Behaviour of the electric conductivity of the precipitations

A. Murri (*)

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Summary. — Continuing the researches on the precipitations, at the Meteorological Observatory of Macerata and the Physics Institute of the University of Camerino, during the years 1967/68/69/70 measurements have been carried out on the conductivity of liquid and solid precipitations with the aim of determining the behaviour at different altitudes.

It has thus been found that the conductivity decreases linearly in the course of the rain fall if the wind remains constant in direction, while the linear behaviour ceases with the changing of direction of the wind. The behaviour of the conductivity is analogous at the two quotas of observation, except the value of the same which appears minor at higher quota. The law of distribution of the conductivity, considered as regards the density of rainfall, may be represented with a particular function which has been calculated. However the dispersion of the values appears considerable, in relation to the different meteorological situations that accompanied the fall of the precipitations, object of the research. There have also been fixed the transport equations that regulate, in turbulent air masses, the diffusion of the particles to which the conductivity of the rains is due.

Riassunto. — Continuando i lavori di ricerca sulle precipitazioni, presso l'Osservatorio Meteorologico di Macerata e l'Istituto di Fisica della Università di Camerino, durante gli anni 1967/68/69/70 sono state compiute misurazioni sulla conduttività delle precipitazioni liquide e solide, allo scopo di determinarne il comportamento a diverse altitudini.

È stato così trovato che la conduttività diminuisce linearmente nel corso della caduta della pioggia se il vento si mantiene costante in direzione, mentre l'andamento lineare cessa al cambiare della direzione del vento. Il comportamento della conduttività è analogo alle due quote di osservazioni, salvo il valore della stessa che appare minore a quota più alta. La legge di distribuzione della conduttività, considerata rispetto alla densità di caduta

(*) Istituto di Fisica – Università di Camerino – Osservatorio Meteorologico, Macerata.

della pioggia, si può rappresentare con una particolare funzione che è stata calcolata. Appare però notevole la dispersione dei valori, in relazione alle diverse situazioni meteorologiche che hanno accompagnato la caduta delle precipitazioni, oggetto della ricerca. Sono state anche stabilite le equazioni di trasporto che regolano, in masse di aria turbolente, la diffusione delle particelle alle quali è dovuta la conduttività delle pioggie.

Introduction.

In the course of the researches carried out on air pollution at Macerata (Meteorological Observatory) and at Camerino (Physics Institute of the University) there has been made, as already done previously for the *Ph* [17] a systematic measure of the electric conductivity of liquid and solid precipitations.

The researches have been made using conductometres on which it is possible to read simultaneously the values of the conductivity and the resistivity. The instruments were initially calibrate and the control measures repeated every week; the collection of the precipitations was made with the methods already described elsewhere [17].

The precipitations collected were examined immediately so as to avoid any modifications in their composition; the solid ones were reduced to liquid state leaving them to melt at room temperature in containers protected from any pollution.

The precipitations on which measures have been carried out are many, of different intensity, form and duration and above all they appear determined by every possible meteorological situation that is normally determined in the region of observation. They cover a time arch of four years.

In almost every case the conductivity has been examined at brief regular interval, measuring the precipitation fallen in the interval between one measure and the following one, as well as at the beginning and at the measures were different according to the intensity of the precipitations.

The aim of such research is to investigate how the conductivity behaves in relation to the quantity of precipitation in the time and to the evolving of the meteorological situation on the ground.

It has also been held useful to consider the observations in their complex and to try to find a mathematical expression capable of expressing the experimental law of relation between the quantity of rain fallen reported to the time and the electric conductivity. Finally the transport equations [2,4,14] have offered a means for expressing the distribution of the concentration C of the aerosol that have given place to the conductivity found, in relation to the turbulent motion of the air masses that carry them.

2. – At the moment in which the rain drops begin their fall in the free atmosphere, two processes begin simultaneously; one of evaporation, another of the capture by these of the aerosol present in the atmosphere crossed through. The concentration of aerosol in the rain may increase for such pre-existing causes. If the total concentration is indicates with C, it could be written in the form:

$$C = f_1 C_1 + f_2 C_2$$

where C_1 and C_2 are the partial concentrations due to the above-mentioned causes and $f_1 \geqslant 1$ is a variable coefficient dependent on the evaporation undergone by the drop, while the coefficient f_2 results almost unitary it being possible to show experimentally that for it the two effects of collection of humidity and evaporation compensate each other frequently almost equal between themselves [9,10,13,15].

If we admit now that the aerosol crossed by the rain is formed by particles of r ray, distributed in space with any law of distribution what so ever $n(r)$, that the section of collision of the rain drops is πa^2 with a ray of the drops and $\eta \leqslant 1$ a collection coefficient, it has already been shown that the value of C_2 may be expressed with sufficient approximation by [·10,15];

$$C_2 = \frac{\pi}{a} H \int \eta r^3 \, n(r) \, dr$$

where H indicates the height of the base of the clouds from which the rain detaches itself.

The experimental calculation of the coefficient η is due to different authors, but Langmuir has given an expression (1957) in function of the a ray of the drops. If we assume the simplificative hypothesis that the rain develops in a windless atmosphere it has been shown that the fraction of concentration due to the washout may be indicated approximately with [9,10,13,15]:

$$C_2 = \frac{4}{3} \frac{\pi}{\varrho} H \, \eta \cdot r^3 \left[1 - \exp\left(- \frac{3}{4} \frac{\eta p}{a} \right) \right]$$

where p indicates the quantity of rain and C_2 is expressed in its function.

