$$

$\mathbf{y}=(y_1, \ldots, y_m)$, $\mathbf{g}=(g_1, \ldots, g_m)$, $\mathbf{w}=(w_1 \ldots, w_k)$, $\mathbf{e}=(e_1, \ldots, e_m)$

with $\mathbf{e}$ distributed according to a multivariate normal distribution with mean zero and covariance matrix $\Sigma = [s_1^2, \ldots, s_m^2]$ I, I being the identity matrix.

2) Let $\mathbf{g}(\mathbf{w})$ be adequately fitted in a neighborhood U of the true value $\mathbf{w}^*$ by the first order Taylor expansion

$$\mathbf{g}(\mathbf{w}) \simeq \mathbf{g}(\mathbf{w}^*) + J(\mathbf{w} - \mathbf{w}^*) \qquad \mathbf{w} \in U \qquad [3]$$

where J is the m x k Jacobian matrix with elements $D_j g_i(\mathbf{w}^*)$

3) Let the probability that the least squares estimate $\hat{\mathbf{w}}$ of $\mathbf{w}^*$ falls outside U be very near to zero.

Thus $\hat{\mathbf{w}}$ can be considered as the least squares solution of the equation

$$\mathbf{y} = \mathbf{g}(\mathbf{w}^*) + J(\mathbf{w} - \mathbf{w}^*)$$

and hence $\hat{\mathbf{w}}$ can be considered normally distributed with mean $\mathbf{w}^*$ and covariance matrix $A^{-1} = (J^T \ \Sigma^{-1} J)^{-1}$.

This implies that $(\hat{\mathbf{w}} - \mathbf{w}^*)^T A (\hat{\mathbf{w}} - \mathbf{w}^*)$ has a chi-square distribution with k degrees of freedom. If $X^2_{p/k}$ is the p percentage point of such distribution then, by definition

$$\Pr \{ (\hat{\mathbf{w}} - \mathbf{w}^*)^T A (\hat{\mathbf{w}} - \mathbf{w}^*) \leq X^2_{p/k} \} = p$$

Therefore, the region

$$\{ \mathbf{w} : (\hat{\mathbf{w}} - \mathbf{w})^T \quad A \quad (\hat{\mathbf{w}} - \mathbf{w}) \leq X^2_{p/k} \} \qquad [4]$$

is a confidence region for $\mathbf{w}^*$ at a 100 p % level.

We remark that, by assumptions 2 and 3, J can computed indifferently evaluating the derivatives of $\mathbf{g}$ at $\mathbf{w}^*$ or at $\hat{\mathbf{w}}$. In the latter case, it doesn't depend on the unknown $\mathbf{w}^*$.

The region (4) is a k-dimensional ellipsoid with centre at $\mathbf{w}^*$, whose more relevant geometrical features are the following:

a) the axes of the ellipsoid lie along the eigenvectors $\mathbf{v}_i$ of A;

b) the length of the i-th semi-axis is $\sqrt{X^2_{p/k} / l_i}$, where l_i is the eigenvalue corresponding to $\mathbf{v}_i$.

The maximum elongation within the ellipsoid is therefore in the direction of the eigenvector corresponding to the minimum eigenvalue l_{min};

c) considering the standard deviation $\hat{s}_i = \sqrt{(A^{-1})_{ii}}$ of the estimate $\hat{\mathbf{w}}_i$, the interval

$$R_i^p = \{ \hat{\mathbf{w}}_i - \mathbf{s}_i \sqrt{X^2_{p/k}} \leq w_i \leq \hat{w}_i + \hat{s}_i \sqrt{X^2_{p/k}} \}$$

is the range of maximum variation of w_i within the ellipsoid.

Fig. 1 illustrates the situation in the case of $k = 2$.

