The detection of *PKIIKP* and damping in the inner core

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SUMMARY - The phase *PKIIKP*, reflected once from the lower surface of the inner core boundary, was most probably recorded by *LASA* from the Faultless underground nuclear explosion at $10^0.9$ angular distance. The array beam detected a *P* onset, with the correct slowness, within 2 sees of the theoretical travel time. The ground amplitude is 3 ± 1 millimicrons at a dominant period of 1.5 sec. By comparison, the *PKiKP* ground amplitude from Faultless is 75 millimicrons at a dominant period of 1.1 sec.

This detection again confirms the sharpness of the inner core boundary and the 1968 core travel times for *PKiKP*. The additional attenuation of *PKIIKP* over *PKiKP* yields, from direct and spectral measurements, $\bar{Q}=450 \pm 100$ for *P* waves in the inner core. This value sharpens the earlier estimates of *I.* Sacks, G. Buchbinder, A. Qamar, and A. Eisenberg and is almost independent of core structure and rigidity.

On the assumption of a constant velocity in the inner core, the observed differential *PKIIKP* minus *PKiKP* travel time requires an average *P* velocity of 11.4 ± 0.02 km/sec for an inner core radius of 1216 km.

RIASSUNTO - La fase *PKIIKP*, riflessa una volta dalla superficie più bassa del nucleo interno, è stata registrata molto probabilmente dal *LASA* mediante l'esplosione sotterranea nucleare di Faultless, alla distanza angolare di $10^0 9'$.

L'apparato ha sorpreso l'inizio di una *P*, con il ritardo corretto entro 2 sec del tempo teorico di tragitto.

L'ampiezza al suolo è 3 ± 1 millimicrom al periodo dominante di 1.5 sec. Dal confronto, l'ampiezza al suolo delle *PKiKP* da Faultless è di 75 millimicron al periodo dominante di 1.1 sec.

Tale scoperta conferma ancora una volta la precisione del limite del nucleo e dei tempi di tragitto per il nucleo, del 1968, per le *PKiKP*. L'attenuazione addizionale delle *PKIIKP* sulle *PKiKP* produce, dalle misurazioni dirette e

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spettrali, una $\bar{Q} = 450 \pm 100$ per le onde *P* nel nucleo interno. Questo valore precisa le valutazioni più approssimate di I. Sacks, G. Buchbinder, A. Qamar e A. Eisenberg, ed è per lo più indipendente dalla struttura del nucleo e dalla rigidità.

Con l'assunzione di una velocità costante nel nucleo interno, il tempo di tragitto differenziale osservato *PKIIKP* meno *PKiKP* richiede una velocità media *P* di 11.14 ± 0.02 km/sec per un nucleo interno di raggio pari a 1216 km.

1. Previous Work.

This paper describes some inferences that follow from the detection of the weak core phase *PKIIKP*. This longitudinal phase is reflected once on the inside surface of the inner core boundary (see Figure 1). On the assumption that my identification is correct, the amplitude measurements, when compared with those of the allied phase *PKiKP*, provide a rather precise estimate of the average velocity and attenuation parameter *Q* for longitudinal waves in the inner core. It seems particularly appropriate to present these results in this special publication because Professor Caloi has over the years published important results on seismic core weves (e. g., Caloi 1961); his observations have cften stimulated the present writer.

After it had become clear that the inner core boundary (*ICB*) was a sharp interface (see Figure 1) from which short-period reflections *PKiKP* were commonly observed, Bolt and O'Neill (1965), made a theoretical study of the relative amplitudes of the two least complicated reflections, *PKiKP* and *PKIIKP*, at the *ICB*. With conventional seismographs in mind, the calculations were « not favorable to the recording of the phase *PKIIKP* ». At $330^0 < \Delta < 360^0$, for example, the amplitude of this once-reflected phase was predicted to be only about 3 per cent of the amplitude of the direct refracted core phase *PKIKP* that had the corresponding ray parameter ($170^0 < \Delta < 180^0$) even in the case of the most favorable plausible physical contrast at *ICB*. This theoretical ray calculation did show, however, that amplitudes of *PKIIKP* might be expected to be of essentially the same order of those of *PKiKP* at the same epicentral distance *for perfectly elastic conditions with no damping.*

(*) Referred to as Paper I below.

Specifically, for $0^0 < \Delta < 20^0$, the amplitude ratio of *PKIIKP* to *PKiKP* was calculated to be about 0.5 (Paper I, Tables 3 and 4).

