Velocity and direction of plate displacements by latitude observations

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Riassunto. — Con riferimento ad un precedente lavoro (6) con il quale sono state dedotte le velocità di scorrimento dei blocchi America–Aurasia ed America–Pacifico, secondo la ricostruzione di Le Pichon, utilizzando la lunga serie (70 anni) di osservazioni di latitudine delle Stazioni del SIL, gli autori prendono in esame il problema dell'effetto che la indeterminazione sulla conoscenza delle coordinate dei poli di rotazione dei blocchi, determina sul calcolo della velocità di scorrimento.

Facendo ricorso ad un procedimento di ottimizzazione dei dati vengono determinate poi le velocità di scorrimento e le coordinate del polo di rotazione dei blocchi America-Eurasia. I risultati si accordano con quelli in precedenza determinati (6) e risultano nello stesso ordine di grandezza di quelli dedotti da rilevamenti geologici e geofisici.

Summary. — The results obtained by the authors in a foregoing paper (6), providing completely independent evidence on the plate tectonics hypothesis, are discussed. How critical is the choice of the rotational pole position of the plates is carefully analysed.

The location of the centre of rotation and the rate of rotation of America-Eurasia plate has been successively obtained by the last-square fit method. These results are in good agreement with geophysical and geological measurements and confirm the possibility of using astronomical data in the study of plate tectonics.

Introduction

Using Le Pichon's (2) reconstruction, the A. A. (6) computed the rotation rate of the Eurasian–American and Pacific–American

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plates from astronomical latitudes observed in the five international astronomical stations of Mizusawa (Japan), Kitab (USSR), Carloforte (Italy), Gaithersburg and Ukiah (USA) situated on the $+39°8'$ parallel. The data analysed cover a period of 70 years. From this analysis it results that in general, disregarding local distorsions which undoubtedly occur, the Kitab and Carloforte stations can be considered as lying on the Eurasian plate. Mizusawa also lies on this plate, although close to the boundary. The fourth and fifth stations, Gaithersburg and Ukiah, must be on different plates. Gaithersburg lies near the center of the American plate and Ukiah must be considered as moving with the East Pacific plate. This plate is almost entirely consumed by the American plate and only a small part remains, whose motion is essentially north-west.

To determine the value of the rotation rate ω_k of different plates (k) by the secular variations of the latitude b_i of the station (i) as well as the secular motion of the Earth's pole (u), condition equations of the type

$$b_{(j)} = u \cos (\lambda_j - v) - \omega_k \cos \Phi_k \sin (L_k - \lambda_j) \qquad [1]$$

were used, in which the first term of the second member represents the effect of secular polar motion on observed latitude variations, while the second term represents the effect of the rotation of the station (j) of longitude λ_j around the rotation pole (k) of coordinates L_k and Φ_k.

The resulting absolute rates of the different plates, averaged on the basis of various cases treated (6) are found to be

$-0''.0027$/year $\pm 0''.0002$	(Eurasian plate)
$+0''.0015$/year $\pm 0''.0002$	(American plate)
$+0''.0018$/year $\pm 0''.0002$	(NE Pacific plate)

These results are in excellent agreement with the plate tectonics theory and show that long-term variations in the motion of the stations can be found by careful analysis of very long series of astronomical data.

INTERNAL AND EXTERNAL ERRORS BY DETERMINING POLE AND ANGULAR RATES

The polar coordinates Φ_k and L_k, which are obtained by determining the relative motions of different plates, are generally affected by observation errors which have repercussions on the determination of the absolute rates calculated by means of relation [1].

The polar coordinates of different plates are usually established directly from spreading rates and fracture zone azimuths (azimuths of the transform faults). In particular, the positions of the center of rotation adopted by Le Pichon (2) are obtained from the azimuths of the fracture zones. The standard deviations of measured from computed azimuths given by Le Pichon (2) are $\pm 5°.7$ for the American–Pacific pole and $\pm 9°.1$ for the American–Eurasian pole. These quantities only indirectly give an idea of the imprecision with which rotation pole coordinates are determined. Because of the procedure of calculating the coordinates of the pole used by Le Pichon (2), the accuracy of these coordinates can be computed only "a posteriori".

