More on the Somigliana waves

($C_{i,j}$ and *PL*)

P. CALOI – G. ROMUALDI

Ricevuto il 28 Aprile 1968

SUMMARY. — Previously Caloi has shown how a few corrections and rectifications of a theory on surface waves published by Carlo Somigliana in 1917, permit a quite new interpretation of the classic Rayleigh equation. In the light of this new interpretation, contrary to the Rayleigh theory, the real roots greater than 1 of the velocity equation have a correct physical meaning. More than a period of thirty years of useless attempts, one of use [1,2,7], finally, have reached a strict theoretical justification about conspicuous systems of typical surface waves, which start at the bottom of the Earth' crust layers, when the angles of incidence of the longitudinal waves or transverse waves are effective. These are the so-called $PL_{i,j}$ and $C_{i,j}$ waves.

These waves are very interesting because they evince a new mechanism of elastic waves propagation, and they permit to study the fundamental characteristics of the crust stratifications. Owing to the considerable wave-lenghth, Somigliana waves present appreciable advantages as compared to ordinary longitudinal waves or transverse waves, both they are not subjected to phenomena of anomalous dispersion (see the case of the ordinary body waves of high frequency), as well as for they indicate the existence of discontinuities in the likeness of transition-zones, which generally escape the investigations when the same are performed by ordinary longitudinal waves and transverse waves.

Besides the new very clear examples of these particular types of waves — very often of remarkable amplitude — the Authors study the limits in which the Somigliana waves rise: then waves may start also at the greatest distances (like the $C_{0,2}$ waves), and sometimes they form the most large phase of a seismogram.

RIASSUNTO. — In precedenti lavori, Caloi ha provato che, alcune correzioni e rettifiche di una teoria sulle onde superficiali, pubblicata da Carlo Somigliana nel 1917, consentono un'interpretazione del tutto nuova della classica equazione di Rayleigh. Alla luce di questa nuova interpretazione,

contrariamente a quanto si verifica nella teoria di Rayleigh, le radici reali superiori all'unità dell'equazione delle velocità, hanno un preciso significato fisico.

Ciò ha consentito di dare finalmente [1,2,7], dopo oltre un trentennio di vani tentativi, una rigorosa giustificazione teorica a cospicui sistemi di onde di tipo superficiale, che nascono alla base delle stratificazioni della crosta terrestre, quando le onde longitudinali o trasversali vi incidono sotto angoli efficaci. Sono queste le onde $PL_{i,j}$ e $C_{i,j}$.

L'interesse di queste onde è notevole sia come testimonianza di un nuovo meccanismo di propagazione di onde elastiche, sia per lo studio delle caratteristiche fondamentali delle stratificazioni della crosta. Attesa la grande lunghezza d'onda, le onde di Somigliana presentano sensibili vantaggi nei confronti delle onde longitudinali e trasversali ordinarie, sia perché risultano esenti da fenomeni di dispersione anomala (cui sono soggette le onde spaziali ordinarie di elevata frequenza), sia perché avvertono l'esistenza di discontinuità sotto forma di zone di transizione, che generalmente sfuggono alle indagini con onde longitudinali e trasversali ordinarie.

Oltre a portare nuovi chiarissimi esempi di questi particolari tipi d'onda (molto spesso di nôtevole ampiezza) si studiano qui i limiti di insorgenza delle onde di Somigliana, che possono formarsi anche alle più grandi distanze (come le $C_{0,2}$) e costituire talvolta le fasi più cospicue di un sismogramma.

1. – It has already been proved [1,2] that the Rayleigh equation tolerates real roots greater than 1 only for values of σ between 0 and 0.26305. In relation to this last value of σ, the real roots greater than 1 coincide at the value 3.5754. For values of σ between 0.26305 and 0.5, the Rayleigh equation has a single real root, namely the one smaller than 1 (and to which, notably, correspond the so-called Rayleigh waves). The other two roots become complex. The Fig. 1 shows the real roots greater than 1 corresponding to the field of variability of σ.

Basing on experience, the question may be raised whether each root greater than 1 actually corresponds to Somigliana waves. Theoretically, this should occur when, according to the Poisson ratio, we have the given limits:

$$0 < \sigma < 0.26305 .$$

In fact, with regard to the Earth, the limit is much narrower. We know that σ only rarely drops to values less than 0.25 (*). Therefore,

(*) Although, values up to 0.1 have been found for single rocks [3].

practically, the roots which correspond to conducive angles are the ones offered by the Rayleigh equation for the following range of variability of σ:

$$0.25 < \sigma < 0.26305 .$$

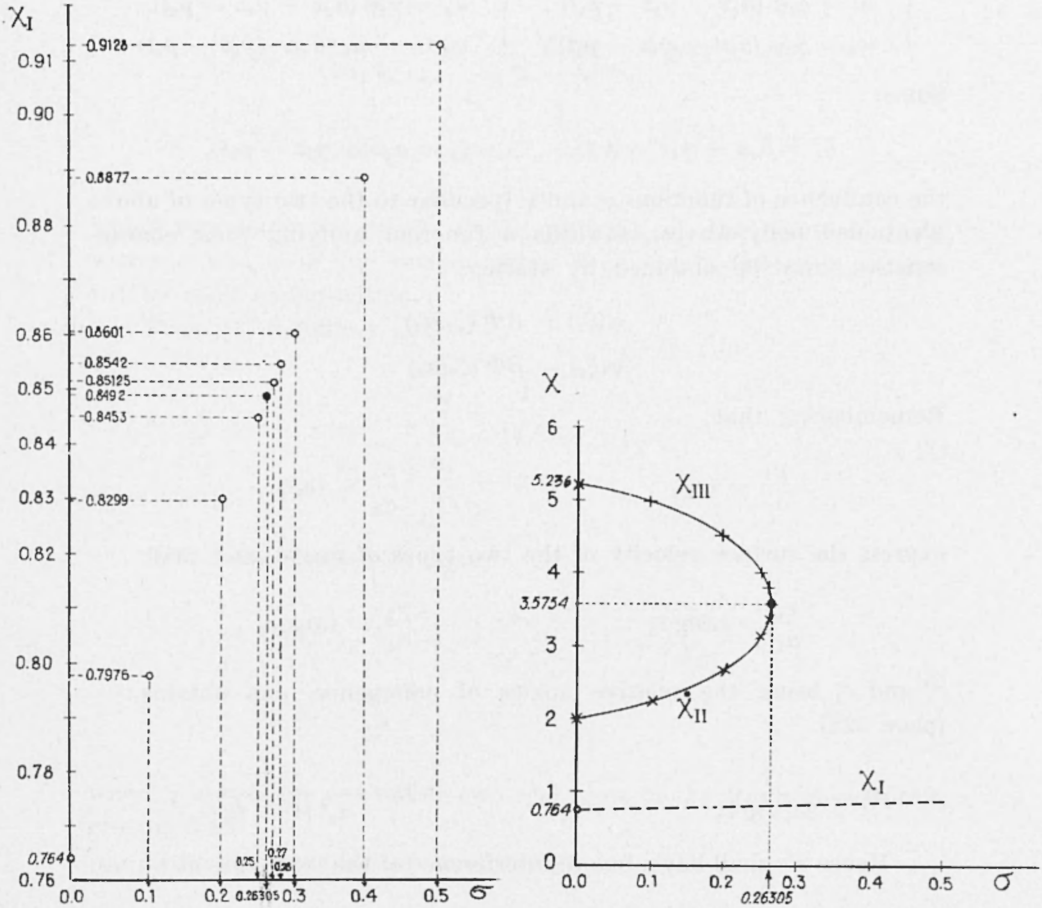

Fig. 1 – Real roots of Rayleigh equation for $0 < \sigma < \frac{1}{2}\left(\chi = \frac{v_3^2}{v_2^2}\right)$.

At this point, another question arises. Is the value greater than 1 for the real root sufficient reason to generate the surface waves of the Somigliana type?

The answer requires a consultation of the expression assumed on the surface layers by the displacements associated with the mentioned waves.

It has been shown (¹) that by designating with u_1, w_1 and u_2, w_2 respectively, the horizontal and the vertical components of the longitudinal (index 1) and vertical (index 2) wave, which when paired are responsible for the generation of Somigliana waves, we may write:

$$\begin{cases} u_1 = a_1\varphi\,(a_1x + \gamma_1 z - p_1 t)\,, \\ w_1 = \gamma_1\varphi\,(a_1x + \gamma_1 z - p_1 t)\,, \end{cases} \qquad \begin{cases} u_2 = \gamma_2\psi\,(a_2x + \gamma_2 z - p_2 t) \\ w_2 = -\,a_2\,(a_2x + \gamma_2 z - p_2 t)\,. \end{cases}$$

Since:

$$\zeta_1 = a_1x + \gamma_1 z - p_1 t\,, \qquad \zeta_2 = a_2x + \gamma_2 z - p_2 t\,,$$

the confluence of functions φ and ψ (peculiar to the two types of above mentioned body waves), towards a function unifying their characteristics, may be obtained by stating:

$$\varphi(\zeta_1) = A\Phi\,(\zeta_1/a_1)$$
$$\psi(\zeta_2) = B\Phi\,(\zeta_2/a_2)\,.$$

Remembering that:

$$\frac{p_1}{a_1} = (v_3)_1\,, \qquad \frac{p_2}{a_2} = (v_3)_2\,,$$

express the surface velocity of the two types of waves, and that:

$$\frac{\gamma_1}{a_1} = \text{tang}\,e_1\,, \qquad \frac{\gamma_2}{a_2} = \text{tang}\,e_2\,,$$

e_1 and e_2 being the relative angles of emergence, one obtains (¹) (page 228):

$$A = \frac{1}{2a_1^2\ \text{tg}\,e_1}\,, \qquad B = \frac{1}{a_2^2\,(1 - \text{tg}^2 e_2)}\,.$$

Hence we shall have, before interference of the two types of waves:

$$\begin{cases} u_1 = \dfrac{1}{2a_1\ \text{tg}\,e_1}\,\Phi\,[x + z\ \text{tang}\,e_1 - (v_3)_1\,t] \\ w_1 = \dfrac{1}{2a_1}\,\Phi\,[x + z\ \text{tg}\,e_1 - (v_3)_1\,t] \end{cases}$$

$$\begin{cases} u_2 = \dfrac{\text{tg}\,e_2}{a_2\,(1 - \text{tg}^2 e_2)}\,\Phi\,[x + z\ \text{tg}\,e_2 - (v_3)_2\,t] \\ w_2 = -\,\dfrac{1}{a_2\,(1 - \text{tg}^2 e_2)}\,\Phi\,[x + z\ \text{tg}\,e_2 - (v_3)_2\,t]\,. \end{cases}$$

But on the surface ($z = 0$):

$$(v_3)_1 = (v_3)_2 = v_3 \ ,$$

so that, stated $u_0 = u_1 + u_2$, $w_0 = w_1 + w_2$, for $z = 0$ we will have:

$$\left\{ \begin{aligned} u_0 &= \left[\frac{1}{2a_1 \operatorname{tg} e_1} + \frac{\operatorname{tg} e_2}{a_2 (1 - \operatorname{tg}^2 e_2)} \right] \Phi (x - v_3 t) \\ w_0 &= \left[\frac{1}{2a_1} - \frac{1}{a_2 (1 - \operatorname{tg}^2 e_2)} \right] \Phi (x - v_3 t) \ , \end{aligned} \right. \qquad [1]$$

which represents the result of the unifying of the pairs of body waves φ and ψ in the surface wave $\Phi (x - v_3 t)$, which is generated by their superposition.