The problem thus set up allows us to consider the rain water conductivity, as function of the concentration of aerosol captured by it during the crossing of a volume of air at H height; in final analysis the conductivity becomes function of the C concentration of the aerosol present in the atmosphere at the moment of the rain. Therefore the conductivity would present itself as a certain function of the quantity of water fallen and of the general meteorological conditions, on which depends the C concentration of the atmospheric aerosol, as on the other side we have calculated by means of the function examined in par. 5.

That has been demonstrated by Landsberg in 1954 with the analysis of *Ph* of single drops and by the writer with the analysis of *Ph* of the precipitations at different altitudes (17).

The measures of conductibility have been made at a higher quota to that where the layer of thermic inversion for the region of observation generally forms, as can be seen from the diagram of the potential temperature given as example in case n° 1 examined. For that reason it may be held that the observations are exempt from the anomalies that the distribution function $n(r)$ of the aerosol could undergo in consequence of observations made below the inversion layer.

Here are now shown and examined different cases of measure of conductivity made during rains with winds constant and winds variable in direction and intensity during measure, others in presence of fogs. The examination of the cases seems to allow us to say that, at least in first approximation, the conductivity of the precipitations appears as a linear function of the quantity of rain fallen as regards the time employed in precipitating, as long as the direction of the wind remains constant.

The influence of the wind is highlighted by those cases in which, at a change of direction of the wind there corresponds an increase or a variation of conductivity, coincident in time with the directional variation of the wind.

3. – *Methodology of the cases examined.*

From among all the cases examined four have been chosen at random, precisely that of 12/6/1968, of 18/9/1968, of 5/11/1968, and 30/11/1969, as indicators of all those examined.

a) Case of 12/6/1968. – The general meteorological situation was characterized by an atlantic wedge driven as far as the British isles, while a depression to the North of the Alps generated a secondary depression from the Gulf of Genoa. The Mediterranean was invested by a cold air inflow.

The perturbed conditions in Central Italy were strengthened by this situation. The rain of stormy character spent itself in the observation area in 65 minutes. The wind blew from NW with a medium velocity of 17 km an hour with gusts from 22 to 25 km an hour. The rain began at 12.30 TMEC, and the first examination of conductivity was made at 13.00. Other examinations followed at 10 minute intervals, on samples collected in the interval. The behaviours found are those indicated in Fig. 1. From these it results that the conductivity diminishes almost linearly with the increase of the quantity of total rain fallen. That presupposes that the conductivity diminishes gradually as the washout of the atmosphere increase and the concentration of the nuclei decreases, modifying the distribution function $n(r)\,dr$ only in virtue of the quantity of water precipitated, and not of the other meteorological conditions, remained unchanged during the measure.

b) Case of 18/9/1968 – A depression to the North of the Alps generated on the Ligurian Gulf a secondary depression which, in its displacement towards the East crossed the Adriatic Regions. The passage of the cold front caused heavy rains of stormy character, accompanied by fogs, in the valleys. Moderate winds from NW.

In the observation station the rain began at 16.30 TMEC while the observations of conductivity were begun at 17 and followed at intervals of 15 minutes.

The total quantity of rainfall was 33,2 mm in 215 minutes. The behaviour of the conductivity is that indicated in Fig. 2. It may be considered with best linear approximation. The direction and intensity of the wind remained constant during all the measure.

c) Case of 5/11/1969 – A deep atlantic depression covered the whole european region, bringing perturbed weather to the mediterranean area. In Central Italy there developed a mixture of rains and storms, linked to the above-mentioned depression. The precipitations, because of the lowering of the zero isotherm, acquired a snowy character at low quotas. Formations of thick and low fog arrived up to a quota of 3000 metres. Variable winds from NW and from W-SW.