From the spherical triangle (Fig. 1) having as vertices the geographical north pole P_n, the rotation pole P_k of the plate (k) on which

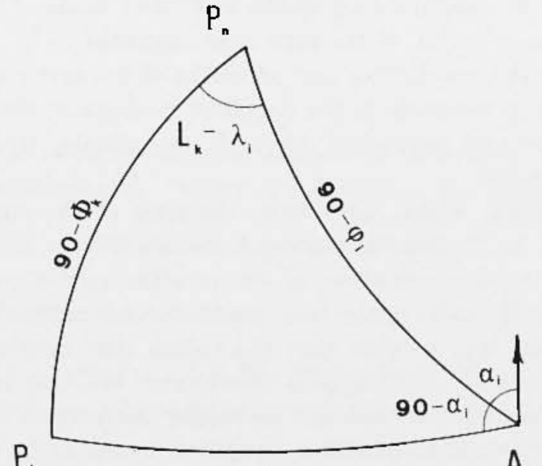

Fig. 1. – Spherical triangle $P_n\, P_k\, A_i$. The points P_n and P_k correspond to the north geographical pole and the rotation pole of the plate.

the point A_i, at which the azimuths a_i of the fracture zone were taken, lies, and the point A_i itself, we can write the following spherical relations:

$$\sin \Phi_k = \sin \varphi_i \cos d_i + \cos \varphi_i \sin d_i \sin a_i$$
$$\cos \Phi_k \cos (L_k - \lambda_i) = \cos \varphi_i \sin d_i - \sin \varphi_i \cos d_i \sin a_i$$
$$\cos \Phi_k \sin (L_k - \lambda_i) = \sin d_i \cos a_i$$

in which $d_{(i)}$ is the angular distance $P_k A_i$, while $\varphi_{(i)}$ and $\lambda_{(i)}$ are the geographic coordinates of point A_i.

From these, we can easily obtain

$$\operatorname{tag} \Phi_k - \cos (L_k - \lambda_i) = \sec \varphi_i \sin (L_k - \lambda_i) \operatorname{tag} a_i$$

from which, in developing and neglecting the second order term, we have

$$\cos \varphi_i \operatorname{cosec} (L_k - \lambda_i) (1 + \operatorname{tag}^2 \Phi_k) \Delta\Phi + \left\{ \cos \varphi_i - \cot (L_k - \lambda_i) \operatorname{tag} a_i \right\} \Delta L = (1 + \operatorname{tag}^2 a_i) \Delta a_i \qquad [2]$$

By means of this relation it is possible to calculate the theoretical values of the uncertainties $\Delta\Phi$ and ΔL of the coordinates of the rotation center P corresponding to the absoluted value of the standard deviations $\Delta a_i = \pm 9^\circ.1$ of fracture zone azimuths.

Utilizing the coordinates and azimuths of fracture zones A_i given by Le Pichon (2) referring to the center of rotation of the American – Eurasian plate and supposing $\Delta\Phi = \Delta L$, we obtain, by formula [2], $|\Delta\Phi| = |\Delta L| \simeq 20^\circ$.

These values, which agree with the sizes of the ellipse of error computed by Le Pichon (2), represent the amount of accuracy to be expected in the determination of the rotation center coordinates of the American–Eurasian plate from fracture zone azimuths.

It is interesting to note that the values thus determined of the indeterminations of rotation pole coordinates are also in agreement with the values of the external precisions determined on the basis of center of rotation coordinates computed by different authors.

In the first four lines of Table 1, the values of the rotation pole coordinates and the relative rate of rotation of the American–Eurasian plate determined by different authors are given.

The standard deviations of the coordinates observed from the average given in Table 1 are:

$$\sigma_{\Phi} = \pm 11^{\circ}$$
$$\sigma_{L} = \pm 19^{\circ}$$

Table 1 – Relative Vector of America–Eurasian Plate Rotation

Φ	L	10^{-7} deg/yr	Authors
78°N	102°E	2.8	Le Pichon (2)
63°N	137°E		Le Pichon (4)
60°N	135°E	2.1	Morgan (5)
48°N	155°E	2.4	Chase (1)
85°N	57°W		Proverbio & Quesada (6)