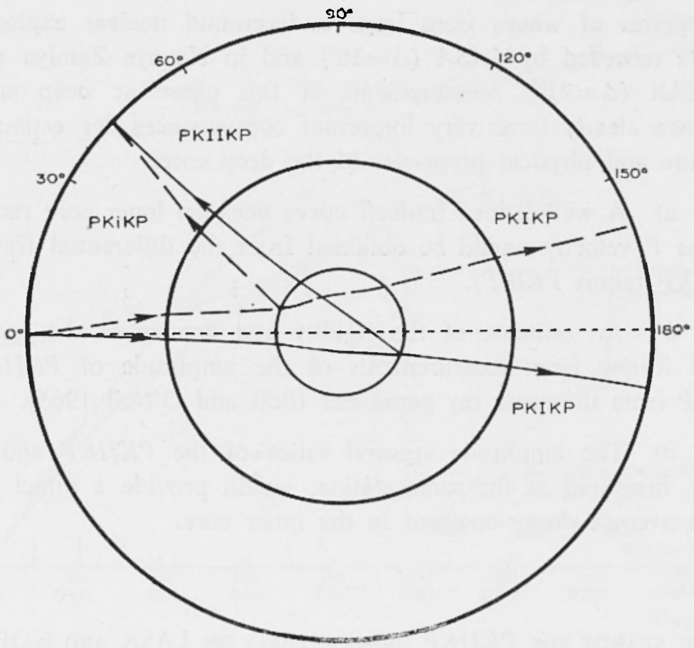

Fig. 1. Diagram showing the paths of the rays for *PKiKP* and *PKIIKP* used in this study. The epicentral distance is taken at about 45^0 for clarity.

At the time, these results made further work with single component seismographs unpromising, but when recordings from the large aperture arrays became available, the search for *PKIIKP* was taken up again. The previous theoretical results had shown (see Tables 3 and 4, Paper I) that the optimum window in the travel-time distance diagram for detection of both *PKIIKP* and *PKiKP* on the same record occurred at epicentral distances of $\Delta < 20^0$. An important step forward was taken with the discovery of the clear recording of *PKiKP* and *PcP* by the *LASA* array from the underground nuclear explosion Faultless in Nevada on January 19, 1968 (Bolt and Qamar, 1970). The epicentral distance was only $\Delta = 10^0.9$ so that the downgoing core phases reflected to *LASA* emerged at almost vertical angles. The amplitude ratio of these two phases was used to obtain a well-constrained upper bound to the density jump at the *ICB*.

Subsequent to the above work, a search for *PKIIKP* was made on seismograms of waves from large underground nuclear explosions in Nevada recorded by *LASA* ($\Delta \simeq 10^0$) and in Novaya Zemlya recorded NORSAR ($\Delta \simeq 21^0$). Measurements of this phase at steep angles of incidence clearly have very important consequences for estimation of structure and physical properties of the deep core:

a) A well-defined tradeoff curve between inner core radius and average *P* velocity would be obtained from the differential travel time (*PKIIKP* minus *PKiKP*).

b) An estimate of the rigidity and density of the inner core would follow from measurements of the amplitude of *PKIIKP* and *PKIKP* from the same ray parameter (Bolt and O'Neil 1965).

c) The amplitude spectral ratios of the *PKIIKP* and *PKiKP* waves, measured at the same station, would provide a direct measure of the average decay constant in the inner core.

2. The search for *PKIIKP* observations on LASA and NORSAR.

Theoretical travel times for *PKIIKP* waves were published in 1964 (Paper I, Table 2) based upon the 1962 times for *PKIKP* waves calculated by the present writer (Bolt, (1962, 1968)). These in turn were calibrated against the standard *P* travel times of Jeffreys and Bullen. A travel time diagram for *PcP, PKiKP, PKIKP* and *PKIIKP* is shown in Figure 2 with core times based on the above tables. The corresponding predicted time for $10^0.9$ is given in column 2 of Table 1. This predicted time provided a search window which was used to scan the LASA vertical-component records from Faultless (origin-time $18^h\ 15^m\ 00.0^s$).