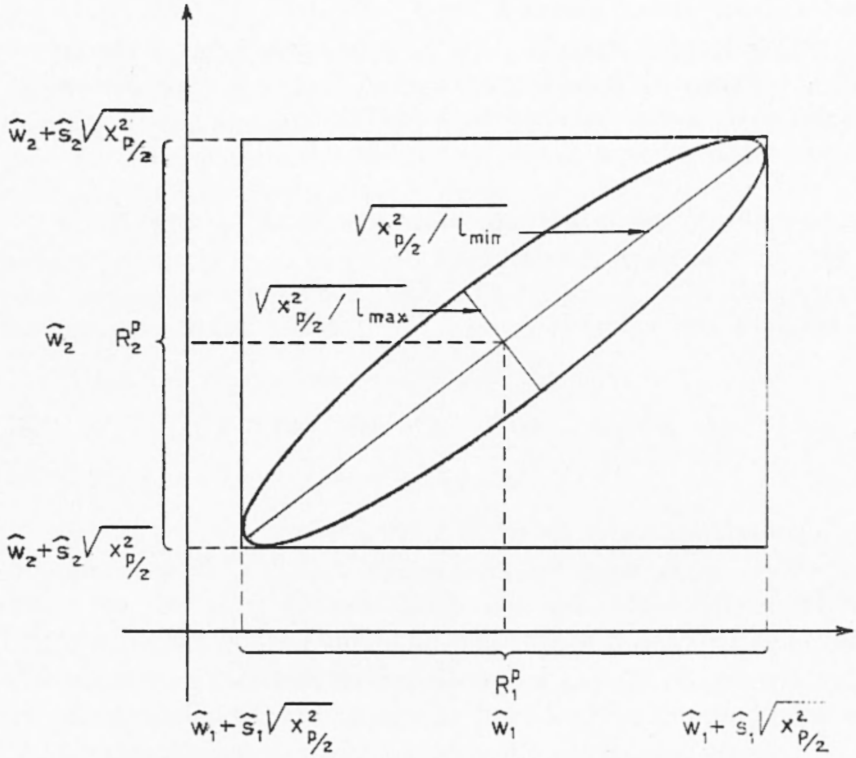

Fig. 1 - Representation of the more relevant geometrical features of an ellipsoid of the form

$$(\mathbf{w} - \hat{\mathbf{w}})^T A (\mathbf{w} - \hat{\mathbf{w}}) \leq X^2_{p/k} \text{ for } k = 2.$$

The ellipsoid [4] gives a confidence region for the whole set of unknown parameters. A confidence region for a reduced set of parameters $\mathbf{w}^*_{ij}$, $j = 1, \ldots, t$ can be built in a quite similar way, considering in place of the whole covariance matrix the matrix with elements $(A^{-1})_{i_r i_s}$, $r, s = 1, \ldots, t$. In particular, the confidence interval at the level p for the i-th component turns out to be

$$S_i^P = \{ \hat{w}_i - \hat{s}_i \sqrt{X^2_{p/1}} \leq w_i \leq \hat{w}_i + \hat{s}_i \sqrt{X^2_{p/1}} \}$$

We remark that S_i^p is smaller than R_i^p, as $X^2_{p/i} < X^2_{p/k}$, and while for the hyperrectangle $\mathbf{R}^p = \{\mathbf{w}: w_i \in R_i^p\}$ the confidence level is greater than p, for the hyperrectangle $S^p = \{\mathbf{w}: w_i \in S_i^p\}$ the confidence level may result lower than p. It can only be proved (Wilks, 1962, p. 291) that, if $q = 1 - k(1 - p)$, then

$$\Pr\{\mathbf{w}^* \in S^p\} = \Pr\{w_i^* \in S_i^p,\ i = 1, \ldots, k\} \geq q$$

For example, for $p = 0.9$, $k = 3$, $\Pr\{\mathbf{w}^* \in S^p\} \geq 0.7$.

As far as assumptions 2 and 3 are concerned, no a priori criterion can be given to ensure their validity, which however can be argued if the residuals are differentiable in $\mathbf{w}^*$ for sufficiently small variances s_i^2. In the case of the hypocentre location problem, the question was discussed by several authors (for example Flinn, 1965; Evernden, 1969; Buland, 1976) for both worldwide and local networks.

They also discussed the related problem of giving the variances s_i^2 of the errors at the stations. In the case that variances can be assumed equal, say to s^2, an estimate of s^2 could be derived directly from the observations by the formula

$$\tilde{s}^2 = \sum_{i=1}^{m} \frac{(t_i - t_o - f_i[\hat{\mathbf{x}}_o])^2}{m - 4}$$

If m is small, as usual for local networks, $\tilde{s}^2$ is not a good estimate of s^2. In such case the F distribution should be used rather than the chi-square distribution in the computation of confidence ellipsoids (Flinn, 1965). However, as pointed out by Evernden, 1969, such confidence regions are much larger than the ones built from variances based on previous experience about the reading errors at the stations.

The various authors generally agree that confidence ellipsoids are a good representation of the uncertainty in the location, at least as far as the epicentre is concerned.

The validity of the probabilistic statement connected to confidence ellipsoids for hypocentres can be tested via Montecarlo simulation, as will be described in the next section for the network of Ancona. Some problems, due to the unboundedness of the ellipsoids for null depth, will be shown to arise when the uncertainty in the hypocentre depth is explicitly considered.