From [1] it follows that:

$$\frac{a_1}{a_2} = \frac{1}{2} \frac{\dfrac{u_0}{w_0} - \dfrac{1}{\operatorname{tg} e_1}}{\dfrac{u_0}{w_0} + \operatorname{tg} e_2} (1 - \operatorname{tg}^2 e_2) \ . \qquad [2]$$

It is to be recalled [1] that:

$$\begin{aligned} \operatorname{tg}^2 e_1 &= \frac{v_3^2}{v_1^2} - 1 = \frac{v_3^2\, v_2^2}{v_2^2\, v_1^2} - 1 = \chi \frac{v_2^2}{v_1^2} - 1 \\ \operatorname{tg}^2 e_2 &= \frac{v_3^2}{v_2^2} - 1 = \chi - 1 \ , \end{aligned} \qquad [3]$$

where χ stands for one of the two real roots of the Rayleigh equation greater than 1.

From the equation of condition [1]:

$$[\lambda + (\lambda + 2\,\mu) \operatorname{tg}^2 e_1] (1 - \operatorname{tg}^2 e_2) = 4\,\mu \operatorname{tg} e_1 \operatorname{tg} e_2 \ ,$$

it follows that it has to be:

$$\operatorname{tg} e_1 = \pm \sqrt{\chi \frac{v_2^2}{v_1^2} - 1} \quad ; \qquad \operatorname{tg} e_2 = \mp \sqrt{\chi - 1} \ , \qquad [4]$$

that is, as has already been observed [1] (page 230) only couplings of a direct longitudinal wave and its own reflected transverse wave, or of a direct transverse wave and its relative reflected longitudinal wave satisfy the Rayleigh equation: this is required by the starting conditions of the Somigliana theory [see ([1]), page 227, formula [3]].

Hence, the [1] are to be written:

$$\left\{\begin{aligned} u_0 &= \left[\pm \frac{1}{2a_1\sqrt{\chi \frac{v_2^2}{v_1^2}-1}} \mp \frac{\sqrt{\chi-1}}{a_2(2-\chi)}\right] \Phi(x - v_3 t) \\ w_0 &= \left[\frac{1}{2a_1} - \frac{1}{a_2(2-\chi)}\right] \Phi(x - v_3 t)\ . \end{aligned}\right. \qquad [5]$$

Now, if the $\Phi(x - v_3 t)$ is a periodic function oscillating between $-a$ and $+a$, while $2U_0$ and $2W_0$ are the given amplitudes for u_0 and w_0 respectively, to determine a_1 and a_2, we shall have the linear equations:

$$\left\{\begin{aligned} &\left[\pm \frac{1}{2a_1\sqrt{\chi \frac{v_2^2}{v_1^2}-1}} + \frac{\sqrt{\chi-1}}{a_2(2-\chi)}\right] a = U_0 \\ &\left(\frac{1}{2a_1} - \frac{1}{a_2(2-\chi)}\right) a = W_0\ , \end{aligned}\right. \qquad [6]$$

which may serve the purpose, *provided* the determinant of the coefficients of $1/a_1$ and $1/a_2$ be other than 0.

When $\Phi(x - v_3 t)$ is not periodic, the relation a_1/a_2 may be determined with formula [2] under due consideration of formulae [4].

2. – It is known that the value of σ, for the surface layers of the Earth's crust, does not differ appreciably from 0.25.

B. Gutenbertg([4]) places it between 0.25 and 0.27 for depth up to 50 km, assuming for this depth a mean value of 0.26.

When $\sigma = 1/4$ the Rayleigh equation gives for real χ (greater than 1) the two values:

$$\chi = \frac{v_3^2}{v_2^2} = \left\{\begin{matrix} 4{,}0 \\ 3{,}1547 \end{matrix}\right. .$$

So in the case of an incident longitudinal wave, the determinant of the coefficients of $1/a_1$ and $1/a_2$ is easily seen as being:

$$\frac{1}{2(2-\chi)}\left(\frac{1}{\sqrt{\chi \cdot \frac{v_2^2}{v_1^2}-1}}+\sqrt{\chi-1}\right),$$

(and likewise for incident transverse wave, except substitution of the symbols).

It is quickly seen that for $\chi = \frac{v_1^2}{v_2^2} = 4$, system [6] is not valid since the determinant cancels out; therefore $a_1 = 0$ and $a_2 = 0$. In other words when $\sigma = 1/4$, the Somigliana waves are generated *only* in correspondence with:

$$\chi = \frac{v_1^2}{v_2^2} = 3.1547 .$$

Consequently, the values of χ, with which one can expect waves of Somigliana, may generally vary between 3.1547 when $\sigma = 1/4$ and 3.5754 when $\sigma = 0.26305$. The apparent velocity v_3 of these waves must therefore *always* be inferior to $2v_2$.

As for the effective angles, we may assume (based on values obtained by resolving Rayleigh equation for σ included between the limits 0.0 and 0.5) a mean angle of incidence on the order of 73° for incident longitudinal waves, and an angle of 33° in the case of incident transverse waves.

With regard to the $C_{i,j}$ waves, therefore, the effective angles at the bottom of the various layers should not differ widely from the value of 33°. This calls for a gradually increasing distance from the source of the $C_{2,1}$, $C_{1,1}$ and $C_{0,1}$, as indeed is the case. For Somigliana waves generated by incident longitudinal waves — except for particular cases — the starting distances would be decidedly shorter.

3. - ***The*** $C_{i,j}$ - We shall not devote too much space to the waves of this type, as a great deal has already been published on the subject (2,5,6,7).

Again we want to state that, when the conditions for their formation occurs, these waves may reach remarkable extensions. It is possible to have them recorded, notwithstanding their long period, by instruments of short period and small static magnification (smoked-paper recording).

The examples cited in a study of 1934 ([8]), where the SL are to be understood as $C_{1,j}$ and the SM as $C_{2,j}$, have been almost all obtained by smoked recording instruments, with periods shorter than 10 seconds.

Those reproduced in two notes of 1948 ([9]) and 1949 ([10]) refer to $C_{1,1}$ waves (with periods on the order of 34 seconds), recorded (Coira, Neuchatel, Zurich) by smoked recording instruments having periods of only 2 seconds! Hence, indications are for evident forced waves of remarkable amplitude, capable of impressing even seismographs with almost no sensitivity for them. In fact, the $C_{i,j}$ waves constitute often the largest waves of the whole seismogram, including the surface waves.

Further documentation is reported in Figures 2, 3, 4, 5, 6.

The apparent velocities of the $C_{1,j}$ waves have been determined previously. Also the apparent velocities of the $C_{2,j}$ waves have been the object of inquiry in the development of a doctoral thesis (Ranalli 1963).

The computation of the apparent velocity of the $C_{0,j}$ waves seemed to be somewhat more complex, also because the conducive angle to their generation, if beyond the 8.000 km, is generally associated with PS instead of with S waves (Figg. 4, 5).

The calculation of the velocities of seismic waves presumes the availability of a number of sufficient data obtained from one and the same earthquake. Initially we disposed of data gathered from various earthquakes, obtained by preceding researches and from a recent systematic research, carried out on a certain number of earthquakes properly to collect the greatest possible number of examples of C_0 waves.

Even though keeping in mind the limitations due to the non-homogeneity of the data with regard to the origin of the earthquake, we considered the possibility to utilize them for an attempt to assess the velocities of $C_{0,j}$ waves. Specifically, in view of the wide field of epicentral distances covered by the above mentioned data and considering that for distances beyond the 8.000 km the $C_{0,j}$ waves appear associated with the PS waves, the calculation was carried out by subdividing the collected data in two groups:

a) epicentral distances *below* 8.000 km;

b) epicentral distances *beyond* 8.000 km.