Through the active help of Dr. J. Filson at Lincoln Laboratory, various frequency pass bands were used in the search, together with velocity filtering appropriate for *PKIIKP* at $10^0.9$ and an azimuth of wave approach of 50^0. The slowness of *PKIIKP* at 10^0 is only about 0.2 sec per degree so that the LASA beam was formed for apparent velocities from 900 km/sec to 200 km/sec. Also, the signals of various sub-arrays

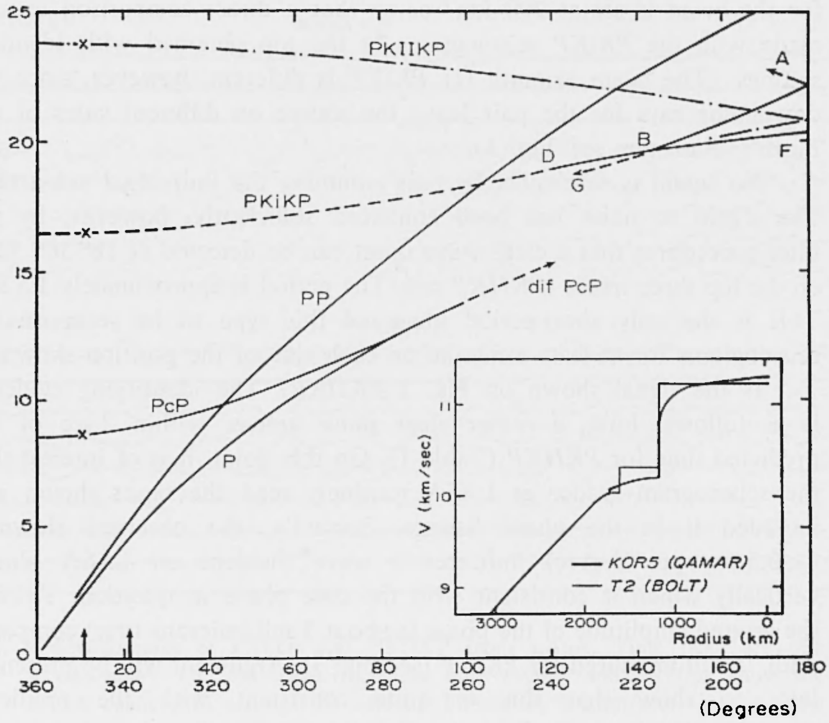

Fig. 2. Two recent solutions for the velocity distribution in the core and the travel times of the phases *PKiKP* and *PKIIKP* consistent with the Jeffreys-Bullen tables and the Bolt, 1968 times for core phases. The arrows denote the distances at which LASA and NORSAR records were searched.

were summed with and without filtering. An example of the resulting seismograms is reproduced in Figure 3. In this case the filter velocity

TABLE 1

Travel Times for Distance Δ^0

Δ	Predicted (Bolt and O'Neill)	Predicted (Massé *et al*)	Observed (*PKIIKP*)	Observed (*PKiKP*)	Observed (*PKIIKP-PcP*)
$10^0.9$	23 50.0	23 58.5	23 51.9	16 35.6	15 17.7
$63^0.0$	23 21.0	23 32.6	23 22 (approx)	—	12 31

for the beam is set at 200 km/sec so that a direct comparison can be made wtih the *PKiKP* seismograms at the top obtained with identical settings. (The beam azimuth for *PKiKP* is different, however, since the downgoing rays for the pair leave the source on different sides of the Earth's diameter, see Fig. 1).

No signal is resolvable by only summing the individual sub-arrays. The signal to noise has been enhanced sufficiently, however, by the filter procedures that a clear wave onset can be detected at $18^h 38^m 52.0^s$ on the top three traces (*PKIIKP* set). The period is approximately 1.5 sec. This is the only short-period phase of this type to be seem on the seismograms for at least a minute cn each side of the position shown.

Is the signal shown on Fig. 3 *PKIIKP*? The identifying evidence is as follows. First, a rather clear pulse arrives within 2 sec of the predicted time for *PKIIKP* (Table 1). On this point, it is of interest that the seismogram reader at LASA routinely read the onset shown and included it in the phase listings. Secondly, the observed slowness (<0.5 sec per degree) indicates a wave incident on LASA almost vertically which is consistent with the core phase in question. Thirdly, the ground amplitude of the phase is about 3 millimicrons (mμ) compared with 75 mμ measured for *PKiKP* (see Fig. 3). Argument will be presented later to show that this is quite consistent with the predicted *PKIIKP*/*PKiKP* amplitude ratio. Fourthly, the observed period of the phase is a little greater (1.5 sec) than that of *PKiKP* (1.1 sec) using the same seismometers and signal enhancement. This difference follows if the *PKIIP* wave has been damped in the ray path not shared with *PKiKP* (see Fig. 1).