From relation [1], neglecting the effect of secular polar motion u on secular latitude variations, for the angular rate of the stations lying on the American–Eurasian plate we find

$$\omega_k = -\frac{[b_j\, a_j]}{[u_j\, a_j]} \qquad (j = 1,\ 2,\ 3,\ 4)$$

where

$$a_j = \cos \Phi_k \sin (L_k - \lambda_j)$$

and $j = 1,2,3,4$ refers to the four stations of Mizusawa, Kitab, Carloforte and Gaithersburg. From the foregoing we immediately find

$$[a_j^2]^2\, \Delta\omega_k = A\,[a_j^2] + B\,[a_j\, b_j] \qquad [3]$$

where the coefficients A and B are

$$A = [b_j \sin \Phi_k \sin (L_k - \lambda_j)]\, \Delta\Phi_k - [\cos \Phi_k \cos (L_k - \lambda_j)]\, \Delta L$$
$$B = [\sin 2\, \Phi_k \sin ^2 (L_k - \lambda_j)]\, \Delta\Phi_k - [\sin 2\, (L_k - \lambda_j) \cos^2 \Phi_k]\, \Delta L_k$$

Utilizing the values for secular variations b_j (6) and considering the rotation center coordinates (2) we find

$$\Delta\omega = -0{,}30\, \Delta\Phi_k + 0{,}42\, \Delta L_k \qquad (10^{-3} \text{ sec of arc/yr}).$$

Although the reliability of this solution is rather scarse (since the computed weight of the quantity $\Delta\omega$ is only 0.006) in any case we have, keeping in mind the maximum uncertainties in calculating the coordinates Φ_k and L_k (order of 10°), an uncertainty in the computed angular rate of the same magnitude of the angular rate by the A. A. ([6]) determined.

As the choice of pole position in rotation calculation by means of the equation [1] seems to be rather critical, we considered it opportune to reconsider the whole problem by turning to a different criterion of data processing.

Determination of rotation center and rate from astronomical latitudes

The location of the center of rotation and the rate of rotation of the American–Eurasian plate was determined by the least-square fit method, similar to the one used by Le Pichon for the determination of rotation parameters of rigid blocks.

The numerical method of fitting minimizes the sum of the squares or the standard deviations of the residuals

$$\Delta\varphi_i = (b_{obs} - b_{cal})_i \qquad (i = 1,2,3,4)$$

between the observed values of secular latitude variations and those calculated by the expression [1]. To calculate this expression, the fixed parameters $u = 0''.0030$ and $v = 69°.6$, determined by the A.A. ([6]) with sufficient accuracy, which characterize the secular motion of the Earth pole, were introduced.

The values of b_{cal} were therefore calculated by means of [1] as a function of the parameter ω_k, changing the coordinates Φ_k and L_k by steps of five degrees over the range

$$0 \leqslant \Phi_m \leqslant 85° \\ 0 \leqslant L_n \leqslant 355°$$

The values of the standard deviation

$$\sigma_{k,m,n}\,(\omega_k, \Phi_m, L_n) = \left[\Delta\varphi_i^2\right]_{m,n}^{1/2} \qquad (i = 1,2,3,4) \qquad [4]$$

are plotted in different diagrams. As an example, in Fig. 2 we represent the values of $\sigma_{k,m,n}$, corresponding to $\omega_k = \pm\, 0''.0012$, with the plus