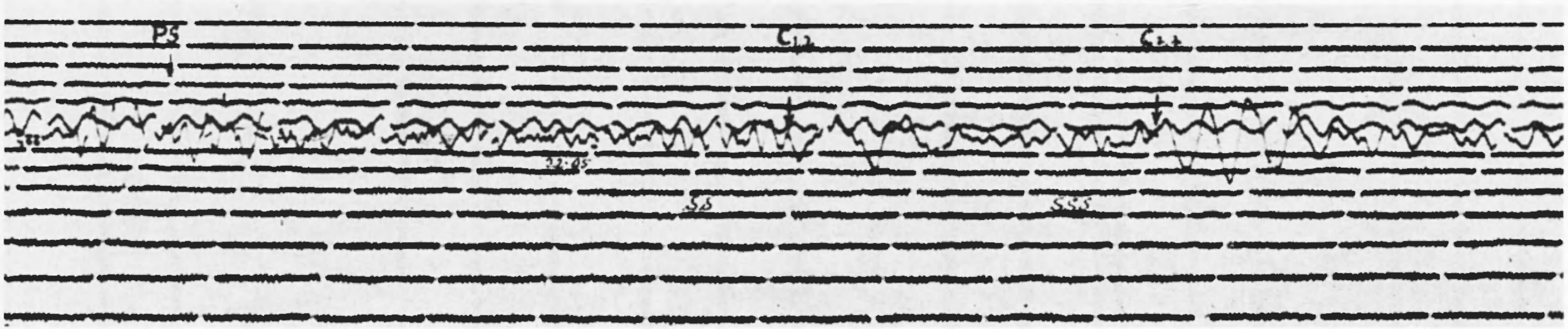

Fig. 2 – With generating conditions observed, the $C_{i,j}$ waves are apt to reach remarkable amplitudes, even in instruments with short period. In the above example (Trieste 18/VI/1933; $\Delta t = + 5^s,7$, NS Alfani, $\varphi = 38^o5$ N; $142^o,8$ E; $H = 21^h37^m36^s$, $\Delta \cong 9400$ km) the $C_{1,2}$, $C_{2,3}$ waves have been recorded by a seismograph with period of about 10 sec. Seismographs, using smoked-paper, with period on the order of 1 second, have supplied clear examples of $C_{i,j}$, waves.

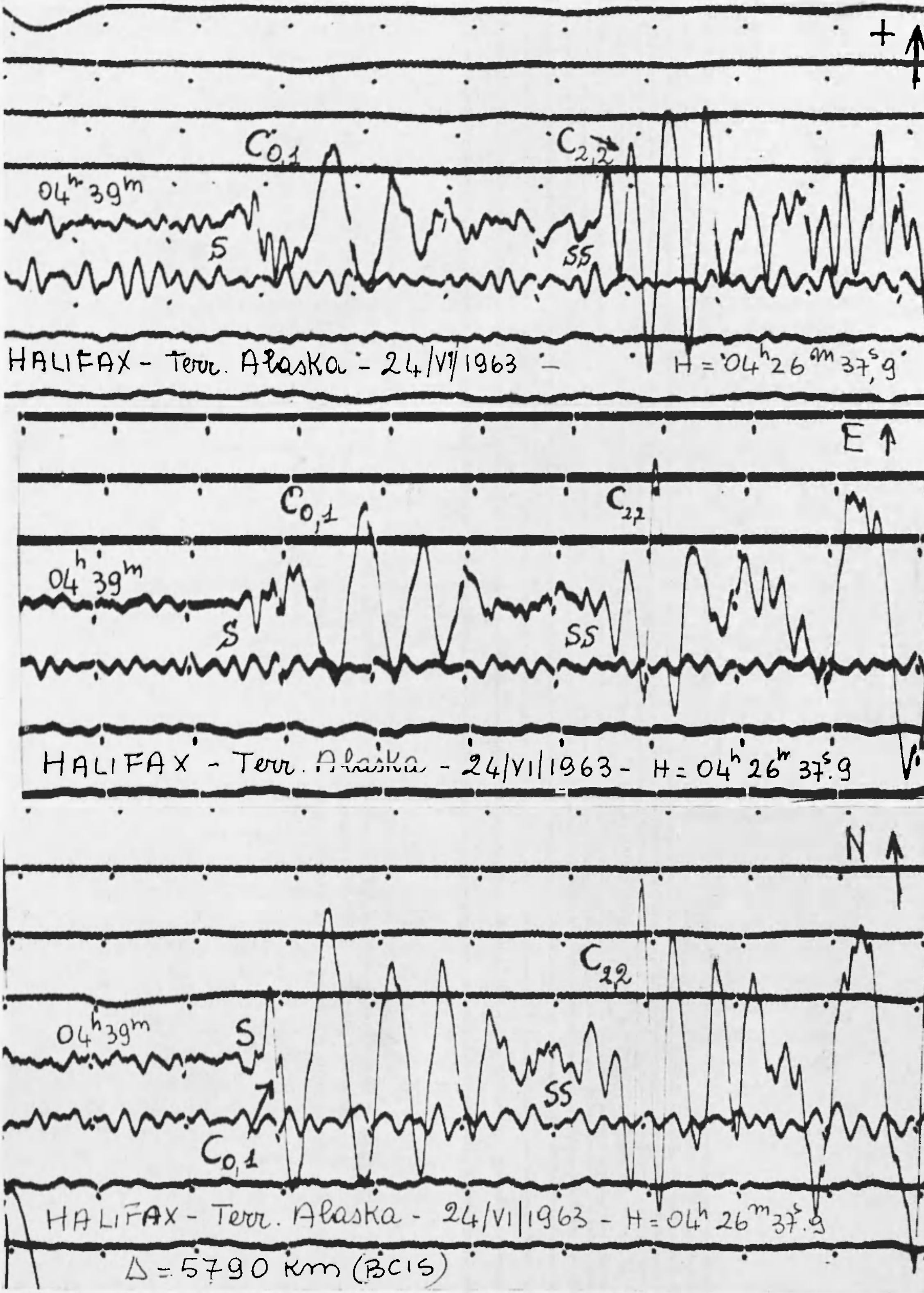

Fig. 3 – When the conducive angles are reached in correspondence to the seismic stations, the Somigliana waves are recorded almost simultaneously with the incident longitudinal waves or transverse waves with which they are associated. The example shown above of $C_{0,1}$ and $C_{2\,2}$ waves, obtained at Halifax is particularly significant.

Fig. 4 – At distances over 8000 km, the $C_{0,1}$ waves are generally associated with PS waves, as shown in the above example (Aquila 28/VI/63; Ep. Kurile Is.; 46°,5 N; 153°,2 E; $H = 21^h55^m38^s$; $h \cong 33$ km, $\Delta \cong 9500$ km), as vell as in figure 5.