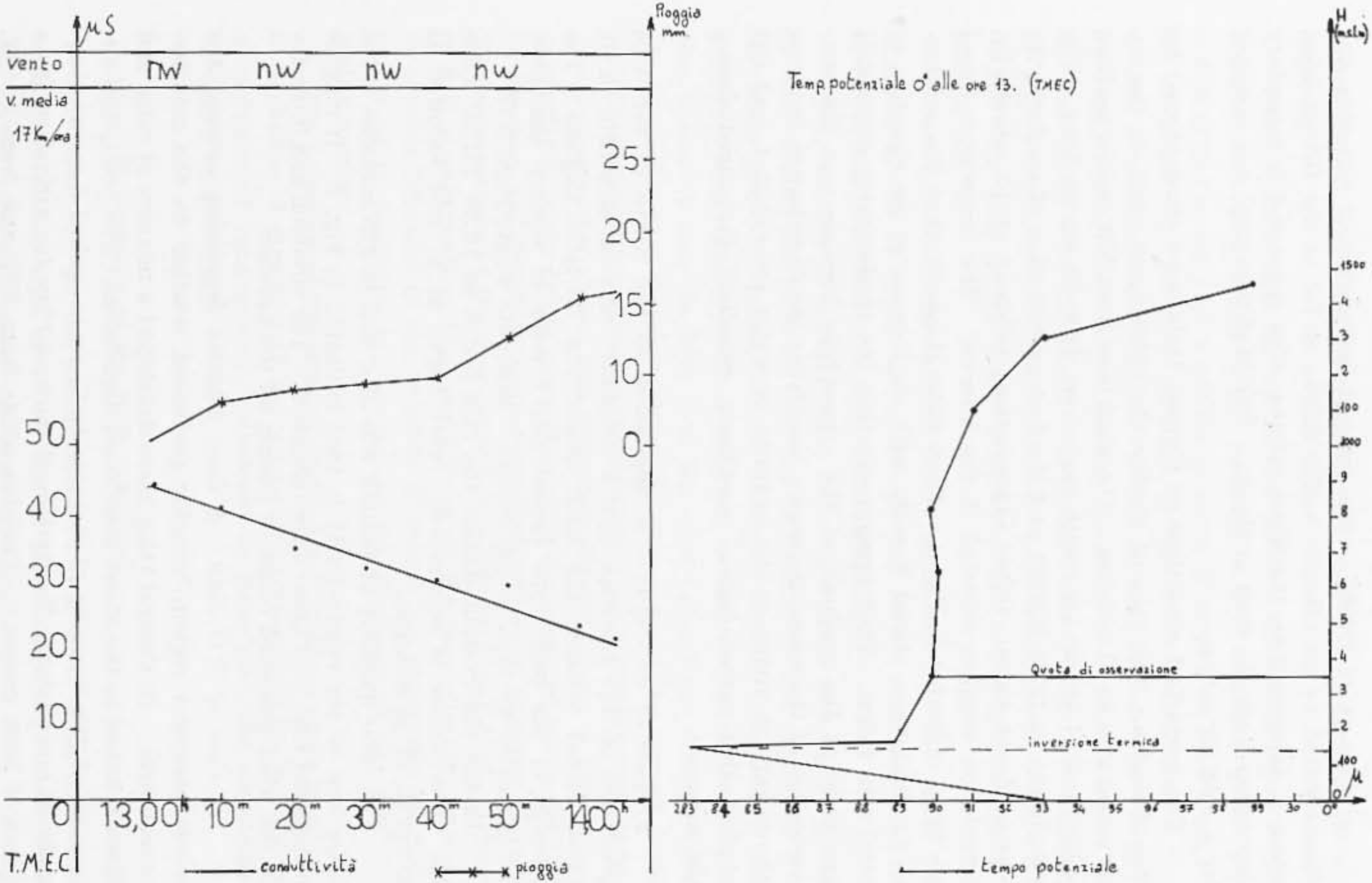

Fig. 1 – Behaviour of the conductivity of the rain, and potential temperature during the case of 12-6-1968.

The rain began at about 8.30 accompanied by NW winds. The first measure of conductivity was made at 8.45 and the following at 15 minute intervals until the end of the rain.

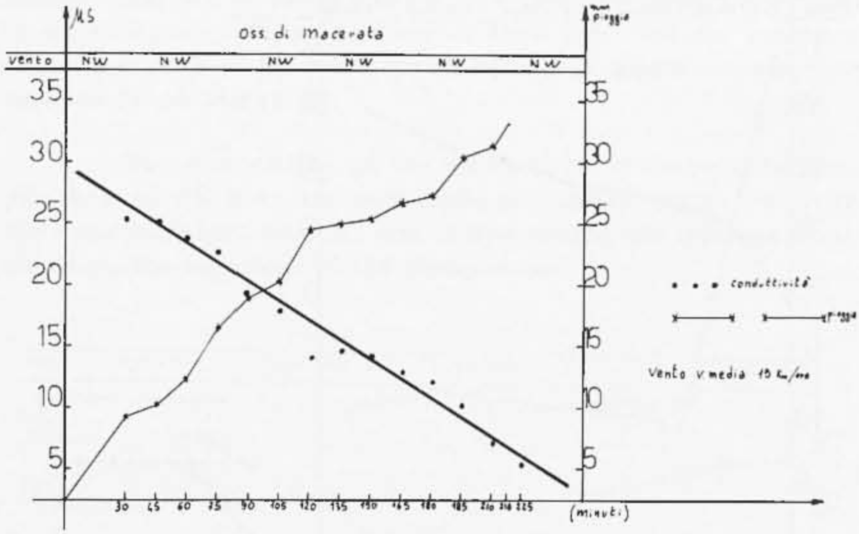

Fig. 2 – Behaviour of the conductivity during the case of 18–9–1968.

The behaviour of the conductivity of the rain was analogous to that described in the other cases, that is to say decreasing in almost linear number, until 10.15. At that hour the conductivity presented at first stationary values, then a met increase, in coincidence with the changed direction of the wind that then blew from SW, with increasing velocity. The increase continued until around 14.00 in concomitance with the gradual intensificating of the wind velocity; at that hour the values of conductivity suffered a new decrease, again in concomitance with the diminuition of wind velocity. The behaviour of the rain during the phenomenon is that shown in Fig. 3.

The behaviour found on that occasion, and verified in other cases, seems typical. It reveals what is the influence of a change of direction of wind on the measures of conductivity, conducted on rains coming from the same cloud system.

The behaviour of the phenomenon is coherent, in the first part, with what was found in the other cases of wind blowing constantly from the same direction. There is in fact in such cases a considerable effect of washout of the aerosol present and a consequent fall of the values of conductivity on the diminuition of the C concentration of the aerosol.