Finally we have to remark that currently available computer programs do not yet treat the statistics in a satisfactory way. HYPO71 (Lee and Lahr, 1972) gives as an output only the standard deviations of the estimates, with the footnote "Statistical interpretation of standard errors involves assumptions which may not be met in earthquake locations. Therefore the standard errors may not represent actual error limits".

The revised version HYPOELLIPSE (Lahr, 1978) gives as an output the lengths and orientation of the axes of the "error ellipsoid", but no probabilistic statement is connected to such "error ellipsoid", thus resulting, in our opinion, of limited use in the interpretation of the results.

Montecarlo simulation for the Ancona network

The accuracy in the hypocentre location was evaluated for the network of Ancona (Ferraris et al., 1975; Crescenti et al., 1977) via Montecarlo simulation. The simulation procedure can be described as follows:

1) a crustal model is assumed for the region and a point in it is taken as simulation hypocentre;

2) according to the model, travel times, from the hypocentre to each station, are computed;

3) a number of simulated reading times r_i are obtained from first arrival times by random perturbations, each drawn from a normal distribution with mean zero and assigned standard deviation;

4) the least squares method is used to estimate the parameters of the simulation hypocentre, considered as unknown, using as r_i as an input;

5) the location error is evaluated comparing the estimated parameters and the simulated ones.

In the simulation performed for the network of Ancona a half space model with a constant velocity of 5 km/sec was assumed. Thus derivatives of the travel time functions could be easily computed at the estimated parameters and confidence ellipsoids for the spatial coordinates built as described in the previous section.

Different simulation hypocentres were placed in different sites of the region in order to represent some really observed situations.

The origin time was assumed to be $t_0 = 0$ without loss of generality.

Fig. 2 shows the distribution of the simulation epicentres: A and D outside the network in the sea, while B and C are inside.

The same standard deviation was assumed for the reading errors. The least squares estimates were computed by the numerical optimization routine OPVM of the Optima Package (Numerical Optimization Centre, 1976). The initial guesses were the smallest reading time for the origin time, the nearest station coordinates for the epicentre and a fixed nonzero depth. Three different values $s = 0.1, 0.05, 0.01$ were selected for the standard deviation. Thus the absolute values of the reading errors e_i were not greater than 0.2, 0.1, 0.02 respectively in about 95% cases and than 0.3, 0.15, 0.03 in about 99% cases. Indeed, from the tables of the normal distribution,

$$\Pr\{-2\,s \leq e_i < 2\,s\} \simeq 0.95$$

$$\Pr\{-3\,s \leq e_i < 3\,s\} \simeq 0.99.$$

Table 1 summarizes the results obtained performing 1 000 trials for each simulation hypocentre:

TABLE 1

RESULTS OBTAINED PERFORMING 1000 TRIALS FOR FOUR SIMULATION HYPOCENTRES AND STANDARD DEVIATIONS s = 0.1, 0.05, 0.01.

	X	Y	D	ID	s	DIV	$\overline{DE}$	$\overline{TE}$	$\overline{A1}$	$\overline{A2}$	$\overline{A3}$	NE	NS1	NS2
A	30.	19.	5.	3.	0.1	0	3.178	0.108	1.356	2.211	$5x10^7$	899	935	996
					0.05	0	1.444	0.031	0.658	1.058	$4x10^5$	972	933	990
					0.01	0	0.279	0.001	0.130	0.207	1.126	990	928	994
B	19.	19.	7.	10.	0.1	0	1.962	0.047	1.471	1.783	$1x10^7$	968	884	975
					0.05	0	0.914	0.004	0.723	0.875	3.497	989	905	995
					0.01	0	0.181	0.001	0.144	0.174	0.690	992	906	995
C	25	15.	4.	6.	0.1	0	1.640	0.027	1.046	1.287	$7x10^7$	996	934	998
					0.05	0	0.764	0.004	0.517	0.635	3.072	997	902	997
					0.01	0	0.148	0.001	0.103	0.126	0.587	992	863	992
D	24.	25.	9.	7.	0.1	3	93.263	18.064	11.890	20.136	$3x10^5$	966	883	969
					0.05	0	1.876	0.063	0.899	1.502	7.330	969	900	976
					0.01	0	0.346	0.004	0.176	0.294	1.385	986	912	994

1) X, Y, D are the epicentre coordinates (referred to the coordinate system of Fig. 2) and the depth of the simulation hypocentre;

   ID is the initial guess for the depth;

   DIV is the number of algorithm divergences;

2) s is the standard deviation of the reading errors;

3) $\overline{DE}$ average distance between the simulation and computed hypocentre;

   $\overline{TE}$ average error in the estimation of origin time;

   $\overline{A1}$, $\overline{A2}$, $\overline{A3}$ average lenghts of the axes of the computed 99% confidence ellipsoid;

4) NE number of 99% confidence ellipsoids containing the simulation hypocentre;

   NS1, NS2 number of hyperrectangles $S^{0.95}$, $S^{0.99}$ (as defined in the previous section) containing the simulation hypocentre.

