A generalization of the frequency-magnitude relation in the hypothesis of a maximum regional magnitude

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SUMMARY. — In this work a new non-linear theoretical cumulative frequency-magnitude relation is proposed, that better fits the experimental data in the high magnitude range than the classical Gutenberg-Richter's relation does. The parameters of the relation are β, which is connected with the parameter b of Gutenberg-Richter's relation, and the value of maximum possible regional magnitude M_p.

The relation is very close to the Gutenberg-Richter's one when the value of M_p is very high.

Finally some application examples are given.

RIASSUNTO. — In questo lavoro viene proposta una nuova relazione teorica cumulativa frequenza-magnitudo, che si adatta meglio della relazione classica di Gutenberg e Richter ai dati sperimentali nel campo delle alte magnitudo.

I parametri della relazione sono due: β, che è legato al parametro b della relazione di Gutenberg e Richter, ed il valore della massima magnitudo regionale possibile M_p.

La relazione proposta si riduce a quella di Gutenberg e Richter nel caso particolare di $M_p \to \infty$.

Infine vengono fatti alcuni esempi di applicazione della relazione proposta a dati sperimentali.

INTRODUCTION

The well known empirical linear relation, given by Gutenberg and Richter [1], for the Californian earthquakes, between the earthquake frequency N and the magnitude M,

$$\log N = a - b\, M, \qquad [1]$$

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extensively used in statistical seismology also for cumulative data (2,3), seems not to be in good agreement with experimental data, when these present convex behaviours. This inconvenience is, moreover, often verified (1,11,12).

Various attempts in order to construct a more appropriate expression have produced some relations (8,10) which make more complex the application for risk evaluations, giving a less economical description of the seismicity (9).

The proposed relation

A more general cumulative relation between N and M is therefore proposed, valid in the high magnitude range.

The relation is obtained on the basis of a simple Poissonian model (9); a further hypothesis is introduced, concerning the existence of a maximum possible value of magnitude M_p for each seismic region, according with many seismologists (7,13,14,15).

From the hypotheses it follows that the normalized distribution function is given by:

$$
\begin{aligned}
f(M) &= \frac{\beta}{1 - e^{-\beta(M_p - M_0)}}\, e^{-\beta(M - M_0)} && \text{for } M_0 \leqslant M \leqslant M_p \\
f(M) &= 0 && \text{for } M > M_p,
\end{aligned}
\qquad [2]
$$

while the cumulative form of the distribution is given by:

$$
F(M) = \int_M^{M_p} f(M)\, \mathrm{d}M = \frac{e^{-\beta(M-M_0)} - e^{-\beta(M_p - M_0)}}{1 - e^{-\beta(M_p - M_0)}} \quad \text{for } M_0 \leqslant M \leqslant M_p, \qquad [3]
$$

where M_0 is the threshold value of the statistics, and β is a parameter connected with b of relation [1] by the expression:

$$
\beta = \frac{b}{\log e} \qquad [4]
$$

From the relation [2] it is possible to deduce the following expression for the mean magnitude $\overline{M}$ and for the magnitude variance σ^2, similar to those obtained in the earthquake sequence statistics (6):

$$\bar{M} = M_p + \frac{1}{\beta} + \frac{M_p - M_0}{e^{-\beta(M_p - M_0)} - 1}, \qquad [5]$$

$$\sigma^2 = \frac{1}{\beta^2} - \left(M_0 + \frac{1}{\beta} - \bar{M}\right)^2 \cdot e^{-\beta(M_p - M_0)}. \qquad [6]$$

The expressions [2] and [3] represent the mentioned generalization of [1], to which immediately reduce for $M_p \to \infty$, with $N(M) = a\,F(M)$ where a is the frequency of the earthquakes with magnitude equal to or higher than M_0.

Some examples

Equation [3] agrees better than [1] with the experimental series of large earthquakes, as it can be observed in the eight examples of world-wide seismic regions reported in the figures 1 and 2.

In these the straight line is the best fitting of the relation [1], while the convex line represents the best fitting of the relation [3].

The experimental data used for the application are after Gutenberg and Richter (5).

Other direct consequences of [3] will be exposed in a next group of papers concerning its sistematic application.

Conclusions

The proposed relation seems to be useful for a more accurate statistical interpretation of the seismic events, giving the possibility to compute without any ambiguity (1) the value of the parameters involved in the frequency-magnitude relation.