In my judgment, this evidence is fairly conclusive that the weak *PKIIKP* wave has been detected. Some indirect support for this conclusion comes from the findings of Massé *et al* (1974). Quite independent research on this phase was carried out from 1971 by these authors using LASA recordings. From the search of seismograms of many events they found one case only that might fit the test criteria for *PKIIKP*. This event was the underground nuclear explosion at Novaya Zemlya on 27 October, 1973 (origin-time $07^h 00^m 09^s$). The distance to LASA is $\Delta = 63^0.0$, much greater than in the case of Faultless. The travel time of the phase identified as *PKIIKP* is given in Table 1; it happens to agree within one second with the predicted time of Paper I. This result was published after the announcement in early 1974 of the probable detection of the phase from Faultless (Bolt, 1974).

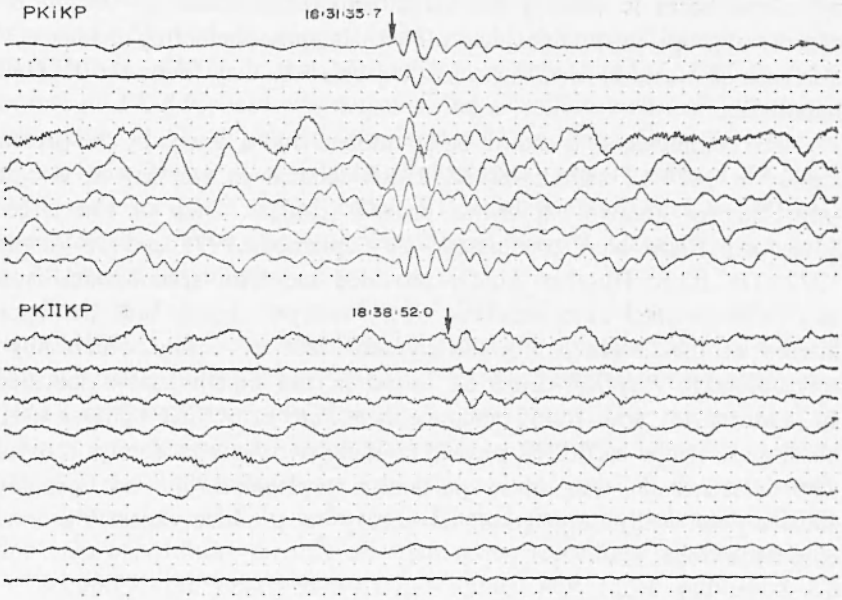

Fig. 3. Seismograms of *PKiKP* and *PKIIKP* from LASA. Time ticks are one second apart. The top three traces for both *PKiKP* and *PKIIKP* show the LASA beam with settings (*a*) velocity filtered at 200 km/sec, (*b*) the same but frequency filtered below 0.7 Hz, and (*c*) the same but frequency filtered below 0.4 Hz. The remaining traces are signals from various subarrays. The beam azimuths are 230^0 and 50^0, respectively.

Unfortunately, the evidence for the identification of the phase from Novaya Zemlya is not strong. Theoretically, the expected amplitude of *PKIIKP* at 63^0 is about half that expected at 10^0. Secondly, at $\Delta = 63^0$ there is some signal contamination possible from such multiply-reflected phases as *pSS*, although the explosive source makes this relatively unimportant. Thirdly, the seismogram from the LASA beam published by Massé *et al* does not clearly demark a separate phase for the *PKIIKP* energy arrival and « a precise onset time cannot be picked for the phase ». The maximum amplitude at the time selected is about 6 mμ compared with background noise of about 2.5 mμ. Finally, no *PKiKP* phase was detectable from this event, although both theory and the Faultless observations entail a more energetic *PKiKP* signal than *PKIIKP*.

Observations supporting identification as *PKIIKP* are the close agreement with predicted travel time and enhancement of the signal when the beam is velocity filtered at small slownesses ($\sim$0.8 sec per degree) appropriate to this phase. There is thus conflicting evidence in this case as Massé *et al* make clear. On balance, they favor the *PKIIKP* hypothesis.