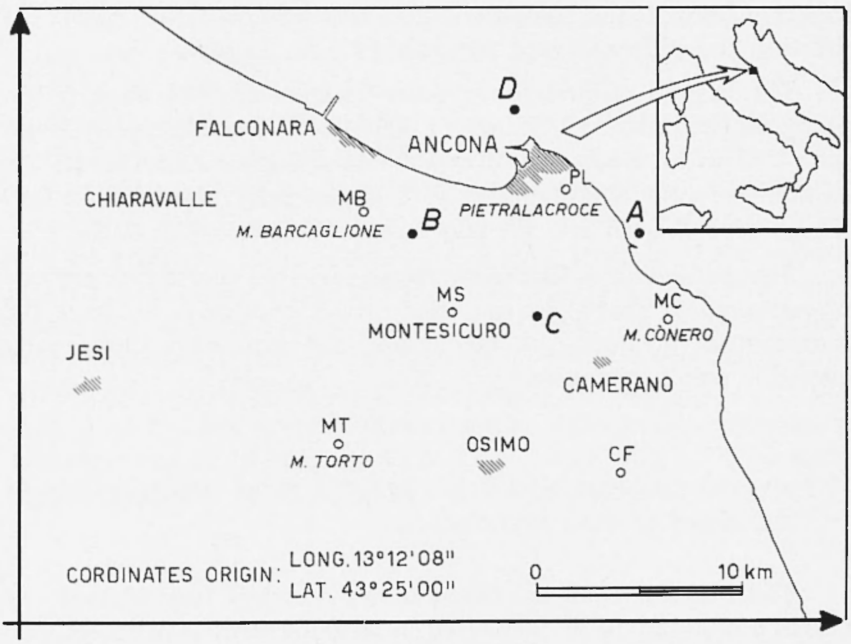

Fig. 2 - The region of Ancona. Black dots indicate simulation epicentres, white dots indicate the stations of the network.

From Table 1 it can be seen that the average error in the location is small in most cases except for the case s = 0.1, for which in 3 trials no minimum of the least squares function could be found by the minimization routine in a neighborhood of the hypocentre.

As far as confidence ellipsoids are concerned, a significant deviation of the computed confidence level NE/1 000 from the expected one 0.99 can be observed for A with s = 0.1, 0.05, for B with s = 0.1, for D with s = 0.1, 0.05.

For the value s = 0.1, in all cases (for s = 0.05 in case A too), a consistent number of ellipsoids, about 10%, turn out to be unbounded along one axis (see $\overline{A3}$ in Table 1) giving no estimation of the error along one direction.

A closer analysis of these cases showed that such anomalous feature was due to the fact that the minimum of the least squares function lays on the surface of null depth, where the matrix J'J defining the ellipsoid (see formula [4]) is singular.

The results quoted for hyperrectangles are not easy to explain: the confidence level NS1/1 000 of $S^{0.95}$ is considerably lower than 0.95 in all cases, but it is not increasing as s decreases; the confidence level NS2/1 000 of $S^{0.99}$ is generally higher than 0.99 except case B with s = 0.1 and case D with s = 0.1, 0.05.

This behaviour is likely due to the fact that confidence regions of rectangular shape do not take into appropriate account the real mutual positions of the actual and estimated parameters even for small variances.

HYPOCENTRE LOCATION AS A MEASUREMENT PROBLEM: FEATURES OF THE MEASUREMENT INSTRUMENT

The measurement problem, in its classical formulation, requires a quantity to be measured, a measurement instrument and in general the possibility of repeating the trials, under the same conditions, any number of times. Let us examine the hypocentre location situation.

Generally, hypocentral coordinates are considered as the quantities to be measured: as far as the instrument is concerned, it is generally viewed as the set of seismometers, or seismographs, equipped with a clock. Let us assume in the regression scheme that the functions f_i are exactly known; then the terms e_i represent the "reading errors" related to fluctuations in the observer-instrument system.

In such case $f_i(\mathbf{x}_0)$ can be viewed as a part of the instrument a part whose response is known and does not give origin to fluctuations. The problem of increasing the precision of the measurement instrument, that is of decreasing confidence regions volume at the same level p, can be solved in two ways: increasing the accuracy of the readings, that is reducing s_i^2 or, at least in principle, increasing the number of the stations. This first claim is obvious; we will discuss now in detail the second one which needs, in our opinion, some mathematical evidence.