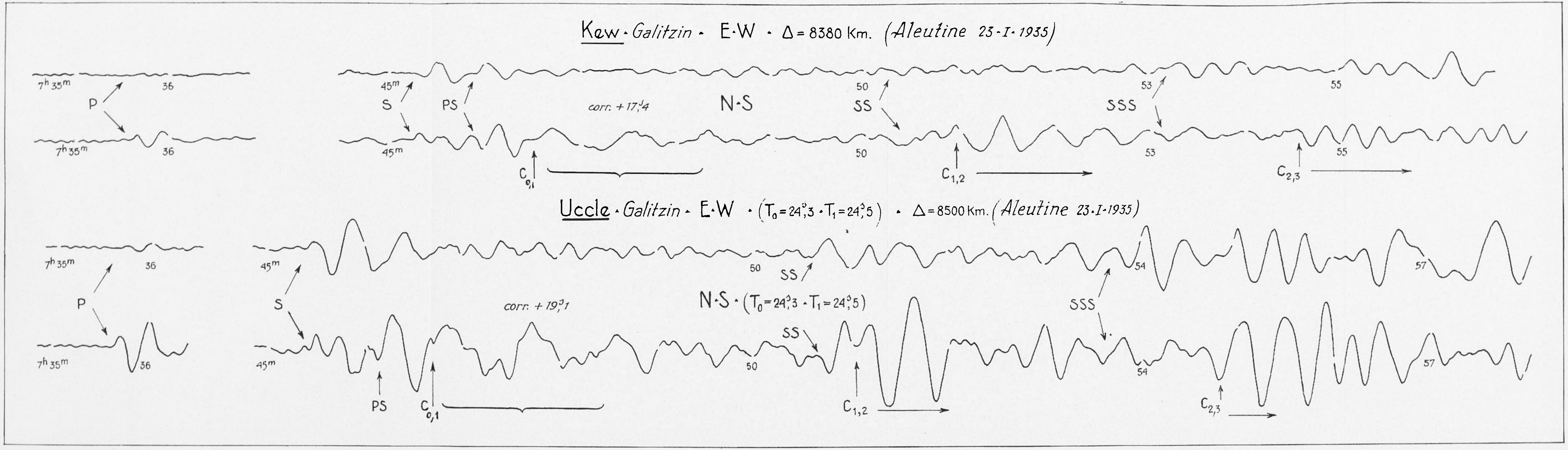

Fig. 5 – See Fig. 4

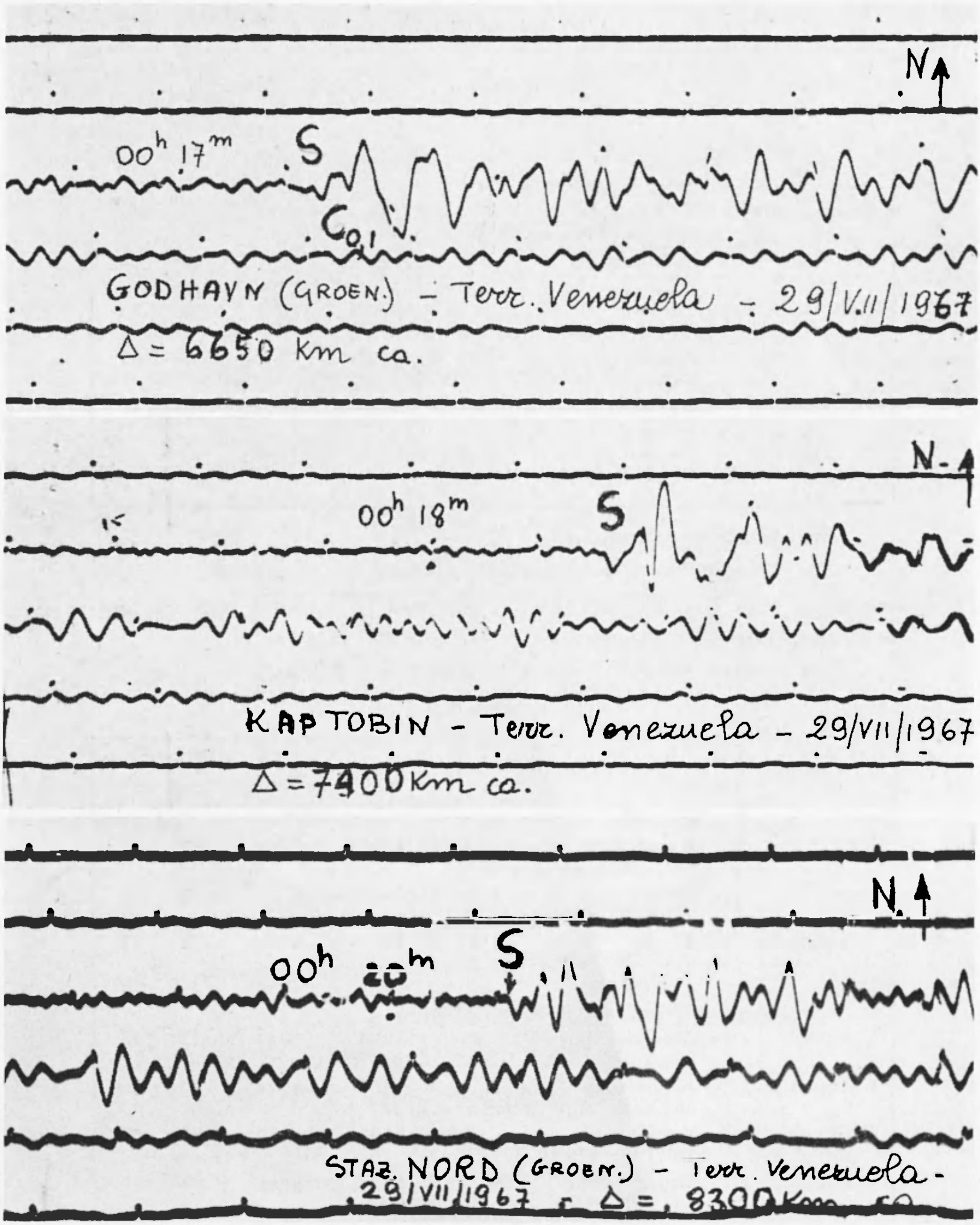

Fig. 6 – Where the Earth's crust is of lesser thickness, there the $C_{i,j}$ waves result of much shorter periods than those reached in continental areas. The example above has been obtained on the coasts of Greenland.

a) Data utilized.

Station	Earthquake	Δ km	Travel-time min.	sec.
De Bilt	Kansu 25.XII.1932 39°,2 N; 96°,4 E (*)	6790	19	15
Bloomington	Aleutian Is. 30.XII.1961 52°,3 N; 177°,6 E; h = 56 km (**)	6876	19	03
Phu-lien	Solomon Is. 3.X.1931 10°,6 S; 161°,7 E (*)	6940	19	25
Halifax	Turkey 23.V.1960 36°,4 N; 28°,3 E; h = 49 (**)	7417	20	16
Kew	Alaska 24.VI.1963 59°,5 N; 151°,7 W; h = 52 (**)	7543	20	22
Palisades	Aleutian Is. 30.XII.1961 52°,3 N; 177°,6 E; h = 56 (**)	7649	20	40
De Bilt	Central China 25.VIII.1933 31°,7 N; 103°,4 E (*)	7790	21	

The equation obtained is:

$$Y = (0.20857 \pm 0.02025)\,X + (6.35149 \pm 1.33848)$$

where Y and X represent the travel-times in minutes and the epicentral distances in degrees.

The relative velocity results of about 8.9 km/sec.

b) Data utilized

Station	Earthquake	Δ km	Travel-time min.	sec.
Kew	Kamciatka 17.III.1933 54°.8 N; 161°,7 E (**)	8090	22	04
Kew	Aleutian Is. 22.VII.1933 52°,9 N; 169°.3 W (**)	8355	22	32
L'Aquila	Alaska 31.IV.1964 61°,6 N; 147°,6 W; h = 40 (**)	8363	22	30
Uccle	Aleutian Is. 22.VII.1933 52°,9 N; 169°.3 W; (**)	8455	22	48
L'Aquila	Alaska 24.VI.1963 59°,5 N, 151°,7 W; h = 52 (**)	8641	22	56
L'Aquila	Kamciatka 26.V.1963 55°,2 N; 159°.9 E; h = 47 (**)	8748	23	20
Taipheih	Kermadec Is. 20.V.1963 30°,7 S; 178°,3 W (**)	8882	23	13
L'Aquila	Kurili Is. 22.V.1963 48°,6 N; 154°,7 E; h = 22 (**)	9246	24	17

(*) According to Int. Seism. Summary.
(**) According to U.S.C.G.S.

The equation obtained is:

$$Y = (0.20149 \pm 0.00101)\ X + (7.36965 \pm 0.78065)$$

with a velocity of about 9.2 km/sec.

With regard to what has been stated in nº 2, the values of the apparent velocities obtained appear excessive; anyways, they may be considered as approximated by excess, especially the second. Other determinations, effectuated with data of diverse origin, led to apparent velocities on the order of 7.7 km/sec.

This is indubitably to be ascribed to the heterogeneity of the material utilized, regarding the diversity of the earthquakes as well as the recording instruments employed.

Actually, a subsequent computation with data obtained from the Venezuelan earthquake of July 29, 1967 (10°, 6 N; 67°, 3W; $H = 23^h59^m58^s,7$; $h = 10$ km), led to a value close to the one called by the theory.

Data Utilized

Station	Δ km	Travel-time min.	sec.
Kap Tobin (Greenl.)	7344	20	25
Strasbourg	8028	21	41
Stuttgart	8141	21	53
Rome	8381	22	21
Copenaghen	8393	22	37
Uppsala	8707	23	05

The equation obtained is:

$$Y = (0.21997 \pm 0.01382)\ X + (5.839359 \pm 1.01480)$$

for a velocity of about 8.4 km/sec.

This value for v_3 fully confirms what has been observed in nº 2.

4. – The source of the $C_{i,j}$ waves, as was exposed in the preceding notes (1, 2, 7), does not appear indoubt. The fact that in the Oceanic islands, they appear for periods much shorter than the mean one observed in continental stations — and, generally, *only* one of the three possible types (6) — was already significant.

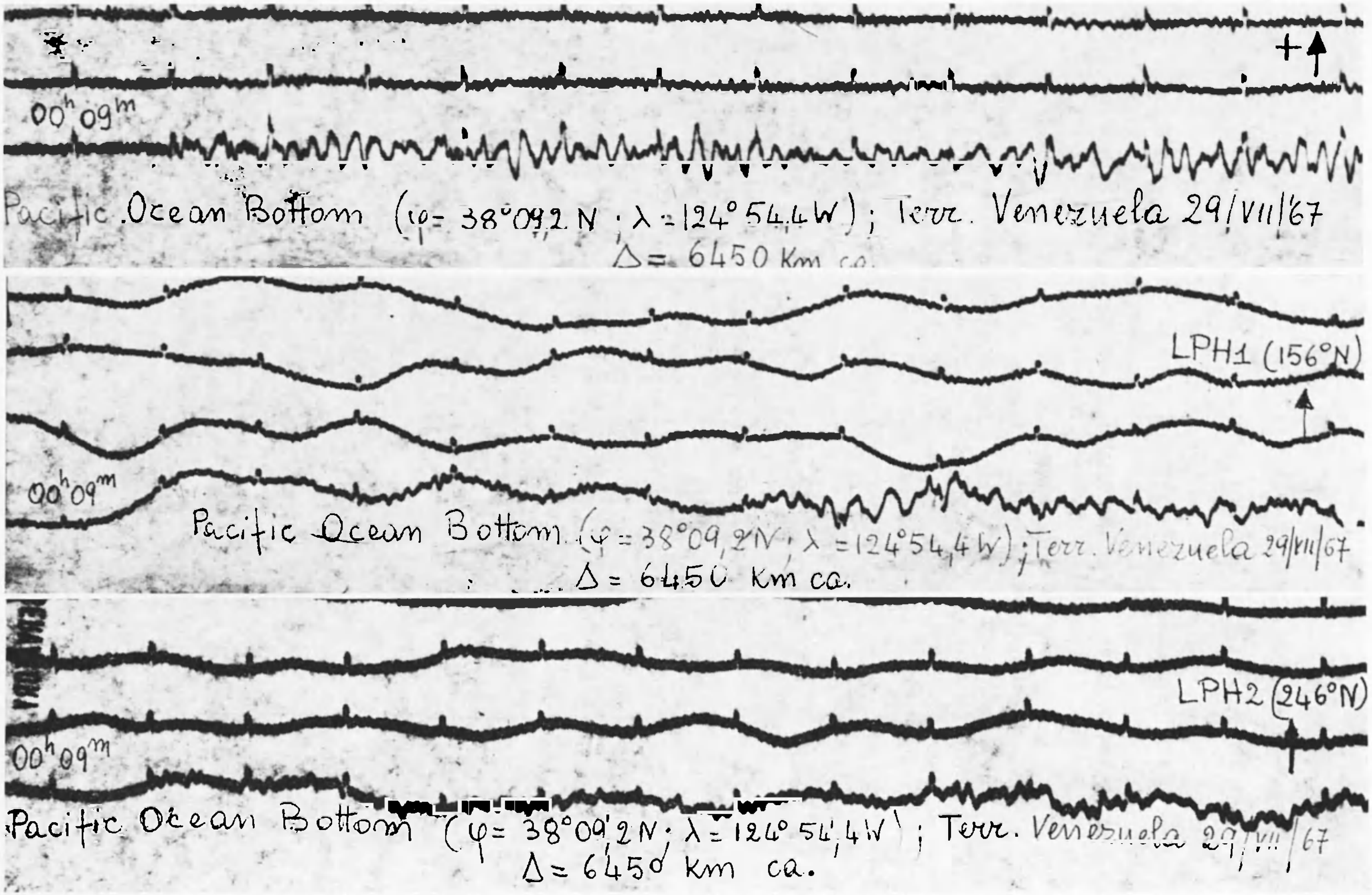

Fig. 7 – If the records are obtained on the bottom of the Ocean, where the Mohorovicic surface is at only a few km depth, the recording of the $C_{t,j}$ waves is almost missing. This is shown by the above recording obtained with a seismograph placed on the bottom of the Pacific Ocean, at the point of coordinates 38°09',2 N; 124°54',4 W, at a depth of 3860 m ab.