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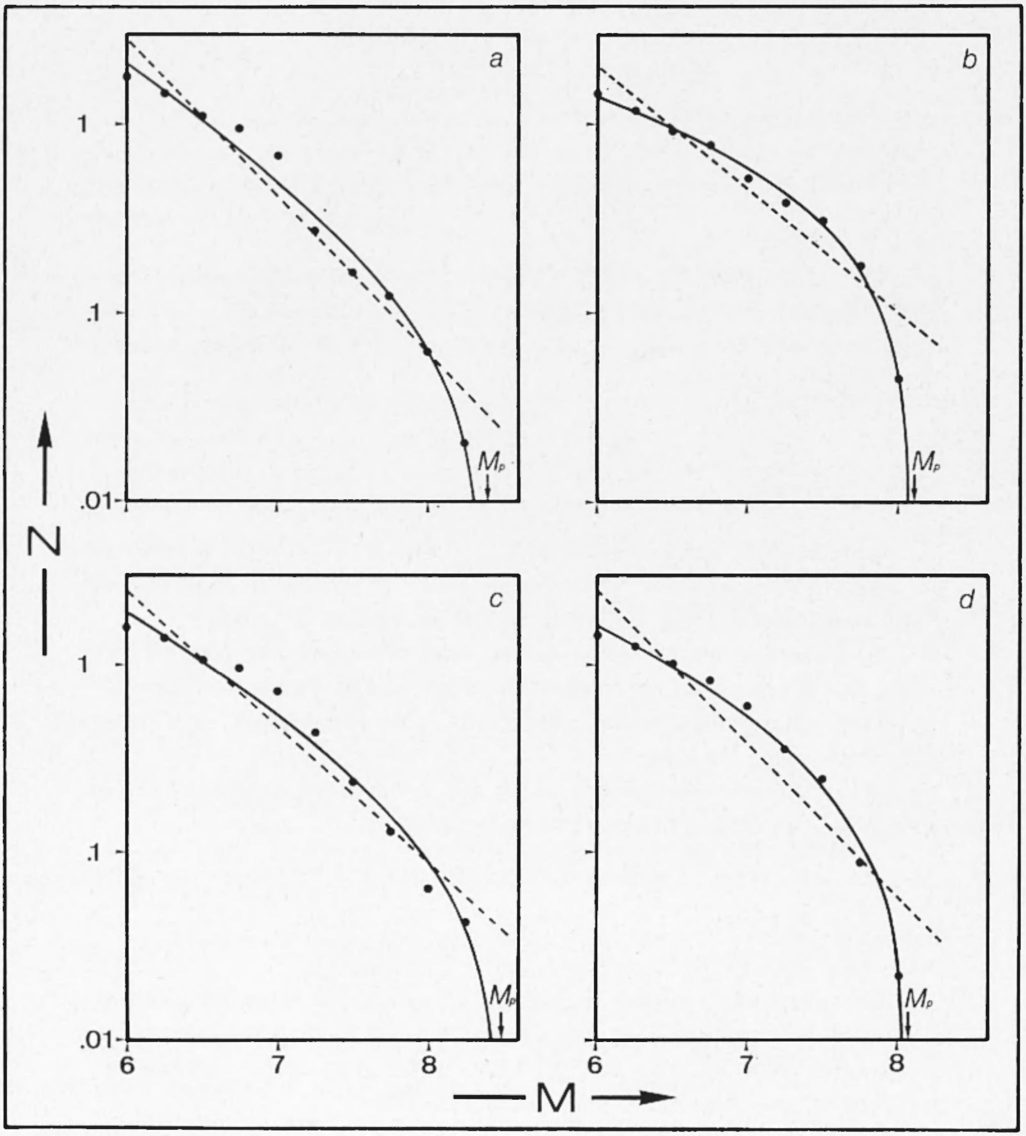

Fig. 1 – Examples of least squares best fitting of the relations [1] — hatched lines — and [3] — full lines — to the experimental data referring to four world-wide seismic regions characterized by subduction of oceanic crust. *a*) Aleutian Arc, *b*) Mexico, *c*) Philippines, *d*) New Guinea. Data after Gutenberg and Richter (1965).