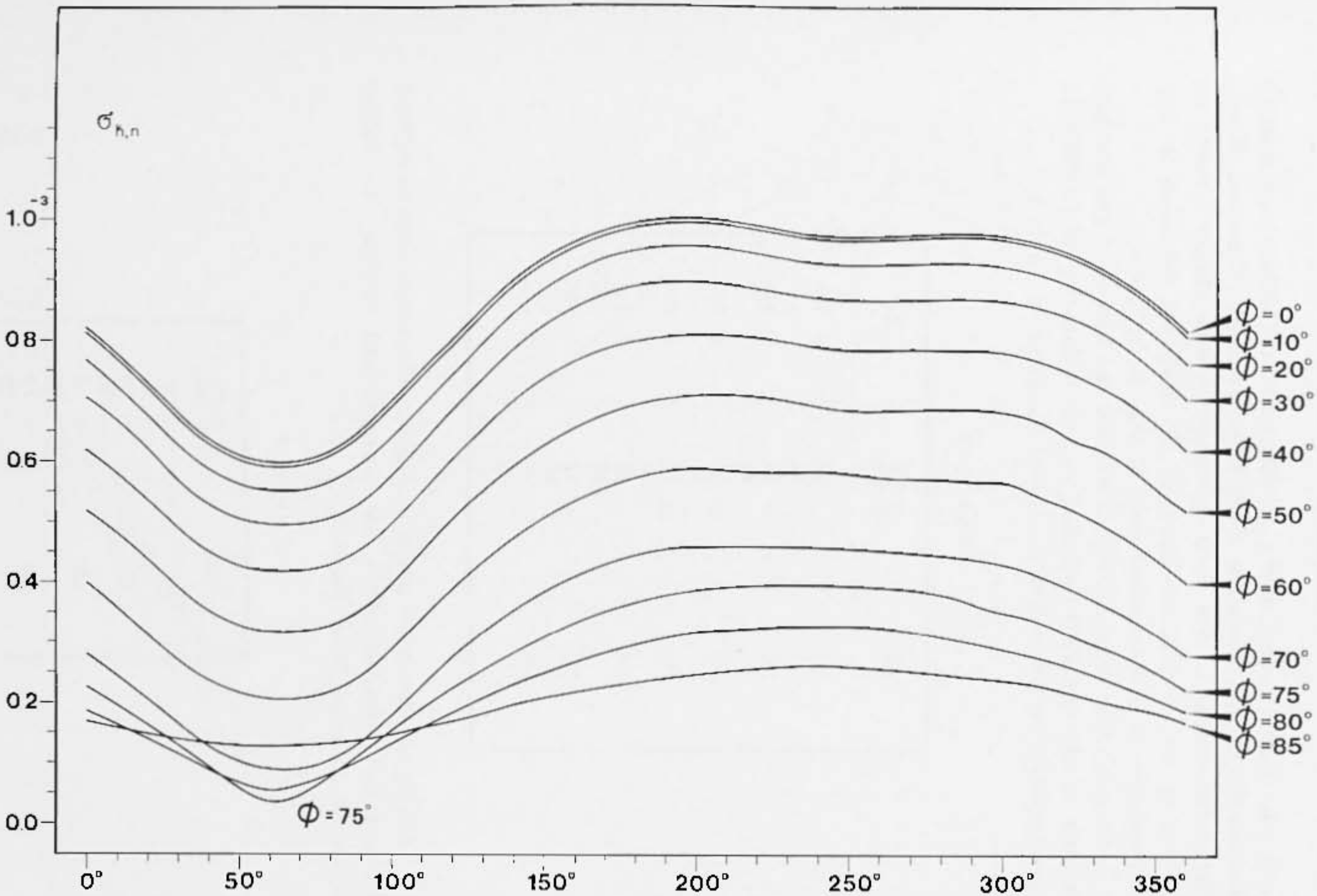

Fig. 2. – Representation of the values of the standard deviation $\sigma_{k,m,n}$ corresponding to $\omega_k = \pm\ 0''.0012$, with the positive sign referring to motion of the American plate and negative sign to motion of the Eurasian plate.

sign referring to motion of the American block (Gaithersburg station) and the minus sign to motion of the Eurasian block (Mizusawa, Kitab, Carloforte stations). For each ω_k value we thus obtain a curve representing an absolute minimum of the standard deviation in correspondence to a determined pair of Φ_m, L_n values.

On the other hand, in Fig. 3 we find the curves of the absolute minima of the standard deviation $\sigma_{k,m}$ as a function of latitude L_n corresponding to the values of ω_k and Φ_m given in Table 2.

Table 2

Code	$\pm\ \omega_k$ (0''.001)	Φ_k
(1)	$\pm$ 12 12	+ 75°
(2)	$\pm$ 14 14	+ 80°
(3)	$\pm$ 16 16	+ 80°
(4)	$\pm$ 12 20	+ 80°
(5)	$\pm$ 15 35	+ 85°
(6)	$\pm$ 12 50	+ 85°
(7)	$\pm$ 12 0	+ 85°

Analogous curves are given in Fig. 4. The latter were calculated keeping the parameter $\Phi_k = 82°$ constant and varying the values of ω_k, as appears from Table 3.