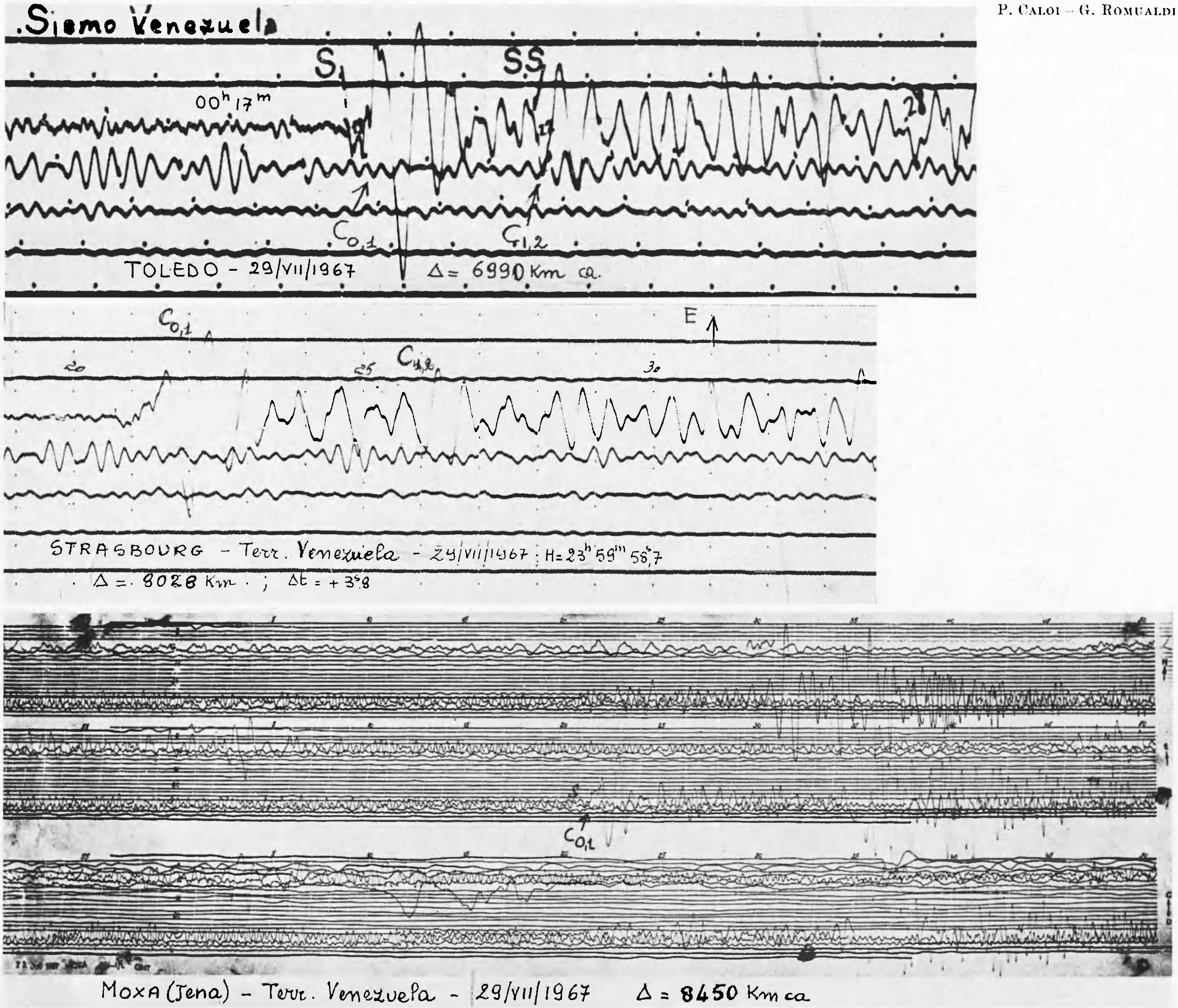

Fig. 8 – Some Venezuela eartquake records, as Fig. 7, obtained in continental stations, with large Somigliana waves ($C_{0,1}$).

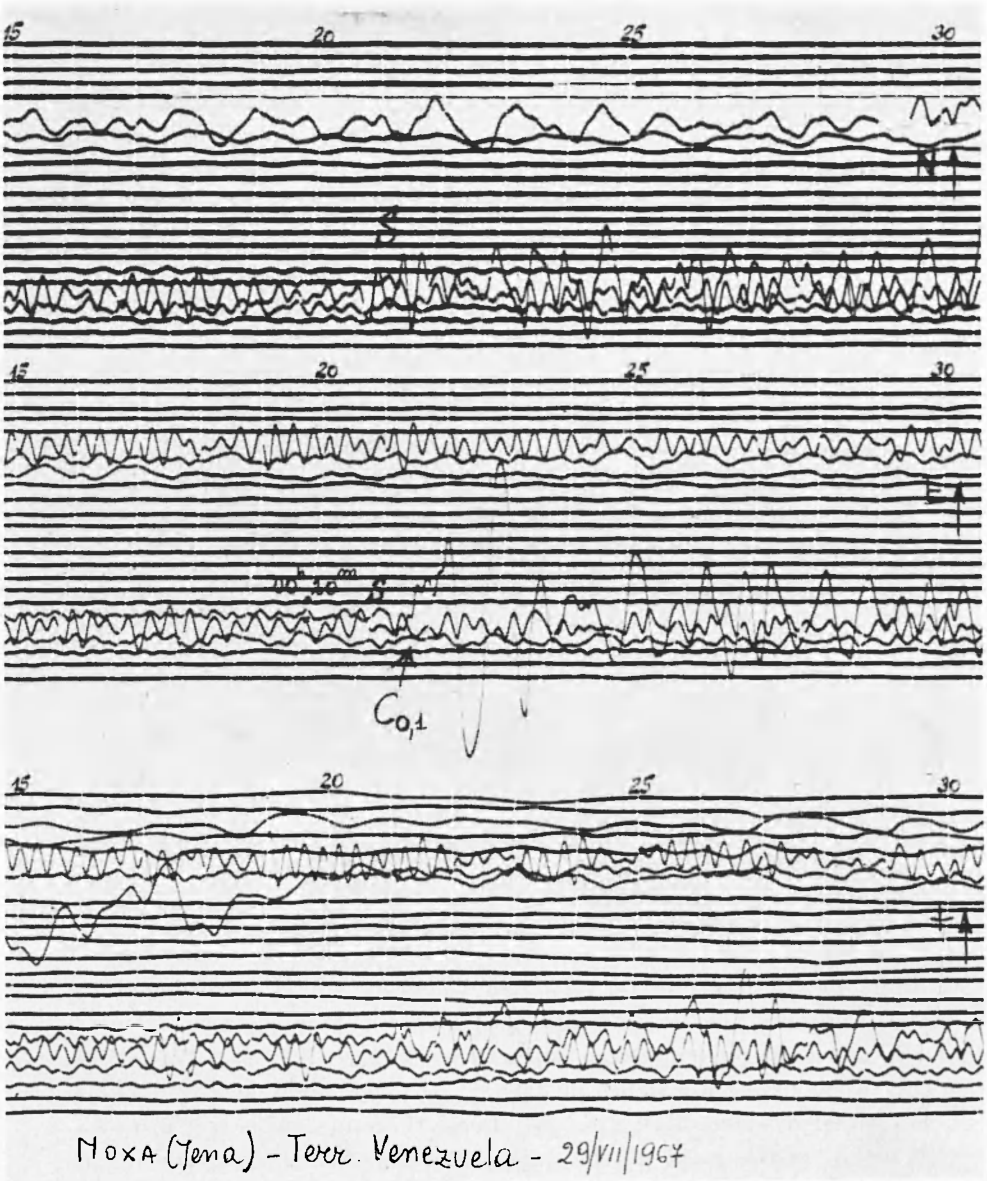

Fig. 8a – See Fig. 8

In view of these facts, it was to be expected that the recording of $C_{i,j}$ waves on the bottom of the oceans would have been hardly possible, if not in particular circumstances and with periods decidedly shorter. This is exactly what happened (see Fig. 7). The Venezuelan earthquake of July 29, 1967, from which is exceptionally large continental records of $C_{i,j}$ waves (Figg. 8, 8*a*), permitted confirmation of what had been anticipated. The amplitude of these waves, and their relative periods, appeared already reduced corresponding to the Greenland stations.

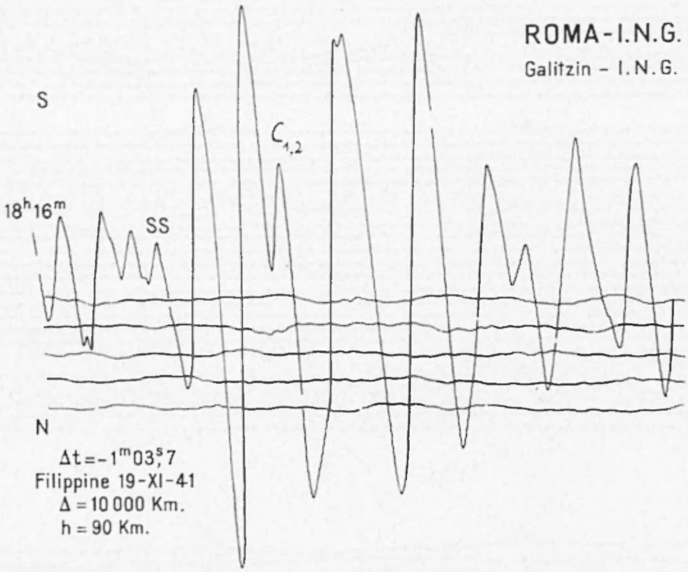

Fig. 9 – With increasing distance between seismic station and the "optimum" point for the setting up of $C_{i,j}$ waves, an increase is noted also in the time interval between recording of the body wave and the one of its associated Somigliana wave, as in example reported above.

Here, the Earth's crust is of minor thickness and without apparent stratification. The waves are almost *totally* lacking on the bottom of the Pacific – at the point of coordinates 38° 09',2 N; 124° 54',4 W – where the Ocean presents a depth of 3860 m ab. and the crustal thickness is estimated about 5-10 km. At almost the same epicentral distance (Toledo), instruments of the same type supplied $C_{0,1}$ waves of an uncommon amplitude (see Fig. 8). In Halifax, at the same distance of the cited suboceanic station from Venezuela, the $C_{0,1}$ and $C_{2,2}$ waves connected with the Alaskan earthquake of 24.VI.1963 hawe been recorded with an exceptional amplitude (Fig. 3). Halifax is a continental station, where the crust presents the layers called for to generate strong Somigliana waves.