Confidence regions obviously depend, in shape and dimensions, also on the number and on the distribution of the seismic stations. Confidence regions should be finite and should get smaller, in some sense, when the number of stations increase; in the limit, they should shrink to a single point.

In this section we list some conditions under which such properties hold for confidence ellipsoids. More details and proof will be given in the appendix. We do not treat here the strictly related problem of the optimality of the distribution of a network. Optimality criteria are discussed in Archetti, Betrò, 1979; Kijko, 1978, who also gives several examples. We assume, for sake of simplicity, that reading errors have equal variances $s_i^2 = s^2$, that is $\Sigma = s^2 I$, I being the identity matrix. $J_m(\mathbf{x})$ will represent the Jacobian matrix of the residuals given by [1] for an hypocentre placed at $\mathbf{x}$. We remark that J_m can be written as

$$J_m = [\bar{J}_m \mid -\mathbf{1}],$$

where $\bar{J}_m$ contains only spatial derivatives and $\mathbf{1}$ is a m-dimensional vector with components 1, and hence doesn't depend on t_0. We can state the following propositions.

Proposition 1: $J^T_m J_m$ is singular if and only if one of the following two situations occurs: a) there $\mathbf{u} \neq \mathbf{0}$ such that $J_m\mathbf{u} = \mathbf{0}$; b) there exists $\mathbf{z} \neq \mathbf{0}$ such that $J_m\mathbf{z} = \mathbf{1}$.

Proposition 2: If $J^T_m J_m$ is not singular for some m, then for any positive k, $J^T_{m+k} J_{m+k}$ is not singular and det $(J^T_{m+k} J_{m+k}) >$ det $(J^T_m J_m)$.

Proposition 3: If $0 \leqslant e^m_1 \leqslant e^m_2 \leqslant e^m_3 \leqslant e^m_4$ are the eigenvalues of

J_m, then the eigenvalues of $J^T_{m+1} J_{m+1}$ are such that

$$0 \leqslant e^m_1 \leqslant e^{m+1}_1 \leqslant e^m_2 \leqslant e^{m+1}_2 \leqslant e^m_3 \leqslant e^{m+1}_3 \leqslant e^m_4 \leqslant e^{m+1}_4 .$$

If $J^T_m J_m$ is not singular then $e_i^m < e_i^{m+1}$ for some $1 \leq i \leq 4$.

Proposition 4: If an infinite sequence of stations is considered, then, if there exists a sequence of groups of p stations (for some $p \geq 4$) such that $J_p^{(k)T} J_p^{(k)}$ doesn't tend to be singular for $k \rightarrow +\infty$ where $J_p^{(k)T} J_p^{(k)}$ represents the Jacobian matrix relative to the p stations of the R-th group, then $e_1^m \rightarrow +\infty$ for $m \rightarrow +\infty$.

Propositions 1-2-3-4 have the following geometrical interpretation:

1) $J^T_m J_m$ singular means that the confidence ellipsoid is unbounded along some direction.

2) As the volume of the ellipsoid is proportional to $1/\sqrt{\det(J^T_m J_m)}$, proposition 2 is equivalent to the geometrical statement "the volume of the ellipsoid decreases".

3) The length of each axis is not increased and, if the ellipsoids are bounded, at least one axis is shortened.

4) When the number of stations increases, if there is a sequence of groups of p stations which doesn't tend to a singular configuration, according to proposition 1, then the ellipsoid shrinks to a single point.

In the simple case when a half space model can be assumed as crustal model, then Proposition 1 has the following interesting geometrical picture. For the half space model $f_i(\mathbf{x}_0) = \frac{1}{v}$ dist $(\mathbf{x}_i, \mathbf{x}_0)$ and hence

$$J_m = \frac{1}{v} \begin{bmatrix} (\mathbf{x}_1 - \mathbf{x}_0)^T \; / \; \text{dist} \; (\mathbf{x}_i, \; \mathbf{x}_0) \\ \\ (\mathbf{x}_m - \mathbf{x}_0)^T \; / \; \text{dist} \; (\mathbf{x}_m, \; \mathbf{x}_0) \end{bmatrix}$$

Then

a) $J_m \, \mathbf{u} = \mathbf{0}$ means

$$\mathbf{x}_i^T \, \mathbf{u} = \mathbf{x}_0^T \, \mathbf{u} \qquad i = 1, \ldots, m.$$

which is the complanarity condition for $\mathbf{x}_i$ and $\mathbf{x}_0$.