Table 3

Code	$\pm\ \omega_k$ (0''.001)
(1)	0 — 30
(2)	+ 5 — 30
(3)	+ 10 — 30

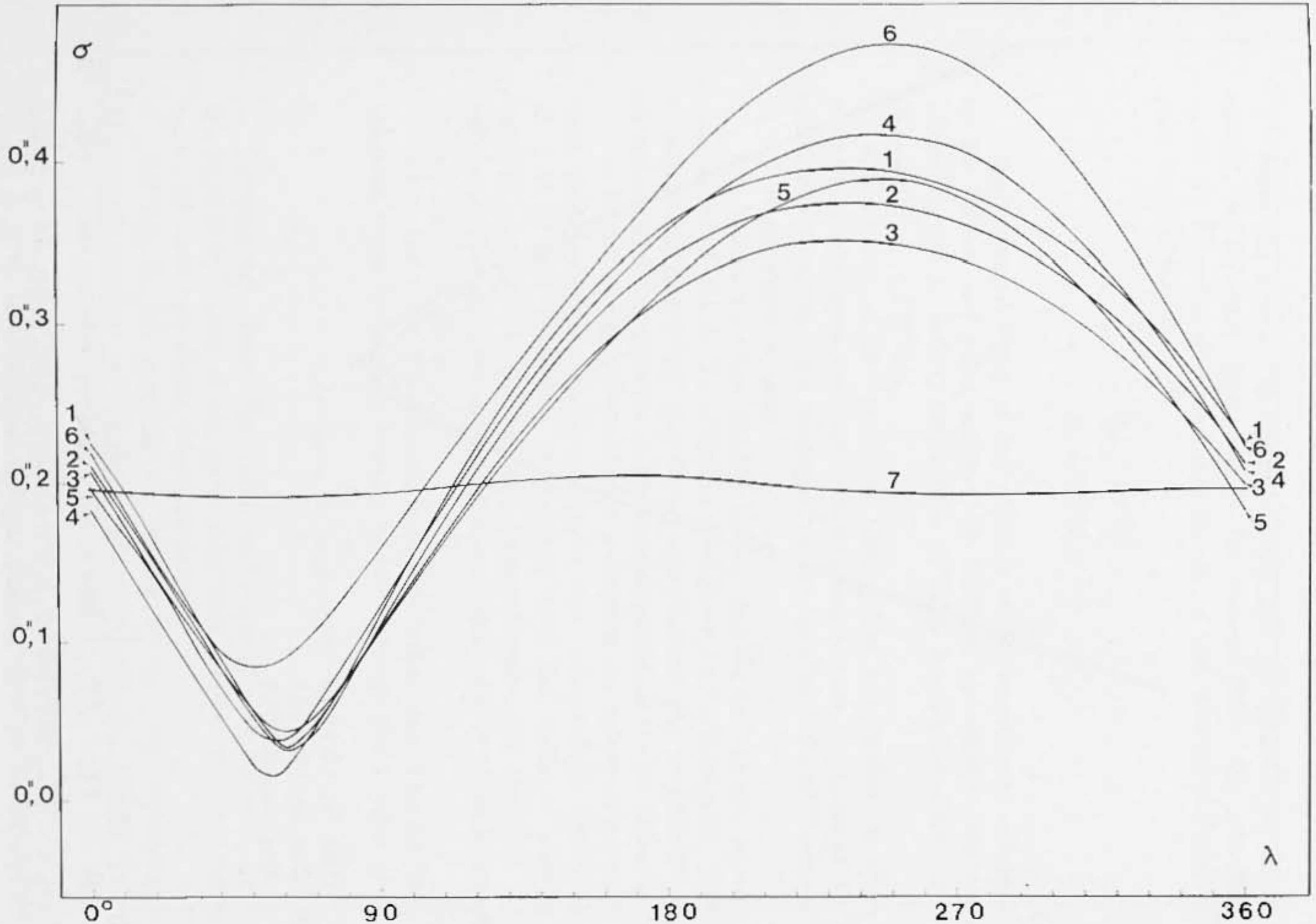

Fig. 3. – Diagram of the curves of the absolute minima of the standard deviation $\sigma_{h,n}$ as a function of latitude L_n corresponding to the values of ω_k and Φ_m given in Table 2.