5. – ***PL waves.*** The problem of long-period waves that followed closely after the first phase of a seismogram (or its reflections) was attacked for the first time in 1930 by Somville [11]. He distinguishes them as *PL* waves. Again in 1931 this was confirmed by Somville and by Landsberg [12, 13]. In these notes, the Authors supply unequivocal examples of *PL* waves for epicentral distances below 2500 km, and they determine periods (> 10 seconds) and travel-time curves. In particular, Somville [12] points out that these waves, whose apparent velocity results equal to 8/10 of the velocity of the direct ones, are to be sharply distinguished from the first reflections of the longitudinal waves.

In 1960, Oliver and Major [14] supplied an interpretation as to the nature of the waves. Accordingly to the *PL* waves constitute a group of waves with normal dispersion rate and periods larger than 10 seconds, whose propagation procedes via the " leaking mode ", as would be derived from the correlation between dispersion of the *PL* and of the Rayleigh waves (*). The movement of the surface particle is always elliptical and progressive.

Furthermore, it is not to be excluded that such waves may be recorded at epicentral distances larger than 25 degrees.

Based on this interpretation, studies by Phinney [15] and by Gilbert and Lester [16] are carried out both theoretically and with the use of models.

Subsequently Oliver [17], using studies of phase velocities of *PL* waves, recorded with long-period equipment in the United States, in connection with two earthquakes in Mexico of May 1962 ($\Delta <$ 4000 km), puts forth the possibility of utilising these waves to obtain information on the structure of the Earth's crust, as it occurs with the Rayleigh and Love waves in a more obvious manner.

As has been already observed elsewhere [2,7], with regard to the *PL* waves, our explanation of their origin may well find its place in the modified theory of Somigliana.

The effective angle for their generation is high [1,2,3].

It follows that particular conditions are required for their generation. Therefore, because of the particular conditions of *PL* propa-

(*) This propagation is supposed to be associated with the crustal wave guide.

gation, the near earthquakes are the most qualified for their formation. This, is — undoubtedly — the reason for the greater frequency of this type of waves with short epicentral distances.

The earthquake of Mistretta, occurred on October 31, 1967, supplied evidence of *PL* waves at Rome ($\Delta \simeq 490$ km, Fig. 10), at L'Aquila ($\Delta \simeq 500$ km, Fig. 11), and at Trieste ($\Delta \simeq 870$ km, Fig. 12). It is our opinion that these are Somigliana waves generated by the longitudinal waves. With regard to the hypocentral depths (about 70 km) as well as to the structure of the crust relative to the Tyrrhenian sea, the propagation of the *P* waves towards Rome should have proceded as shown on Figure 10.

By analogy this fact must be valid for the recording at L'Aquila, but not at Trieste. In the latter the same type of wave is to be attributed to *PP* waves, incident at the bottom of the intermediate layer (or "granite" layer), under the same conducive angle.

This type of wave is rarely observed in connection with distant earthquakes. Sometimes, however, even in case of teleseisms, the *P* waves appear followed by remarkable *PL* waves (see Fig. 13). We have observed that such records are associated, generally, with earthquakes beginning in the asthenosphere (zone where the velocity undergoes a flexion). This fact may cause the *P* (or *PP*) waves to present themselves at the bottom of the crust with very high angles of incidence (see Fig. 13) and thus permit the generation of Somigliana waves at the bottom of one of the layers of the crust by incidence of longitudinal waves under conducive angles. Considering the mechanism of propagation for seismic rays, influenced by the asthenosphere, it is possible that the *PL* waves, associated with earthquakes of distant origin, may be started — as Somigliana waves — by longitudinal waves that reach the seismic station slightly later than the normal *P* waves, which have crossed layers characterized by a higher propagation velocity.

With this mode of reason, we can also explain the long-period *P* waves which are sometimes recorded in connection with very near earthquakes. We refer to the example reproduced in Figure 14. The unusual length of the period of the *P* waves may be explained by the generation of Somigliana waves in the alluvial layer underneath the seismic station of Rome (Tiber valley), connected with the incidence under conducive angles of longitudinal waves at the bottom of the same layer.

The suggested theory justified also for the setting up of the long waves, which on great distances are associated with the *SS*,

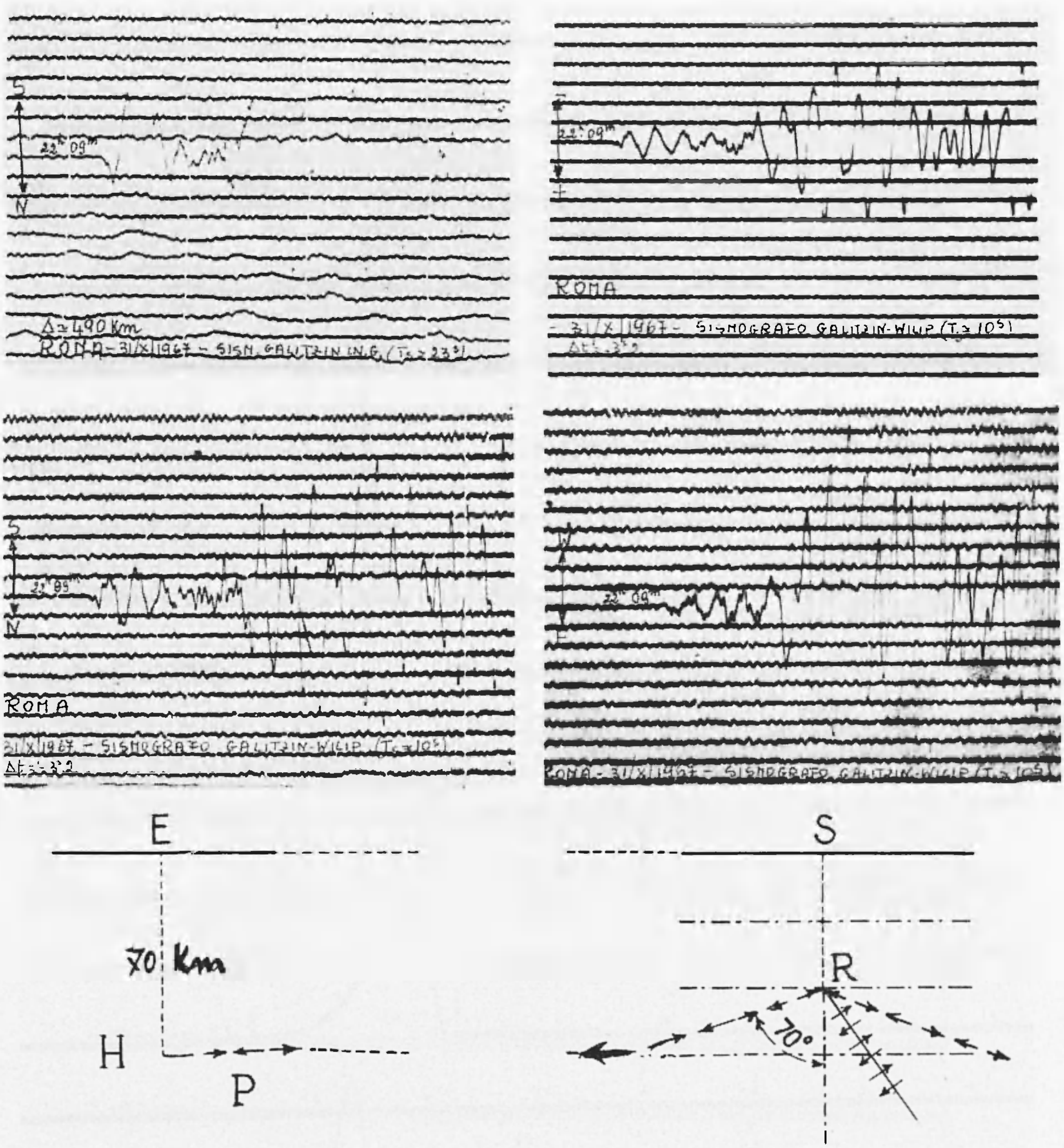

Fig. 10 – The hypocentral depth influences the starting conditions of the waves. The examples of *PL* waves reported above and in figures 11, 12, are due to the fact that hypocentral depth of the source of the earthquake was of about 70 km. Therefore it was possible to reach the conducive angles at Rome and at L'Aquila in correspondence to the *P* waves, and at Trieste in correspondence to the *PP* waves.