This fact explains the difficulties in solving the hypocentre location problem for shallow earthquakes. The complanarity condition is always satisfied if the stations are on a straight line.

b) $J_m \, \mathbf{z} = \mathbf{1}$ means

$$\frac{(\mathbf{x}_i - \mathbf{x}_0)^T \, \mathbf{z}}{\text{dist} \; (\mathbf{x}_i, \; \mathbf{x}_0)} = v \qquad i = 1, \ldots, m.$$

which is the complanarity condition for the points

$$\mathbf{w}_i = \mathbf{x}_0 + (\mathbf{x}_i - \mathbf{x}_0) \; / \; \text{dist} \; (\mathbf{x}_i, \; \mathbf{x}_0) \, .$$

When the points $\mathbf{x}_i$ lie on a plane, $\mathbf{w}_i$ lie on a plane if and only if dist $(\mathbf{x}_i, \mathbf{x}_0)$ = cost., that is the stations are on a circle centered at $\mathbf{x}_0$.

We can therefore come to the conclusion that, as far as the accuracy of the location is concerned, there is a sort of symmetry in the results between the operation of increasing the accuracy of the observations (i.e., increasing signal-to-noise ratio, increasing chart speed or sampling rate, and so on), and the operation of increasing the number of observation points.

We wish to stress that this statement holds in principle only under the above conditions, that is f_i are known exactly. Out of this case, the error in the location is not automatically reduced when the number of the stations is increased; in presence of large travel-times deviations, due to structural anomalies related to the new station ray-paths, an even worse location may be obtained.

This fact can be seen as a "weakness" in the conventional approach: before discussing how it could be overcome, we want to analyze another weakness.

On the probabilistic meaning of confidence regions in the hypocentre location problem

We have introduced confidence regions, in particular confidence ellipsoids, as a probabilistic measure of the accuracy in the hypocentre and origin time estimates. Confidence regions are random, sets, in which the random element depends somehow on the observations. In region (4) the random element is $\hat{\mathbf{w}}$, which is a function of the reading times t_i.

The straightforward interpretation of the probabilistic statement connected to a confidence region at a 100 p % level is as follows:

"If the observations t_i, $i = 1, \ldots, m$ where indipendently performed a certain number n of times, for any set of observations t_i, $i = 1, \ldots, m$ a different confidence region would be obtained, but the 'true' parameters would be contained, when n approaches infinity, in 100 p% of such regions".

It looks very easy to prove that in the traditional approach to the hypocentre location problem such interpretation contrasts with the fact that observations cannot be repeated. If the set of seismographs is to be viewed as the instrument, the measure can be performed only once, that is when the earthquake occurs. It follows that for practical purposes such interpretation of confidence regions, even if correct from a probabilistic point of view, can be of restricted use. Actually in those problems where single hypocentres, fitted up with the accuracy of the estimates, play a relevant role (in some seismotectonic maps, for example), the impossibility of being '100% sure' that the computed region will cover the 'true hypocentre' might invalidate the application of confidence regions, as well as of any other measure of accuracy.

In our opinion the above difficulties arise because of the attempt to use for classifying single events methods and quantities introduced to classify a population of events.

The only way to overcome the contradiction, apart from a radical change in the approach and in the formalism, is to give a qualitatively different meaning to confidence regions, considering not single seismic events, but a whole set of them.

Actually we can find in the literature authors considering simultaneous location of several events (Douglas 1967, Lilwall and Douglas 1970, Veith, 1975).

But in our opinion this approach has not been exploited enough, and these authors rather consider joint location as a tool for refining individual determinations.

We want to propose another interpretation.

Indeed, the probabilistic statement:

$$\Pr \{ \mathbf{w}^* \in \hat{A}(y_1, \ldots, y_m) \} = p$$

where $y_1, \ldots, y_m$ are the observations of $\mathbf{w}^*$, can be read in the following way:

"Assume that a distribution is given to $\mathbf{w}^*$, that is each value of $\mathbf{w}^*$ can be viewed as a realization of a random variable. If n realizations $\mathbf{w}_j^*$ are observed, and the observations $y_j = y(\mathbf{w}_j^*)$, j = 1, ..., m are statistically independent on $\mathbf{w}_j^*$, then, when n approaches infinity, for 100 p % of realizations $\mathbf{w}_j^*$, $\mathbf{w}^*_j \in A\ [y_1\ (\mathbf{w}_i^*), \ldots, y_m\ (\mathbf{w}^*)]$, regardless the distribution of $\mathbf{w}^*$".