For completeness it should be mentioned that a search by the present author in 1973 of signals on NORSAR located in Norway at $\Delta = 21^0$ from Novaya Zemlya explosions proved fruitless. Two of the largest Soviet explosions were considered: 27 September 1971 and 28 August 1972. Dr. E. S. Husebye kindly provided me with seismograms from NORSAR beamed at a slowness of 0.5 sec per degree and band-pass filtered at 0.8-2.8 Hz. Unfortunately, no trace of onsets which might correspond to *PKIIKP* could be found in the time window specified. In addition, it was found that neither *PcP* or *PKiKP* phases were observable on the NORSAR records of these two Novaya Zemlya sources. The inference is that these particular explosions did not generate sufficient vertically propagating *P* energy to produce detectable inner core reflections.

3. The Average *P* Velocity in the Inner Core.

The observed travel time difference, *PKIIKP* minus *PKiKP*, for Faultless to LASA was 436.3 sec. As suggested by Fig. 1, for $\Delta = 10^0.9$ these phases have almost identical ray paths along the Earth's diameter. Thus, if v is the mean *P* velocity in the inner core of radius r,

$$v = \frac{4r}{436.3} \text{ km/sec.} \qquad [1]$$

On the interpretation of this study, [1] gives a strong trade-off relation between mean velocity and radius. For a radius $r = 1216$ km (Bolt, (1972)), we find $v = 11.14$ km/sec. Because the *II* portion of the ray paths in the inner core diverges somewhat from the diametral ray, [1] should contain a factor like cos θ, where θ is the apparent angle of incidence upon the inside of the ICB. Even for $\theta = 5^0$, an extreme value, v is reduced by only 0.02 km/sec, which is probably also the range of error from uncertainty in reading onset times and ellipticity effects.

Massé *et al* (1974) also estimated the average *P* velocity in the inner core from their measurement of the arrival times of the phases they identified as *PKIIKP* and *PcP*. They obtained 11.0+0.05 km/sec. Because no *PKiKP* phase was detected, the dependence on *PcP* introduces assumptions on the *P* velocity distribution in the outer core. To assist in this analysis, Massé *et al* calculated theoretical travel times of *PKIIKP* based upon an Earth model called B1 (Jordan and Anderson, (1974). Unfortunately, this comparison introduces internal inconsistencies into the inferential arguments.

First, in Table 1 the predicted *PKIIKP* times of Massé *et al* are compared with the observed times of the two phases that have been identified independently as *PKIIKP*. For the Faultless observation the residual is 6.6 sec; for the Novaya Zemlya observation the residual is 10.6 sec. In other words, the table given by Massé *et al* agrees neither with their own observation or with the present one. In contrast, both observations agree closely with the predicted times by Bolt and O'Neill which are based more directly on seismogram recordings of core phases.

Secondly, the observed *PKIIKP* minus *PcP* times published by Massé *et al* (see Table 1) seems unaccountably long compared with the theoretical value, $12^{m}\,13.3^{s}$, predicted by the observed *PKIIKP* travel time and the Jeffreys-Bullen value for *PcP*. Even allowing for adjustments of two or three seconds in the latter, a residual of ±15 sec results. The same calculation for the Faultless time yields a residual of +6 sec which is more easily explicable.

Until these discrepancies are resolved, it seems better to use the *PKIIKP* times in Paper 1 and place a larger uncertainty than indicated on the estimate of average velocity in the inner core given by Massé *et al*.

4. The *Q* of the Inner Core for *P* Waves.

The measurement of relative period of the *PKiKP* and *PKIIKP* observations can be used to yield an independent estimate of $\bar{Q}$ in the inner core. As Figure 3 shows, the *PKIIKP* phase has a longer period than the *PKiKP* phase on the corresponding upper three beams. On the corresponding second traces (both filtered below 0.7 Hz) the *PKIIKP* wave period is 1.5 sec compared with 1.1 sec for *PKiKP*. This spectral shift suggests an effect from attenuation of higher frequency components in the inner core.

Let the amplitude ratio of *PKIIKP* to *PKiKP* be R. Then, to the first order in frequency f (in Hz),

$$ln\, R\,(f) = A - \frac{\pi t}{2Q} f, \qquad [2]$$

where the constant A is a function of the geometrical spreading and reflection coefficients of the *PKIIKP* and *PKiKP* waves at the inner core boundary, and t is the total travel time of *PKIIKP* in the inner core.