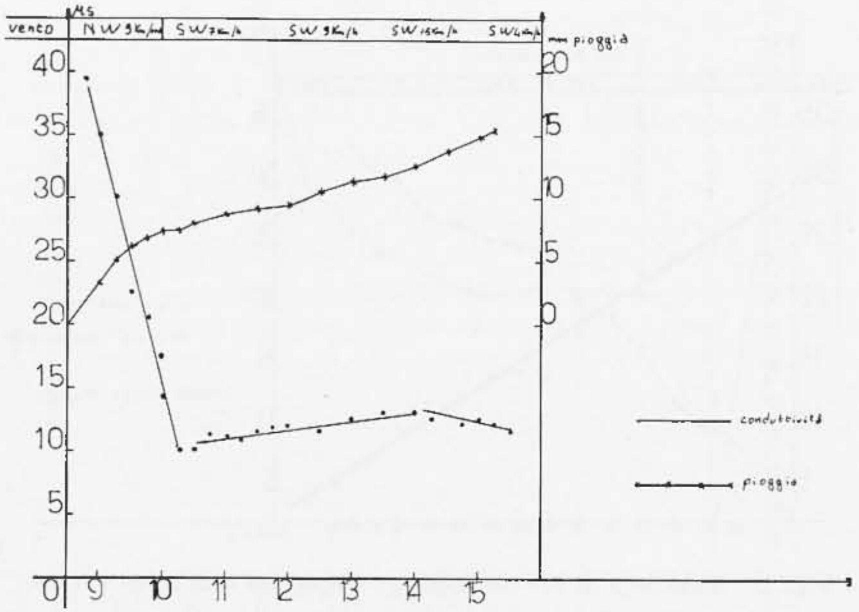

Fig. 3 – Conductivity of the rain, during a case of variable Winds (5–11–1968).

The prevailing of the SW wind, modifying the concentration of the aerosol and their function of distribution with the bringing of new material, produces an increase of conductivity. It diminishes again with the diminishing of the wind intensity and on the continuing of the effect of washout caused by the rain.

d) Case of 30/11/1969 – A deep depression with centre in the West Mediterranean moved East and in its slow passage affected all Italian regions. On Central-Adriatic Italy cold air from N-NE. Sky with very heavy and low cloud; very heavy precipitations at low quota. Winds of very variable intensity with a tendency to wheel from the North quadrants to the South quadrants.

The rain began about 4.00 in the morning and first measure of conductivity made at 6.00. Since the density of the rainfall was small the measures were made every hour. Up to 11.00 the behaviour of the conductivity was that already found on the other occasions exposed. At about 11.00, in coincidence with a diminuition of the wind intensity, there was a formation of thick fog. The measures of conductivity, taken every hour on the fractions of rain fallen in the interval, show a tendency to a stabilization of the values of the element measured on about 22,5 μS. The same condition was verified in all analogous cases in presence of thick fogs, and the minimum values measured of the conductivity on such occasions are comprised between 24 μS and 15 μS.

4. – The observations on the conductivity of the precipitations, like those of *Ph*, have also been made at Camerino (quota 700) with the same modality, with the aim of ascertaining the influence of altitude on the behaviour of the phenomenon.

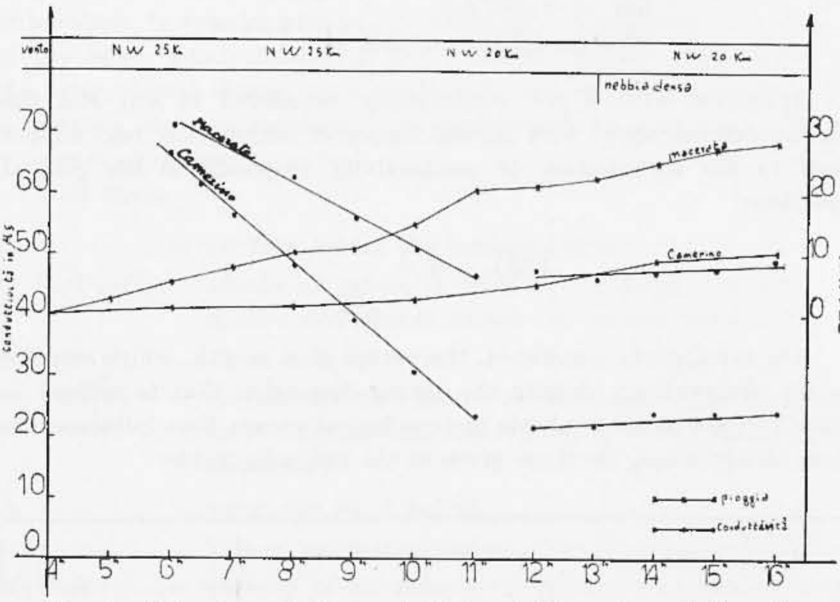

Fig. 4 – Behaviour of the conductivity at Macerata and Camerino (30-11-1969).

In Fig. 4 there is given one of the typical behaviour found in a winter case and from which it is easy to observe how the course of the phenomenon in the station at higher quota has an analogous behaviour to that found for the station at lower quota.

The values of conductivity at Camerino remained, in the case under examination, lower than those found at Macerata, in function of the higher altitude, as always noted also in the other cases. The formation of fog, present in the two observation stations at the same hour, determines also the analagous behaviour of the conductivity in the two places, registering a slight increase.

5. – From the observations we have made an experimental equation which may represent with sufficient exactness the distribution of the conductivity values, found in all the cases taken into consideration.

With that aim 115 observations of conductivity of precipitations, carried out from July 1967 to July 1970 have been considered. The conductivity was brought to a temperature of 0°C and the values obtained have been ordinated according to the density of fall in the time of precipitation, defining such density according to the relation:

$$\frac{\text{mm of water fall}}{\text{effective minutes of fall}} \times 10 = d\,.$$

Indicated with Y the conductivity (measured in μS) and the density defined above with X, the family of curves that best adapts itself to the distribution of conductivity responds to the general equation:

$$Y(X) = a + \frac{b}{\sqrt[n]{X^m}}\,.$$

On calculations completed, the values of n, m, a, b, which respond to the observations, despite the strong dispersion that is noticed in these because of the multiple meteorological causes that influence the same observations, are those given in the following Table:

n	m	a	b	ε
3	2	27,130	9,299	156128,14
4	3	29,163	7,230	183392,96
5	3	25,206	11,388	155396,17
7	4	24,249	12,432	155068,01
7	5	28,289	8,053	156619,32

The values of a and b corresponding to the couples n, m are those which render minimum the value of ε and which resolve the system

$$\frac{\partial \varepsilon}{\partial a} = 0 \qquad \frac{\partial \varepsilon}{\partial b} = 0 \; .$$

The form found of the curve is on the other hand analagous to that found by other authors (5) in researches on the chemical physics of the precipitations. The curve reported in Fig. 5 is that for the values of $n = 5$, $m = 3$ that better responds to the values found experimentally.