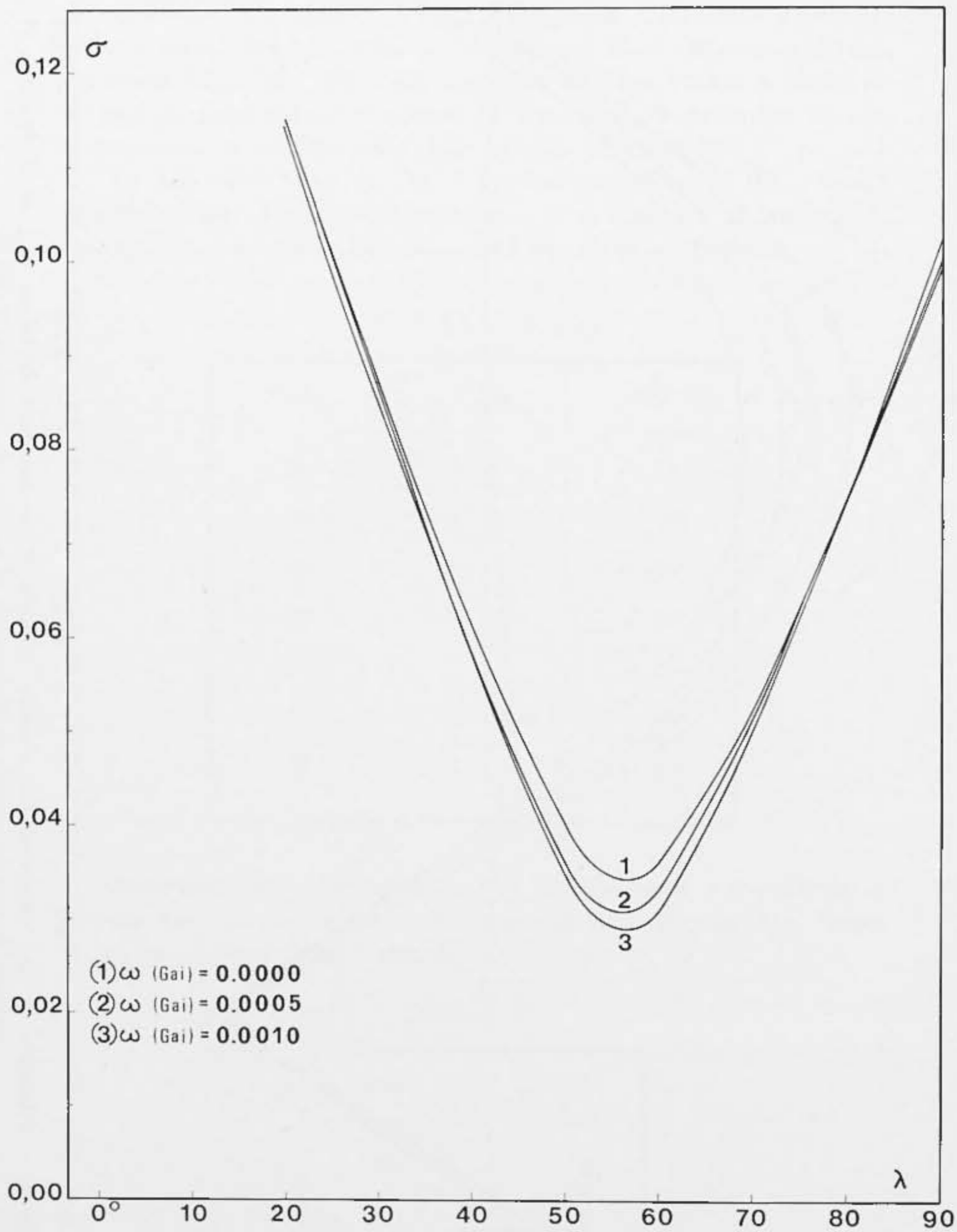

Fig. 4. – Diagram of the curves of standard deviation calculated by keeping the parameter $\Phi_k = 82^\circ$ constant and varying the values of ω_k as appears from Table 3.