The statement follows immediately by the so called total probability formula:

$$\Pr \{ \mathbf{w}^* \in \hat{A}(y_1, \ldots, y_m) \} =$$

$$= E [\Pr \{ \mathbf{w}^* \in \hat{A}(y_1, \ldots, y_m) \mid \mathbf{w}^* \}] = E(p) = p$$

In other words, the proposed interpretation allows to give a judgement on the accuracy of the hypocentre location of a set of events, recorded by the same network, provided that the measurement errors at each station do not depend on each single event. When the number of events increases, the percentage of confidence regions covering the "true" hypocentres approaches 100 p%.

This interpretation looks more useful and more consistent with the initial statements.

More useful because, when we turn to the classification of several rather than single events the influence of negative cases (i.e. confidence regions not covering the "true" hypocentre) does not weaken the entire measure operation, as it could happen classifying single events separately. It looks also more consistent with the proper meaning of the condition "e_i are normally distributed random variables, with s_i^2 variances and zero mean". Even disregarding the actual procedure under which this condition is proved, it's clear that the condition itself can be assumed if, and only if, at each single station, a large number of measurements, that is readings of events, has been globally examined and classified.

Conclusions

The analysis should have focussed some internal limits of the hypocentre location conventional approach, suggesting the following considerations.

— Confidence regions can provide an estimate of the location accuracy, though the probabilistic statement connected with them is seldom clearly defined in most location computer programs, therefore causing difficulties in the interpretation and use of the results.

— If the condition of exact knowledge about travel times is not satisfied, the increase of the accuracy is not automatically accomplished by increasing the number of the station, nor by increasing the precision of the readings.

— As a consequence of the previous points, the problem itself of assessing and increasing the accuracy of the estimates should be considered less dramatically: the difficulties coming from the point-source measurement approach suggest that big efforts in the direction of an increase in the accuracy may provide very poor results.

— A positive solution to the problem can be achieved only changing the approach, that is dropping individual hypocentres as main figures and giving more relevance to joint hypocentres, or hypocentres and travel-times, determination and, more important, to joint accuracy estimations.

There are still some considerations to be made in the frame of the measurement problem.

In the last two sections, we mainly tried to give an answer, in the frame of a measurement problem, to the following questions:

— how to increase the precision of the measurement instrument?

— how to use more correctly the probabilistic statement connected to confidence regions?

In both cases we have pointed out some difficulties which are inside the conventional approach, and which do exist independently from the knowledge of f_i: in both cases we got answers which are satisfactory, but which hold only under the condition that f_i are exactly known. A direct consequence could be the indication that principal efforts are to be made in the direction of increasing the knowledge of f_i, in order to approximate, as close as possible, the condition that they are "exactly known".

The obvious problem is whether this aim is to be pursued independently from the hypocentres location procedures. Though we can find in the literature some works, referring mostly to Douglas, proposing joint hypocentre determination as a method to detect bias in the travel-time functions, the location routines most commonly widespread and used in the ordinary work of most seismologists are normally informed to the following positions:

— each single hypocentre is located individually and it is fitted up with individual accuracy estimates;

— hypocentre location and travel-times determination are considered as separated problems, though the solution of each one of them needs some contribution from the other one (*).

The approach proposed in 1976 by Crosson and by Aki and Lee seems to go further in solving these contradictions. Aki and Lee, when saying:

"In addition to four source parameters (epicenter coordinates, focal depth and origin time), each earthquake event contributes independent observations, as much as the number of observed stations in excess of 4, to the potential data set for determining the earth's structure", point out a definite symmetry between the operation of increasing the number of events and the operation of increasing the number of stations, exceeding 4; that is, is a certain way between the operations of increasing the observations in space and in time.

* It's interesting to observe that in this scheme the measurement procedure of each one of these quantities is usually performed restraining alternatively the other one taken as exactly known, though it's clear that those also come from a measurement operation and are therefore affected by measurement errors.

In our opinion, in such position the measurement instrument has not changed, while the quantity to be measured has turned to be both focal and crustal parameters simultaneously. In this scheme the repetition of the trials is represented both by the stations, exceeding four, and by the seismic events, whose individual relevance is therefore decreased. Some conceptual obstacle could be overcome in this way, like the use of a probabilistic statement connected to confidence regions: the interpretation proposed in the previous section, for instance, could find a more complete application within the development of methods based on such scheme.

It is clear that joint interpretation requires in principle a different formalism and, inside, it, a different assessment and computation of the 'accurary'; moreover, a rather high number of observations is required to obtain good solutions (Crosson and Koyanagi, 1979) (*).

It seems to be the only way, in principle, to build up a more dynamic description of earthquake activity, which could lead to the measure of a quantity, still to be defined (seismic or geodinamic state or something like that), which could provide a more direct and expressive measure of tectonic activity, taking into account focal and crustal parameters together.