6. – The behaviour of the conductivity of the precipitations depends on the concentration C of the aerosol in the same precipitations, while the aerosol are collected by the rain that crosses through the atmosphere, in relation also to the laminary and turbulent movements of the same, which modifying the quantity and the distribution of the same aerosol change their concentration in the precipitations. It is possible then to represent the phenomenon by means of suitable transport equations (2, 4, 14, 15).

Let then:

ϕ	=	air flow across the unitary surface
$U; U_r; U_a; U_b$	=	steady air mass; in motion; in motion upwards; in motion downwards across the unitary surface σ
$t; \Delta t$	=	time; time interval
$C; C_r; C_v; C_t$	=	specific size that the air masses carry across the unitary surface; same size carried by the rain; wind; in the time t
C_z	=	same size at Z height
$a; b;$	=	indices indicating motion upwards and downwards
$W; W_r; (+;—)$	=	velocity of air masses (+ upwards; — downwards); velocity of rain masses
$V; V_H; V_Z$	=	wind velocity and its horizontal and vertical components
$\rho, \bar{\rho}, \rho_r, \rho_v$	=	air density; medium air density; rain density; air density during wind
K	=	coefficient of the general equation of the exchange of air masses $\left[\phi_c = - K \dfrac{\partial c}{\partial z}\right]$.

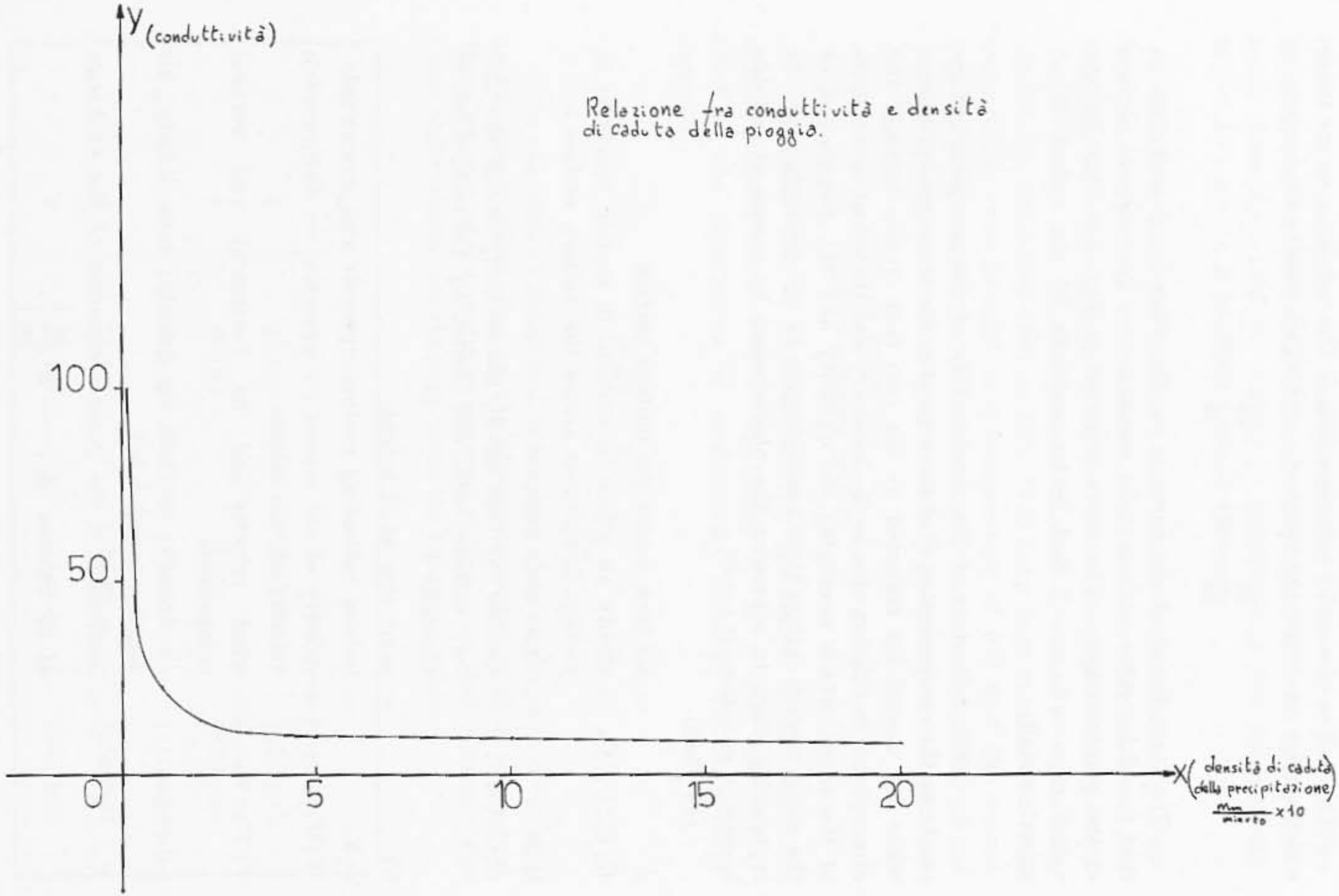

Fig. 5 – Curve of distribution of conductivity.