From the average of the values Φ_m and L_n obtained by minimizing the standard deviations in the various cases, the following values for the rotation center coordinates of the American–Eurasian plate were obtained:

$$\Phi_k = 82°\mathrm{N}$$

$$L_k = 57°\mathrm{W}$$

The averaged value of the rotation center coordinates obtained by Le Pichon and other authors by utilizing spreading rates and azimuth of the transform faults given in Table 1 is, on the other hand, $\Phi = 62°\mathrm{N}$, $L = 132°\mathrm{E}$.

Comparison of the values of these latter coordinates to those we calculated shows that the two rotation centers are situated approximately on the same meridian but on opposite sides with respect to the Earth rotation pole.

The values we found for the coordinates Φ_k and L_k are, in any case, included within the ellipse of error given by Le Pichon (2). This rather comforting result once again demonstrates the possibilities of astronomical data in studies on the secular motion of the Earth's crust.

In Fig. 5 we find represented in a diagram the values of the standard deviations σ_k corresponding to the values $\Phi_k = 82°\mathrm{N}$ and $L_k = 57°\mathrm{W}$ and calculated by keeping fixed the value of angular velocity $\omega_k^{(2)} = +\,0''.0012$ of the American plate and changing the value $\omega_k^{(1)}$ (having a minus sign) of the angular velocity of the stations of the Eurasian plate.

From the curve interpolated from Fig. 5 it is seen that the most probable value for angular velocity of the Eurasian plate seems to be

$$\omega_k^{(1)} = -\,0''.00295/\mathrm{year}.$$

Successively using this value and varying the value of $\omega_k^{(2)}$, the minimum values of standard deviations relative to the various values of $\omega_k^{(2)}$ introduced were determined.

In Fig. 6 the trend of the values of $\omega_k^{(2)}$ are represented. The curve presents a minimum in correspondence to the value $\omega_k^{(2)} = +\,0''.00125$. This value can be considered as being the most probable for the rotation speed of the U.S. American plate.

It is most interesting to observe that the most probable values of the angular velocities

$$\omega_k^{(1)} = -0''.00295 \text{ (Eurasian plate)}$$
$$\omega_k^{(2)} = +0''.00125 \text{ (American plate)}$$

are in excellent agreement with those obtained by an entirely independent way (6) and reported in the introduction of this paper.

The optimization method used in this paper is efficacious in the hypothesis that the number of residuals $\Delta\varphi_i$ is greater than 1. This has allowed the determination, as we have seen, of the most prob-

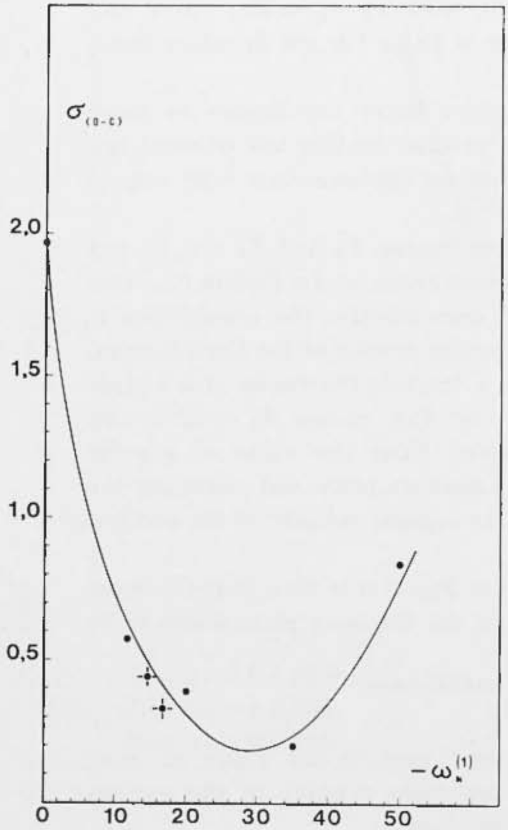

Fig. 5. – Diagram of the values of the standard deviation corresponding to the values $\Phi_k = 82°$N and $L_k = 57°$W and calculated by keeping fixed the value of angular velocity of the American plate and changing the value of the Eurasian plate.