The estimation of $\bar{Q}$ from [2] has been approached in two ways. First, a point estimate can be obtained for a particular frequency so long as a theoretical value of A is adopted. Put $f = 1/1.2$ Hz, $t = 436.3$ sec, $R\,(f) = 3/75$. Also a value for A can be estimated for a plausible core model from the calculations of Paper I. The term A has two factors, one the geometrical spreading and the other the wave reflection ratios (see Tables 3 and 4 in Paper I). For all cases considered, $A \simeq 0.5$. Substitution in [2] then gives $\bar{Q} = 440$. Uncertainties can be assessed by varying the various parameters in [2] through their allowable ranges. This sensitivity analysis indicates $350 < Q < 600$, approximately.

The second approach is to Fourier analyze the *PKIIKP* and *PKiKP* wave packets on the first traces in Fig. 2, and form the ratio. If the logarithm of this ratio is plotted against f, then [2] shows that an approximate straight line can be expected with slope proportional to $1/\bar{Q}$. This spectral analysis yields the curve in Fig. 4. For the LASA instrumentation and processing, the band of significant frequencies passed is about $0.5 < f < 1.2$ Hz. In this range, there is a decrease in the $ln\, R\,(f)$ values with some fluctuations. For comparison, two theoretical lines for $Q = 350$ and 550 are drawn. The suggestion is that a value of $\bar{Q} = 450$ would summarize the gradient adequately.

Overall, a value $\bar{Q} = 450 \pm 100$ appears reasonable. This is a very low value and implies a high rate of damping in the inner core - in striking contrast to the high Q values ($Q > 5000$) found for high frequency P waves in the outer core (Bolt and Qamar, 1970).

The above estimate is in agreement with other recent assessments of high attenuation in the inner core. Buchbinder, (1971) obtained $\bar{Q} = 400$ and Sacks, (1971) inferred $Q = 170$ at the outside of the

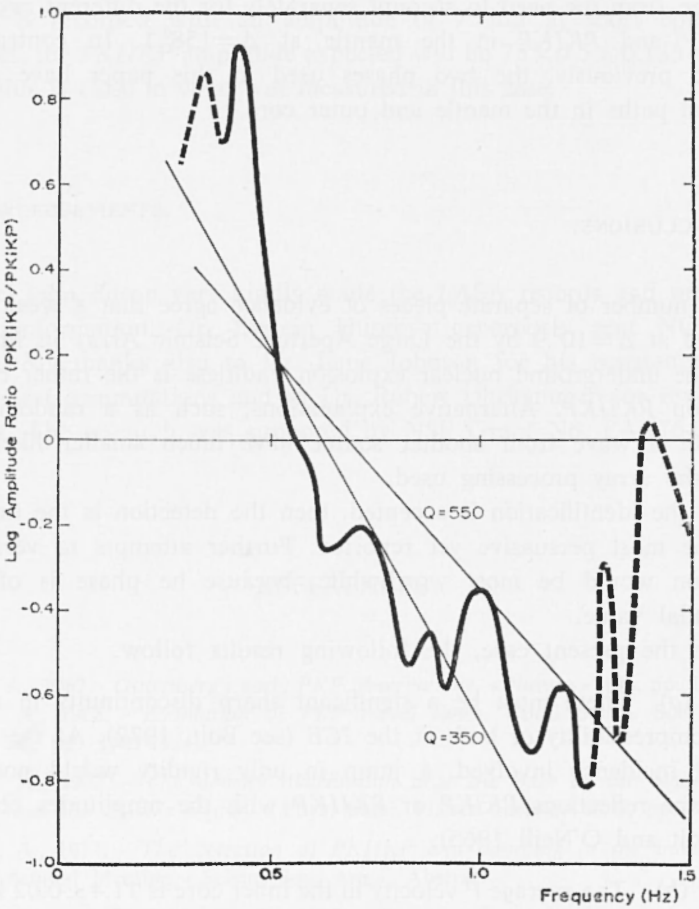

Fig. 4. The spectral amplitude ratio of *PKIIKP* to *PKiKP* from the LASA recordings of the Faultless explosion. The gradients of the straight lines corresponds to Q values of 350 and 550. The dashed lines indicate spectral values outside the signal detection pass-band.

inner core rising to $\bar{Q}=600$ towards the center. These results were obtained using refracted *PKP* waves. Recordings of two branches of these waves (*PKP* and *PKIKP*) were used by Qamar and Eisenberg, (1973) to form a log spectral ratio and estimate Q from the gradient as above. The spectrum indicated that in the outer part of the inner core $120<\bar{Q}<400$, approximately. The precision of the method suffers, however, from the need to account separately for the different ray paths of *PKP* and *PKIKP* in the mantle at $\Delta = 158^0.1$. In contrast, as stressed previously, the two phases used in this paper have almost identical paths in the mantle and outer core.