The equations that regulate the phenomenon are linked to the orientated movements of masses of the atmosphere and of masses in the atmosphere (rain). Of the former we can consider the horizontal and vertical components in the both directions; of the latter we shall have to consider the vertical component from the top downwards, with the hypotheses that the medium values of the meteorological sizes in the horizontal direction do not change. We shall postulate that such masses cross the surface σ, placed at Z height, of dimensions sufficiently great as regards the horizontal dimensions of the masses in vertical movement.

That fixed, we now consider only the vertical transport of specific size C, operated by the masses of turbulent air in the time Δt across the surface σ. The intensity of the flow ϕ of the specific size C in the time Δt will then be, across σ:

$$\phi_{[C, \Delta t, \sigma]} = \left| \sum_i U a_i C_{a(z)} - \sum_i U b_i \ C_{b(z)} \right| + \\ + \left| \sum_i U^{+} a_i \ C^{+}_{a\,(z)} - \sum_i U^{-} b_i \ C_{b(z)} \right| \qquad [1]$$

where with U_a, U_b are indicated the neutral masses of air in motion upwards and downwards across σ, and with $U^{+}_{a,b}$, $U^{-}_{a,b}$ the hot and cold masses also in motion $C^{+}_{a,}{}^{-}_{b}$ the values of the specific size at Z height concerning the air masses now mentioned.

In agreement with the theory of turbulence (1, 2, 4, 6, 12, 13, 15) we consider a particle of air belonging to the mass U_a and which is carried to a heigt $Z_a < Z$ where the specific size C, characteristic of such a particle and which we will indicate with $C_{a(Za)}$, is equal to the medium value of size C of the air mass at such an altitude (10, 15, 18). Taking this into account we could write at least in first approximation, and considering that the functions are continuous:

$$C_{a(z)} = C_{a(za)} + \frac{d\,C_{(z)}}{dZ} \ (Z - Z_a) \qquad [2]$$

that could again be expressed, if we refer to the Z level:

$$C_{a(z)} = C_{(z)} + \left| \frac{\partial\,C_{(z)}}{\partial Z} - \frac{d\,C_{(z)}}{dZ} \right| (Z - Z_a) \qquad [3]$$

where $\frac{dC_{(z)}}{dZ}$ - indicates the specific variation of the size C that takes place during the unitary increase of height of the mass U_a. We could now make different considerations, which would allow us to arrive immediately at the value of the intensity of the flow of quantity C across σ in the unity of time.

It is therefore evident that, adding and taking away $C^+_{(z)}$ we may write:

$$\sum_i U_{ai} \overset{+}{C}_{a,b(z)} = \sum_i U_{i(a,b)} \left| \overset{+}{C}_{(z)} - \overset{+}{C}_{a,b(z)} - \overset{+}{C}_{(z)} \right| = \\ = \varrho^+ W^+ \overset{+}{C}_{(z)} \sigma \cdot \Delta t \qquad [4]$$

where the double index a, b, indicates how the equation serves for the ascending masses as well as for the descending and the values ϱ^+, W^+ represent respectively the medium value of the density, the vertical velocity of specific size C of the ascending air masses (descending) at Z height. The medium value $C^+_{z)}$ is given by:

$$\sum_i U_{ai} \left| \overset{+}{C}_{a(z)} - \overset{+}{C}_{(z)} \right| = 0 \,. \qquad [5]$$

Taken into account then that among the ascending and descending masses there is the evident relation:

$$\sum_i Ua_i - \sum_i Ub_i = \varrho \, W \sigma \cdot \Delta t \qquad [6]$$

where ϱ, W are the medium density and the medium velocity at Z height, from equation [1], taken into account the postulated continuity of the motion and of the space we obtain, for the intensity of the flow of size C, across the horizontal unity surface, in unitary time:

$$\phi_c = K \left| \frac{dC}{dz} - \frac{\partial C}{\partial z} \right| + (\varrho \, W) \, C + (\varrho^+ W^+) \, C^+ + (\varrho^- W^-) \, C^- \qquad [7]$$

where K is defined by the relation:

$$K = \frac{1}{\sigma \cdot \Delta t} \left| \sum_i Ua_i \, (Z - Z_a) + \sum_i Ub_i \, (Z_b - Z) \right|$$

and coincides with the noted coefficient of exchange between air masses (1, 10, 13, 14, 15).

In [7] the velocity W must be considered with the two signs, it having ascending, descending and steady masses, and therefore $W \gtreqless 0$ and also $W^- \leqslant 0$, $W^+ \geqslant 0$. When then is carried because of the rain (case of the experiments) then to [7] will have to be added a term that takes count of that, W_r being the velocity of rain fall and ϱ_r its density, C_r the specific size for the rain. It will then be:

$$\phi_c = K \left| \frac{dC}{dz} - \frac{\partial C}{\partial z} \right| + (\varrho W)\, C + (\varrho^+ W^+)\, C^+ + (\varrho^- W^-)\, C^- + (\varrho_r W_r)\, C_r\,. \quad [8]$$

Besides the action of transport of the specific size C on the part of the rain, it is necessary to consider transport due to wind, as on the other hand is shown by experiments reported in 1ª part. It is known how the wind, in proximity to the ground, does not run parallel to this but can assume a bent more or less accentuated in conformity with the friction met up with, of the form of the isobars, etc.

If then V represents the velocity vector of the wind in unitary time, we consider the parallelepiped having V as diagonal and the corners lying respectively on the horizontal plane parallel to the surface of observation and on the normal to this.