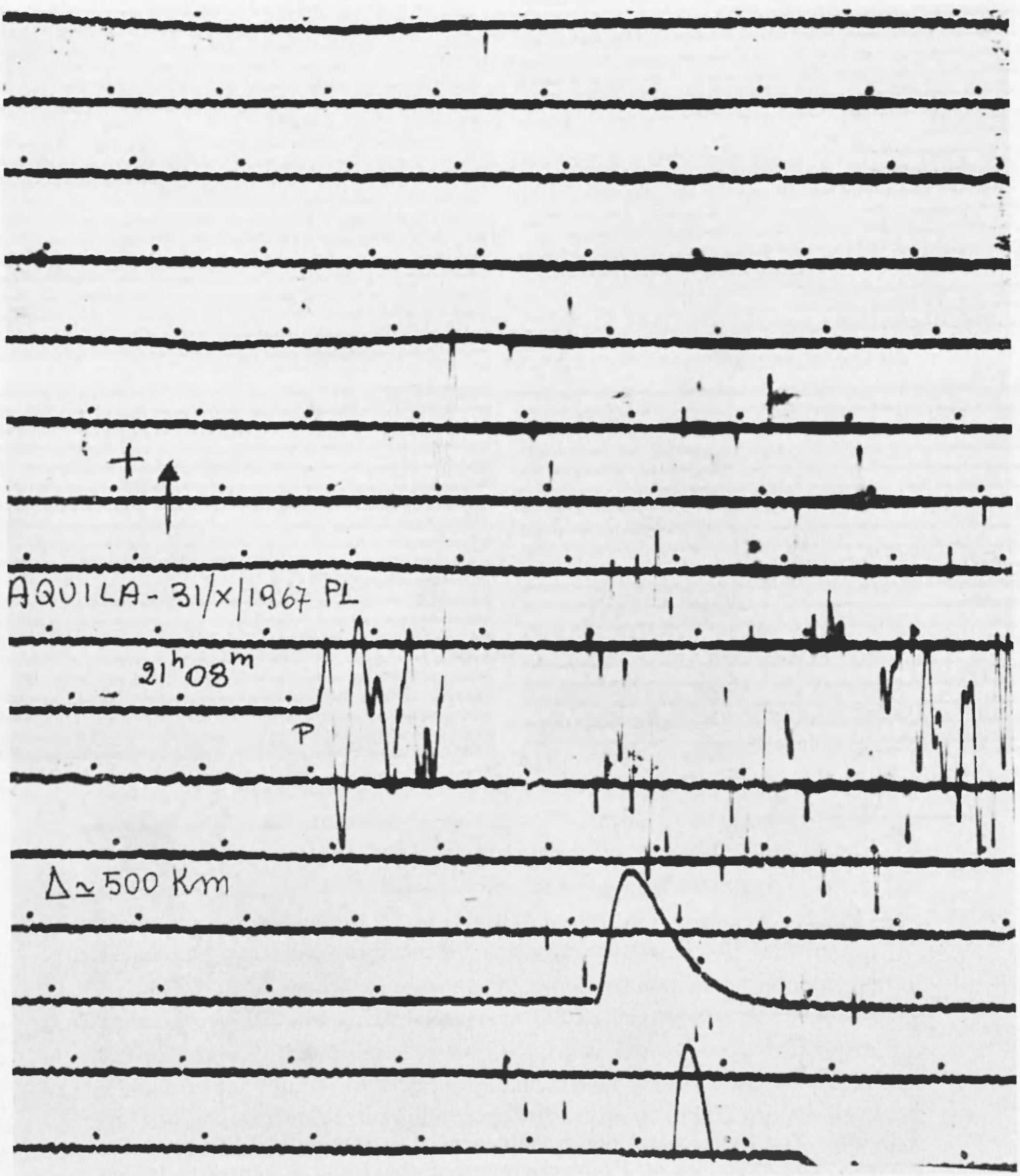

Fig. 11 – See Fig. 10

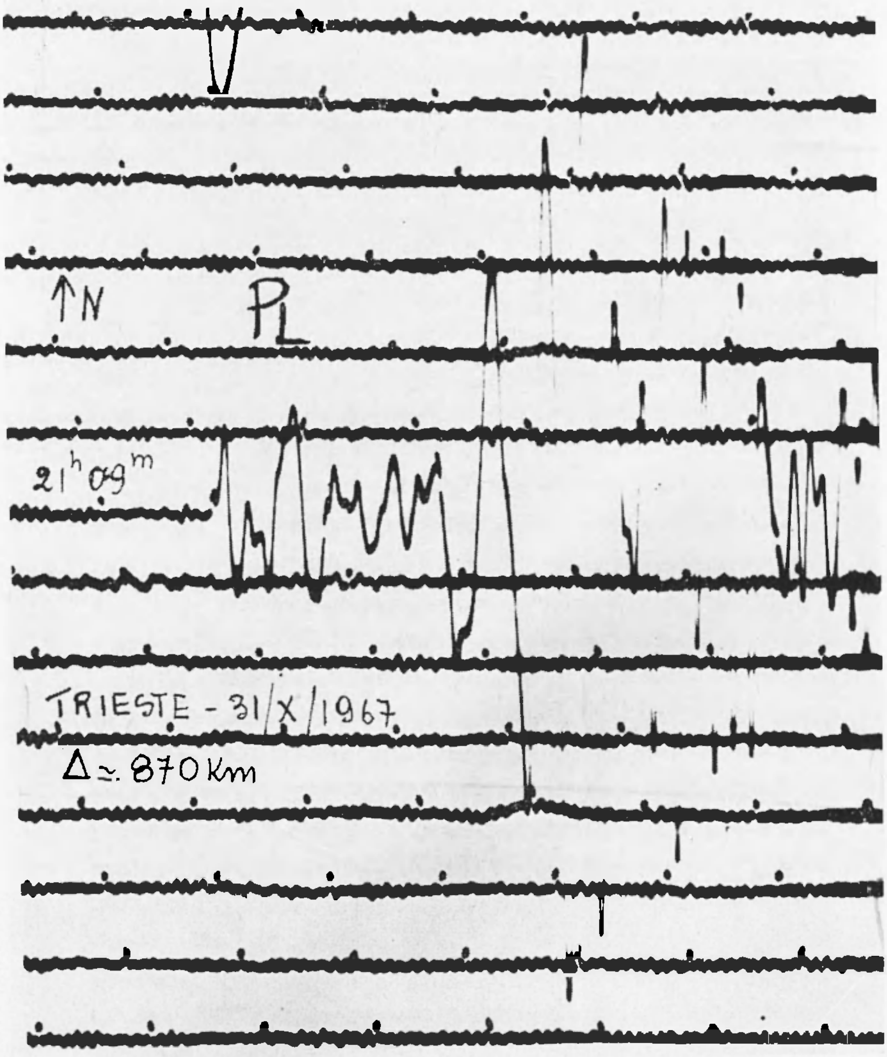

Fig. 12 – See Fig. 10

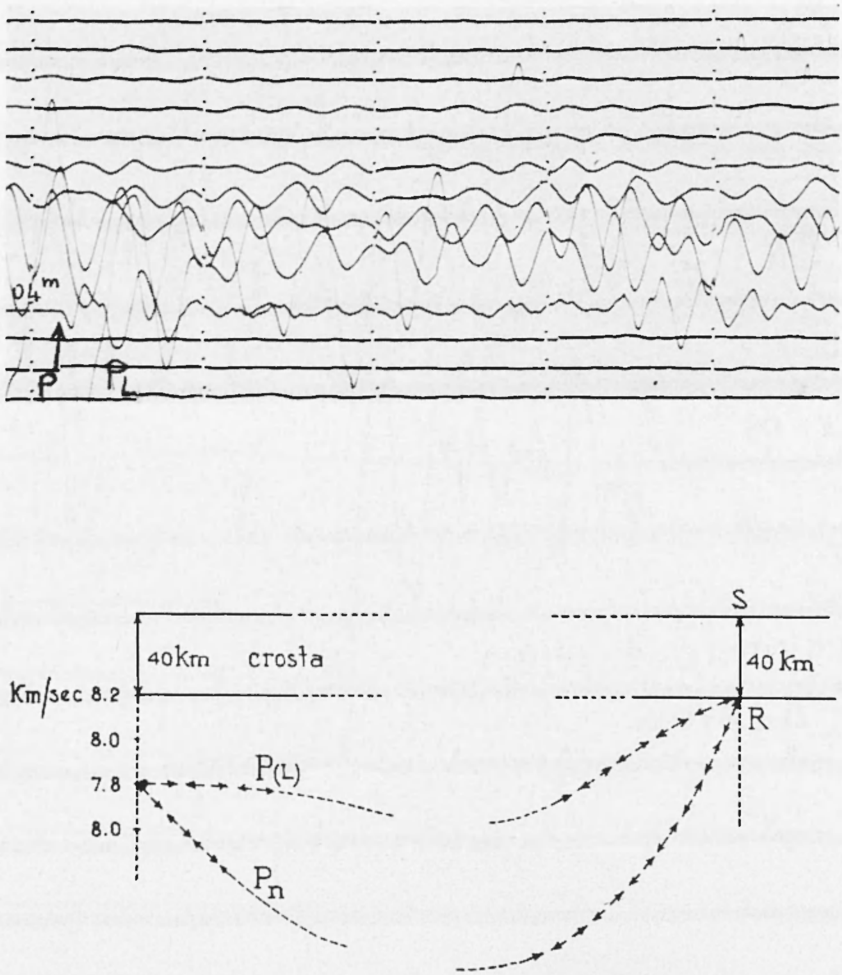

Fig. 13 – *PL* wave in distant earthquake.
(Saint Louis, 11.VIII.1961, $\Delta \simeq$ 8300 km).