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* Such approach, in which it is not compulsory to restrain alternatively focal or crustal parameters, could be, in principle, the only one which allows to follow time variations of the crustal parameters during a swarm or an aftershocks sequence, and, therefore, to re-evaluate critically some observed migrations of the foci during the same periods.

APPENDIX

Proposition 1: We may write

$$J_m^T \; J_m \; = \; \left[\begin{array}{cc} \bar{J}_m^T \; \bar{J}_m & -\bar{J}_m^T \; \mathbf{1} \\ \hline -\mathbf{1}^T \; \bar{J}_m & \mathbf{1}^T \; \mathbf{1} \end{array} \right]$$

If det $(\bar{J}_m^T \bar{J}_m)=0$ then, as $J_m^T J_m$ is semipositive definite, det $(J_m^T J_m)=0$ too. But det $(\bar{J}_m^T \bar{J}_m)=0$ if and only if for some $\mathbf{u}$ $\mathbf{u}^T \bar{J}_m^T \bar{J}_m \mathbf{u}=0$, that is $J_m \mathbf{u}=\mathbf{0}$.

If det $(\bar{J}_m^T \bar{J}_m)\neq 0$ the following equality holds

$$\det (J_m^T J_m) = (\bar{J}_m^T J_m) (\mathbf{1}^T \mathbf{1} - \mathbf{1}^T J_m (\bar{J}_m^T J_m)^{-1} \bar{J}_m^T \mathbf{1});$$

moreover

$$(\mathbf{1}_m^T \mathbf{1} - \mathbf{1}_m^T J_m (\bar{J}_m^T J_m)^{-1} \bar{J}_m^T \mathbf{1}) = (\mathbf{1} - J_m (\bar{J}_m^T J_m)^{-1} J_m^T \mathbf{1})^T \cdot$$

$$(\mathbf{1} - \bar{J}_m (\bar{J}_m^T \bar{J}_m)^{-1} \bar{J}_m^T \mathbf{1})$$

Thus det $(J_m^T J_m) = 0$ if and only if $\mathbf{1} - J_m (\bar{J}_m^T J)^{1} \bar{J}_m^T \mathbf{1} = \mathbf{0}$, that is if and only if there exists $\mathbf{z}$ such that $J_m \mathbf{z} = \mathbf{1}$.

Proposition 2: We can write $J_m^T = [\mathbf{c}_1 \mid \ldots \mid \mathbf{c}_m]$, where $\mathbf{c}_i$ ($\mathbf{c}_i \neq \mathbf{0}$) is the gradient of the residual for the i-th station. Thus

$$J_m^T \, J_m = \sum_{i=1}^{m} \mathbf{c}_i \, \mathbf{c}_m^T$$

and

$$J_{m+1}^T \, J_{m+1} - J_m^T \, J_m + \mathbf{c}_{m+1} \, \mathbf{c}_{m+1}^T \qquad [A1[$$

If $J_m^T J_m$ is not singular then, by (A1), the following equality holds

$$\det(J_{m+1}^T J_{m+1}) = \det(J_m^T J_m)\,(1 + c_{m+1}^T (J_m^T J_m)^{-1} c_{m+1}) \qquad [A2]$$

which implies

$$\det(J_{m+1}^T J_{m+1}) > \det(J_m^T J_m)\cdot$$

Proposition 3: The first part follows from Wilkinson (1965) pp. 95-98. For the second part, we remark that it follows from [A2] and the observation that

$$\det(J_m^T J_m) = e_1^m\, e_2^m\, e_3^m\, e_4^m$$

Proposition 4: By Proposition 3, e_1^m is an increasing sequence, and hence has a limit, finite or infinite, for $m \to +\infty$.
By (A1) we have, for fixed $p \geqslant 4$

$$e_1^m = \inf_{\|x\|=1} [x^T (\sum_{i=1}^{m-p} c_i c_i^T)\, x + x^T (\sum_{i=m-p+1}^{m} c_i c_i^T)\, x] \geqslant$$

$$\geqslant \inf_{\|x\|=1} x^T \sum_{i=1}^{m-p} c_i c_i^T)\, x + \inf_{\|x\|=1} x^T (\sum_{i=m-p+1}^{m} c_i c_i^T)\, x =$$

$$= e_1^{m-p} + e_{min} (\sum_{i=m-p+1}^{m} c_i c_i^T).$$

Should $e_1^m \to a$, then $e_1^{m-p} \to a$ too, and hence $e_{min} (\sum_{i=m-p+1}^{m} c_i c_i^T)$ would tend to be zero, that is $\sum_{i=m-p+1}^{m} c_i c_i^T$ would tend to be singular, which contrasts with the assumption.

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