The vector V will then be able to decompose in two vectors V_H, lying in the horizontal plane, and V_z along the vertical.

To the equation [8] we must therefore add the terms relative to the action of the wind, which are:

$$(\varrho_v V_H)\, C_v$$

for the horizontal component

$$(\varrho_v V_z^+)\, C_v; \quad (\varrho_v V_z^-)\, C_v^-$$

for the vertical components that may be descending (—) or ascending(+)

The equation [8] becomes then:

$$\phi_c = K \left| \frac{dC}{dz} - \frac{\partial C}{\partial z} \right| + (\varrho W)\, C + (\varrho^+ W^+)\, C^+ + (\varrho^- W^-)\, C^- +$$

$$+ (\varrho_r W_r)\, C_r + (\varrho_v V_H)\, C_v + (\varrho_v^+ V_z^+)\, C_v^+ + (\varrho_v^- V_z^-)\, C_v^- \qquad [9]$$

The equation obtained could formally make one think of the well known exchanged equation of the air masses (1, 4, 5, 16, 17).

$$\phi_c = -K \frac{\partial C}{\partial z} \qquad [10]$$

but this latter is valid only in the case that in the observation point there exist only air masses in priviliged conditions, that is without their own regime of motion, devoid of turbulence or such that at least during such motions is $\frac{dC}{dt} = 0$, while all the vertical components of the motion are null. Such conditions were evidently not verified, in our case; however it remains ascertained that the equation [9] is substantially different from [10], even if formally similar.

The medium local variation of the size C may be obtained from the preceding equation [10], indicating with $\bar{\rho}$ the medium density of the air in observation site at the moment of same, deriving the [10]:

$$\begin{aligned} \frac{\partial \bar{C}}{\partial t} = \frac{1}{\bar{\rho}} \Bigg\{ \frac{\partial}{\partial z} \Bigg| K \left(\frac{\partial C}{\partial z} - \frac{dC}{dz} \right) - \varrho W C - \varrho^+ W^+ C^+ - \varrho^- W^- C^- + \\ - \varrho_r W_r C_r - \varrho_v V_H C_v - \varrho_{v^+} V_{z^+} C_{v^+} - \varrho_{v^-} V_{z^-} C_{v^-} \Bigg| + \\ + \varrho W \frac{dC}{dz} + \varrho^+ W^+ \frac{dC}{dz} + \varrho^- W^- \frac{dC}{dz} + \varrho_r W_r \frac{dC_r}{dz} + \qquad [11] \\ + \varrho_v V_H \frac{dC_v}{dz} + \varrho_{v^+} V_{z^+} \frac{dC_{v^+}}{dz} + \varrho_{v^-} V_{z^-} \frac{dC_{v^-}}{dz} \Bigg\} \end{aligned}$$

with which it is taken into account that the medium local variation $\frac{\partial \bar{C}}{\partial t}$ of the size C is influenced by the variations of C due to the different factors enunciated above, or that is to the transport due to wind, rain, etc. The [11] represents in our case the generalization of the equation of diffusion, and it gives count of the changes operated by the movement of air masses in the values of the specific size C, as also of the change caused by rain fall, which acquiring the specific size C or a part of it, leaves a trace that is the phenomenon of washout.

7. – The equations [9] and [11] of the preceding paragraph lend themselves then to the interpretation of the behaviour of the conduc-

RECENSIONI

J.-J. LEVALLOIS, *Geodesie Générale.* Tome I: *Méthodes générales et techniques fondamentales.* « Collection Scientifique de l'Institut Géographique National ». Volume di pp. 404, con 80 figure e 56 tabelle. Eyrolles, editore. Paris – 1969. 98 F.

È un'opera, a largo respiro, divisa in quattro volumi.

Il primo volume — di cui qui si tratta — è composto di quattro capitoli.

Nel primo capitolo, dopo le definizioni generali (gravità, latitudine, longitudine, azimut, ellissoide di riferimento, ecc.), si espongono i metodi della geodesia geometrica (triangolazione, scelta dell'ellissoide di riferimento, coordinate geografiche geodetiche, rappresentazioni piane), le misure di altitudine (livellamento astro-geodetico, geodesia astronomica), il concetto di geodesia dinamica, la gravimetria (determinazione della forma della Terra, il campo esterno, le deviazioni della verticale), la geodesia tridimensionale e i satelliti artificiali, le determinazioni geometriche della forma della Terra.

Il secondo capitolo tratta della teoria degli errori. Si dà il significato di osservazioni dirette e indirette, di equazioni di condizione e di errori di osservazione, di precisione, di errori apparenti e veri, di scarti e di residui. Si passa quindi a trattare degli errori accidentali delle misure dirette, del diagramma delle frequenze, della curva di Gauss e della correlazione, del metodo dei minimi quadrati, dell'uso delle matrici, della precisione delle osservazioni e dei calcoli, del metodo delle equazioni di condizione e del calcolo degli errori medi. Un intero capitolo è dedicato alla rifrazione atmosferica (rifrazione astronomica, rifrazione geodetica, rifrazione laterale, rifrazione delle onde radioelettriche nell'atmosfera). Il quarto capitolo è riservato alle misure geodetiche: triangolazioni, misure angolari (precisione possibile, errori sistematici, metodi operativi, misure di distanze zenitali, ecc.), misure di distanze, misure elettromagnetiche di distanze, misure astronomiche, determinazioni delle coordinate geografiche, livellamento di precisione, misure della gravità, la rete gravimetrica mondiale, ecc.

P.C.