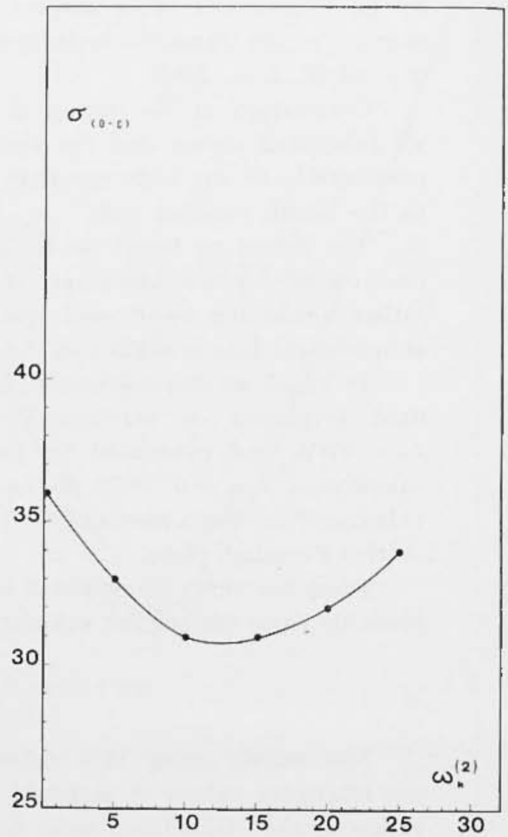

Fig. 6. – Representation of the trend of the values of $\omega_k(^2)$. The curve presents a minimum in corrispondence to the value $\omega_k(^2) = +0''.00125$.

able values of rotation center coordinates and of angular velocities ω_k relative to the two American and Eurasian blocks.

In the case of the NE Pacific plate, we have only one value of the quantities $\Delta\varphi_i$, corresponding to the Ukiah station (U.S.A.). In this circumstance it is thus completely impossible to obtain minima for the standard errors σ. However, it is possible even in this case to carry out an analysis of purely qualitative interest by determining the range of variability of the parameters Φ, L and ω which annul the residuals:

$$\Delta\varphi = (b_{obs} - b_{cal}) = 0 \qquad [5]$$

In Fig. 7, in bold-faced type, the areas of variability of the parameters Φ, L and ω are represented, for which the previous condition

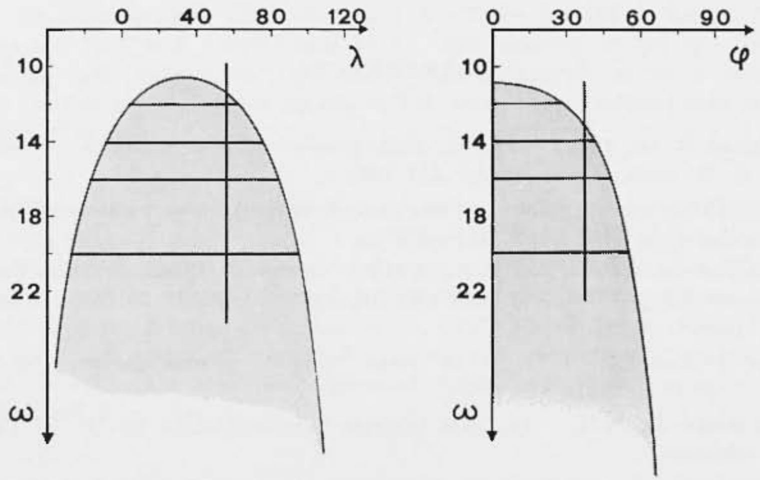

Fig. 7. – Representation of the areas of variability the parameters Φ, L plate, and ω corresponding to the motion of the America-Pacific.

is actually verified. We consider it most significant that also in this case the values of the rotation center coordinates of the American–Pacific plate determined by Le Pichon (2): $\Phi = 53°N$, $L = 47°W$, clearly indicated in the same Fig. 7, fall within the areas of variability which, in a sense, represent the optimal areas of the values of the parameters Φ and L.

Conclusions

To conclude:

a) using a procedure completely independent of the one used by A. A. ([6]), the most probable values of the coordinates and velocity of rotation of the American–Eurasian plate were determined;

b) these values are in full agreement with those previously determined by A. A. ([6]) and are on the same order as those calculated with geophysical and geological measurements;

c) this confirms the possibility of using astronomical (latitude and longitude) observation data in the study of the secular motion of the Earth's crust.

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