5. Conclusions.

A number of separate pieces of evidence agree that a weak phase detected at $\Delta = 10^0.9$ by the Large Aperture Seismic Array in Montana from the underground nuclear explosion Faultless is the rather esoteric reflection *PKIIKP*. Alternative explanations, such as a random noise pulse or a wave from another source have much smaller likelihood, given the array processing used.

If the identification is accepted, then the detection is the first and also the most persuasive yet reported. Further attempts to verify the detection would be most worthwhile, because he phase is of great inferential value.

In the present case, the following results follow.

(*a*) There must be a significant sharp discontinuity in density or incompressibility or both at the *ICB* (see Bolt, 1972). At the almost normal incidence involved, a jump in only rigidity would not yield either the reflections *PKiKP* or *PKIIKP* with the amplitudes observed (see Bolt and O'Neill 1965).

(*b*) The average *P* velocity in the inner core is 11.4 ± 0.02 km/sec for a radius of 1216 km.

(*c*) The average attenuation parameter *Q* in the inner core is 450 ± 100. This low value suggests viscosity of a material near the melting point (Gans, 1972).

The recorded amplitudes of *PKIIKP* and *PKiKP* at LASA from Faultless are in agreement with values earlier predicted for plausible Earth models. From Tables 3 and 4 of Paper I the surface amplitude ratio of $PKIIKP/PKiKP \simeq 0.5$. The spectral decay (Fig. 4) estimated by others independently gives a *Q* value of approximately 450. But *PKIIKP* travels about 4864 km. Thus the amplitude will decrease by a factor exp $(-\pi \times 0.7 \times 4864/11.1 \times 450) \simeq 0.135$. It follows that if

PKiKP is recorded with an amplitude of 75 mμ at short epicentral distances, the *PKIIKP* amplitude expected will be $75 \times 0.5 \times 0.135 = 5 \text{m}\mu$. This value is close to what was measured in this case.

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REFERENCES

BOLT, B. A., 1962. - *Gutenberg's early PKP observations.* « Nature » **196**, pp. 122-124.

BOLT, B. A., 1968. - *Estimation of PKP travel times.* « Bull Seism. Soc. Am », **58**, pp. 1305-1324.

BOLT, B. A., 1972. - *The density distribution near the base of the mantle and near the Earth's center.* « Phys. Earth Planet. Interiors », **5**, pp. 301-311.

BOLT, B. A., 1974. - *The detection of PKIIKP and damping in the inner core.* Annual Meeting « Seism. Soc. Am.», Abstract.

BOLT, B. A., and O'NEILL, M. E., 1965. - *Times and amplitudes of the phases PKiKP and PKIIKP.* « Geophys. J. R. astr. Soc. », **9**, pp. 223-231.

BOLT, B. A., and QAMAR, A., 1970. - *An upper bound to the density jump at the boundary of the Earth's inner core.* « Nature », **228**, pp. 148-150.

BUCHBINDER, G. G. R., 1971. - *A velocity structure in the Earth's core.* « Bull. Seism. Soc. Am. », **61**, pp. 429-456.

CALOI, P., 1961. - *Seismic waves from the outer and inner core.* « Geophys. J. R. astr. Soc. », **3**, pp. 139-150.

JORDAN, T. H., and ANDERSON, D. L., 1974. - *Earth structure from free oscillations and travel times.* « Geophys. J. R. astr. Soc. », **36**, pp. 411-460.

GANS, R. F., 1972. - *Viscosity of the Earth's core.* « J. Geophys. Res. », **77**, pp. 360-366.

MASSE, R. P., FLINN, E. A., SEGGELKE, R. M. and ENGDAHL, E. R., 1974. - *PKIIKP and the average velocity of the inner core.* « Geophys. Res. Letters », **1**, pp. 39-42.

SACKS, I. S., 1971. - *Anelasticity of the inner core.* Carnegie Inst. Annual Rep. of Dir.. Dept. of Terres. Mag., 1969-1970, p. 416.

QAMAR, A., and EISENBERG, A., 1974. - *The damping of core waves.* « J. Geophys. Res. », **79**, pp. 758-765.