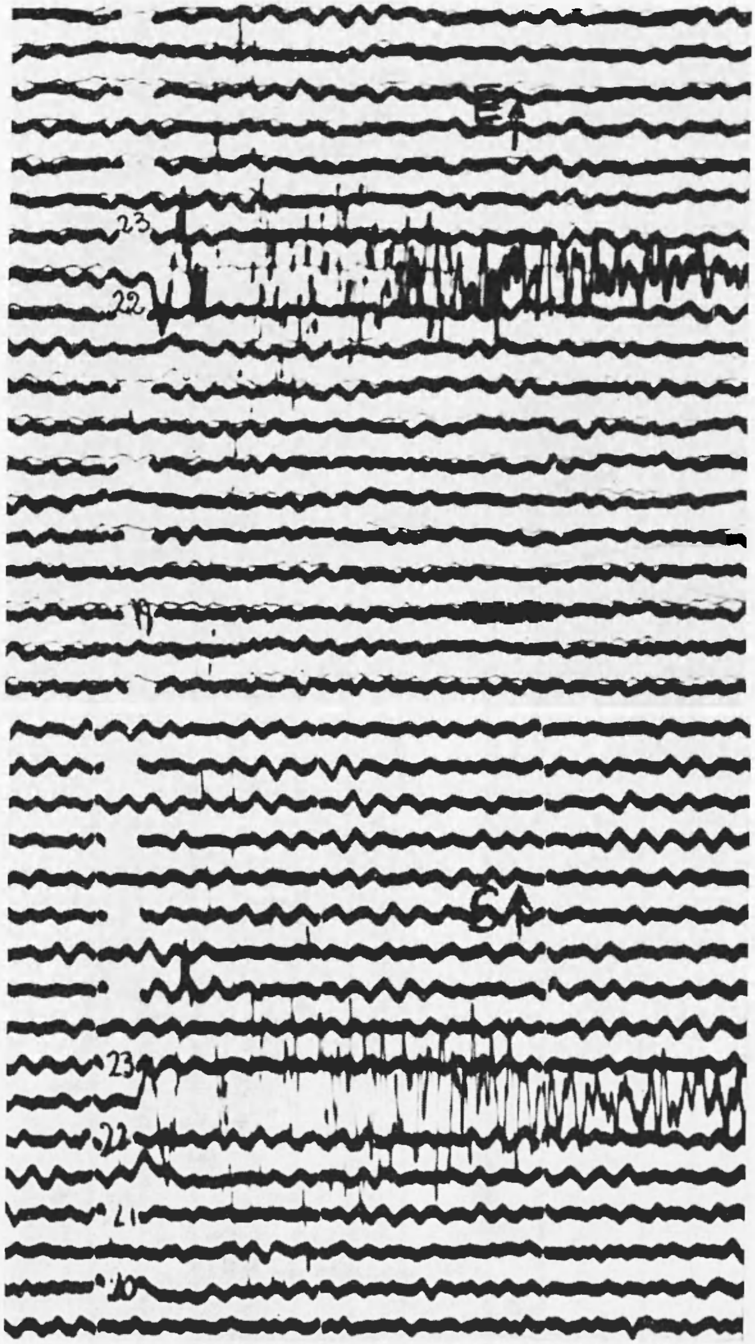

Fig. 14 – Shallow earthquakes may set up *PL* waves at short distance, as in the example recorded at Rome, in connection with the earthquake occurred under the Laga mountains (3–XII–1967), at an epicentral distance of only 100 km. The conducive angle was probably reached at the bottom of the thick alluvial layer of the Tiber valley.

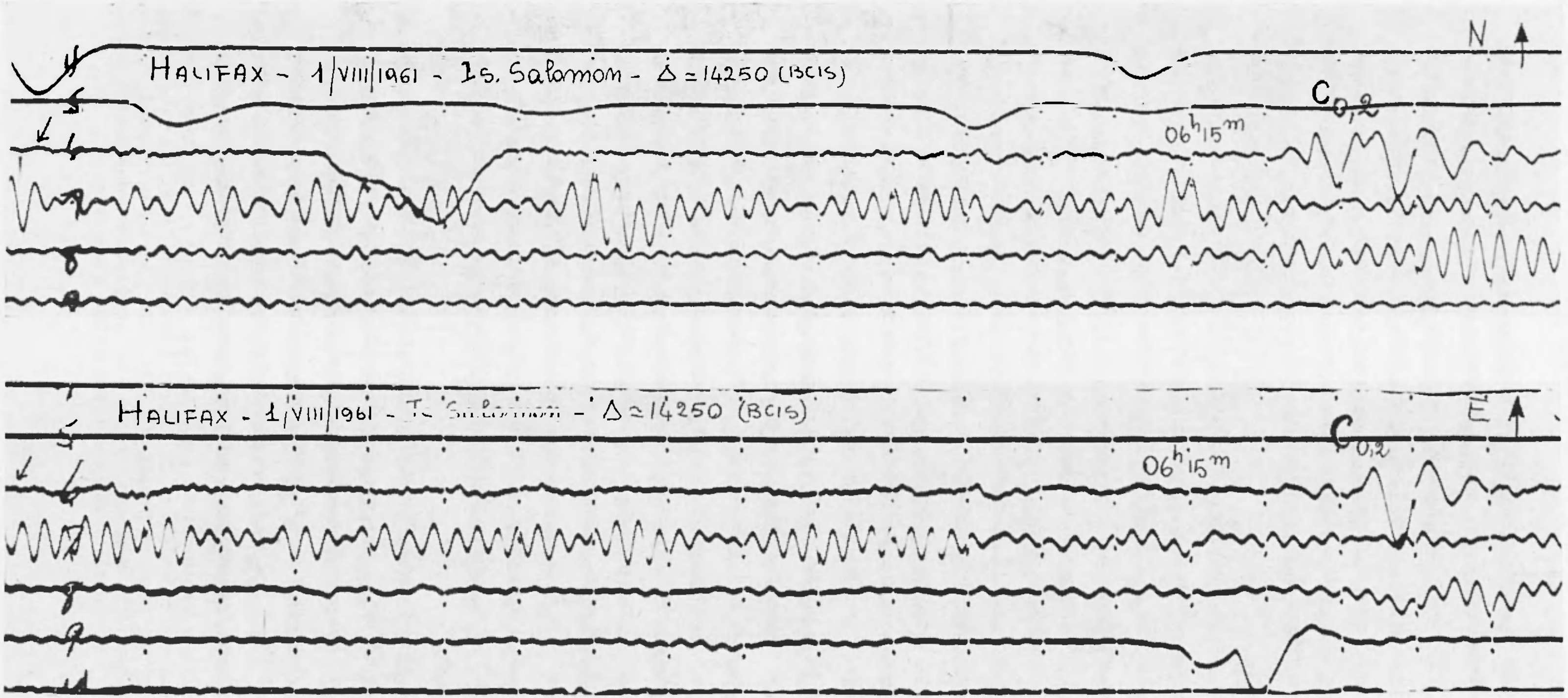

Fig. 15 – The exposed theory justified also the formation of long waves, which on great distances are associated with *SS*, *SSS* ... waves, and for which there seems to be no other possible explanation. See also the figures 16 and 17, in which the $C_{0,2}$ waves represent the largest phases of the whole seismogram.

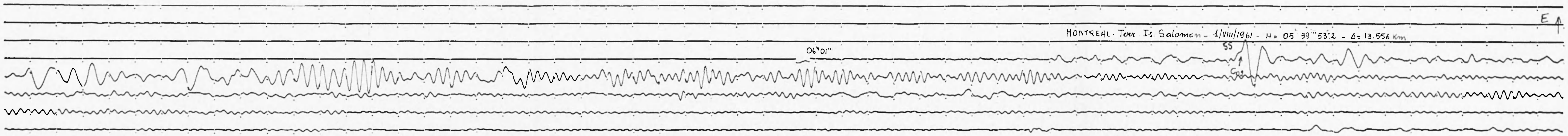

Fig. 16 – See Fig. 15

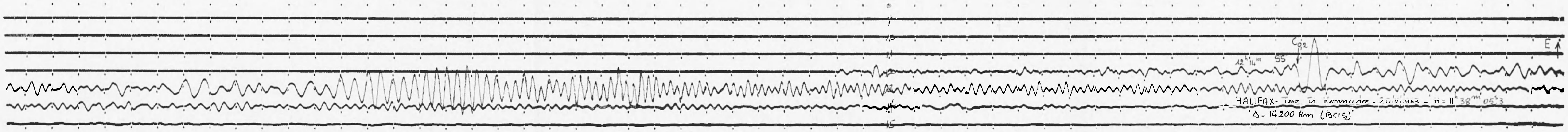

Fig. 17 – See Fig. 15

SSS . . . waves, and for which there seems to be no other possible explanation. The examples shown in figures 15, 16, 17 present the largest phases of the whole seismogram for the $C_{0,2}$ waves. This would prove that $C_{i,j}$ waves have the same free oscillations of the Earth's crust layers, where they take origin.

REFERENCES

(1) CALOI P., *L'equazione di Rayleigh e le onde di Somigliana. II: La teoria di Somigliana; rettifiche, conseguenze.* « Atti Acc. Naz. dei Lincei », Classe Scienze Fis., Mat. e Nat., **XLI**, ser. VIII, fasc. 5, (1966).

(2) CALOI P., *On the Upper Mantle*, pp. 79-211, in « *Advances of Geophysics* », Academic Press, New York and London, p. 127, (1967).

(3) GUTENBERG B., *Physics of the Earth's Interior*, pp. XII + 240, Academic Press, New York and London, p. 181, (1959).

(4) GUTENBERG B., *Wave Velocities at Depths Between 50 and 600 Kilometers.* « Bull. Seismol. Soc. of Am. », **43**, p. 228, (1953).

(5) CALOI P., *Sur l'origine des ondes de type superficiel, associées aux ondes S, SS, SSS.* Bureau Central Séism. Intern., Série A, Travaux Scientific. Fascicule 17, (1948).

(6) CALOI P., « $C_{i,j}$ », « Annali di Geofisica », **VIII**, 3, pp. 293-313, (1955).

(7) CALOI P., *L'equazione di Rayleigh e le onde di Somigliana. III: Le $C_{i,j}$ sono onde di Somigliana. Loro importanza per lo studio della crosta terrestre.* « Atti Acc. Naz. dei Lincei », Classe Scienze Fis., Mat. e Nat. **XLIII**, fasc. 6, (1967).

(8) CALOI P., *A proposito delle onde SL ed SM.* « Boll. Com. Geod. e Geof. del C.N.R. », **IV**, (1934).

(9) CALOI P., PERONACI F., *Onde superficiali associate alle onde S, SS, ... nel terremoto del Turkestan del 2 Novembre* 1946. « Annali di Geofisica », I, 4, pp.520-525, (1948).

(10) CALOI P., PERONACI F., *Ancora sulle onde di tipo superficiale associate alle S, SS, nel terremoto del Turkestan del 2 Novembre 1946.* « Annali di Geofisica », **II**, 2, pp. 290-294, (1949).

(11) SOMVILLE O., *A propos d'une onde longue dans la première phase de quelques seismogrammes.* I. « Gerlands Beiträge zur Geophysik », **27**, (1930).

(12) SOMVILLE O., *A propos* .... II. « Gerlands Beiträge zur Geophysik » **29**, (1931).

(13) LANDSBERG H., *Beobachtungen zur PL-Welle.* « Gerlands Beiträge zur Geophysik », **29**, (1931).

([14]) OLIVER J., MAJOR M., *Leaking modes and the PL phase.* « Bull. Seism. Soc. Am. », **50**, (1960).

([15]) PHINNEY R. A., *Leaking modes in the crustal wave guide.* « Journ. Geophys. Research », **66**, (1961).

([16]) GILBERT F., LASTER S. J., *Experimental investigation of PL modes in a Single Layer.* « Bull. Seism. Soc. Am. », **52**, (1962).

([17]) OLIVER J., *Propagation of PL waves across the United States.* « Bull. Seism. Soc. Am. », **54**, (1964).