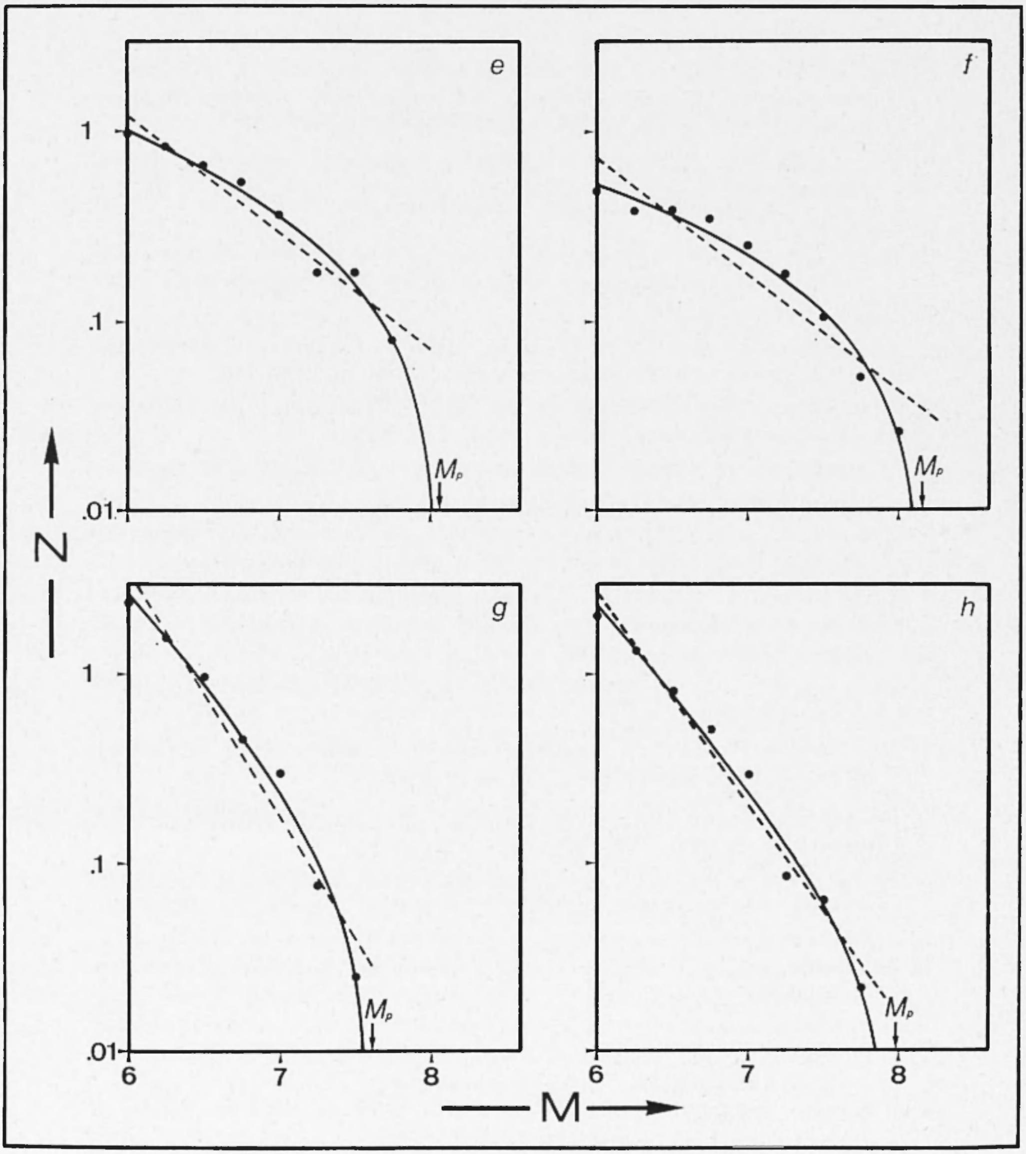

Fig. 2 – Examples of least squares best fitting of the relations [1] — hatched lines — and [3] — full lines — to the experimental data referring to four world-wide seismic regions. *e*) New Zealand and *f*) Burma Arc are characterized by the presence of continental crust, while *g*) Mid-Atlantic Ridge and *h*) Indian Ocean are characterized by oceanic rises. Data after Gutenberg and Richter (1965).

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