J.-J. LEVALLOIS, *Géodésie Générale. Tome II: Géodésie classique bidimensionnelle.* « Collection Scientifique de l'Institut Géographique National ». Volume di pp. XII+408 con 159 figure e 37 tabelle. Eyrolles, editore. Paris – 1970. 134 F.

È il secondo volume della « Geodesia Generale » di Levallois.

Il capitolo con cui si apre il libro (V dell'opera) è dedicato alla geometria dell'ellissoide di rivoluzione e ai calcoli sull'ellissoide. Dopo alcuni richiami di geometria, l'A. si sofferma sulle curve gobbe, sulle proprietà locali delle superficie, sulla curvatura totale e l'eccesso angolare, sui triangoli geodetici, sulle geodetiche delle superficie di rivoluzione e, in particolare, dell'ellissoide di rivoluzione, sui metodi d'inversione numerica, sulle costanti di alcuni ellissoidi, ecc. Il successivo capitolo tratta della rappresentazione dell'ellissoide sulla sfera o sul piano. In esso si espongono i concetti di rappresentazione piana conforme dell'ellissoide, di curvatura delle trasformate piane, di trasformazioni per numeri complessi, e si descrivono le proiezioni di Marcatore, di Lambert e di Mercatore trasversa, indicando le vie per il passaggio approssimativo da una proiezione conforme ad un'altra proiezione conforme. Viene quindi esposto l'impiego delle proiezioni in navigazione e la rappresentazione conforme dell'ellissoide sulla sfera. I metodi di compensazione delle triangolazioni classiche costituiscono l'argomento del 3° capitolo del volume (VII dell'opera). Viene esposta la conformazione dei triangoli e la struttura delle relazioni d'osservazione. Si descrivono gli aggiustamenti grafici e la messa in posto dei luoghi geometrici. Si passa quindi all'applicazione del metodo dei minimi quadrati (metodo delle osservazioni dirette condizionate, metodo delle variazioni di coordinate, metodo delle giaciture; esempi numerici). Accennato alle grandi compensazioni del passato, il volume si chiude con l'esposizione dei metodi spediti (di Bowie, d'Ourmaieff, ecc.) delle compensazioni rigorose (metodo dei gruppi) e dei metodi per sviluppo (metodo di Boltz).

P.C.

J.-J. LEVALLOIS, *Géodésie Générale. Tome III: Le champ de la pesanteur.* « Collection scientifique de l'Institut Géographique National ». Un volume rilegato in tela, formato 16×25, di pp. XII+436, con 173 figure e 27 tabelle. Eyrolles, editore. Paris 1970. 156 F.

Il terzo volume di Geodesia Generale di Levallois si apre con un capitolo (l'VIII dell'opera) sull'introduzione allo studio del potenziale newtoniano. Dopo alcuni richiami generali, l'A. tratta del potenziale newtoniano (di massa isolata, di volume, di superficie o di semplice strato, di strato doppio), del teorema di Gauss e delle equazioni di Laplace e di Poisson, delle funzioni armoniche (formula di Green, problema di Dirichlet, ecc.), delle funzioni sferiche, e si sofferma sul potenziale e l'attrazione di qualche corpo, e sul metodo d'inversione.

Il capitolo successivo è dedicato al campo dello sferoide di riferimento e contiene l'espressione rigorosa del campo dell'ellissoide, i teoremi di Clairaut, la celebre formula di Somigliana, l'equilibrio di una massa fluida in movimento di rotazione uniforme, il valore dello schiacciamento terrestre in conseguenza della precessione degli equinozi e la formula della gravità normale. Segue il capitolo (X dell'opera) sul campo potenziale della Terra reale, con le definizioni dei sistemi d'altitudine, dell'ellissoide generale, del geoide e del quasi-geoide; l'esposizione dei modelli terrestri di confronto, del calcolo delle correzioni; l'espressione dell'equazione fondamentale della gravimetria, le formule di Stokes e di Pizzetti, per finire con lo studio del campo nell'intorno di un punto e la ricerca del gradiente di gravità. Chiude il terzo volume un capitolo sulle deviazioni della verticale. Dopo le definizioni generali, l'A. s'intrattiene sulle deviazioni assolute della verticale per via gravimetrica, sul metodo pratico di calcolo delle deviazioni assolute della verticale, sul livellamento astrogeodetico e su esempi di applicazione. Termina esponendo il significato geometrico dell'equazione di Laplace e il metodo di determinazione dell'azimut di Laplace.

P. C.

J.-J. Levallois et J. Kovalevsky, *Géodésie Générale*. Tome IV. *Géodésie spatiale*. « Collection scientifique de l'Institut Géographique National ». Un volume rilegato in tela, formato 16×25, di pp. x + 268, con 41 figure. Eyrolles, editore. Paris 1970. 104 F.

Il quarto volume della Geodesia Generale, per la redazione del quale Levallois si è valso della collaborazione di Kovalevsky, espone le più recenti e sorprendenti acquisizioni della geodesia: la geodesia tridimensionale e spaziale, la geometria e la meccanica celeste dei satelliti artificiali. Vi sono esposti alcuni dei metodi, il cui uso ha fatto fare alla geodesia nell'ultimo decennio, un progresso maggiore di quello da essa compiuto nel secolo precedente, specialmente per quanto concerne la conoscenza precisa delle costanti dinamiche della Terra.

Il capitolo dedicato alla geodesia tridimensionale (XII dell'opera), dopo un'introduzione di carattere matematico, tratta della triangolazione spaziale su satelliti artificiali e dei relativi calcoli; della compensazione d'insieme e di alcuni metodi di geodesia spaziale su satelliti artificiali (impiego dei Lasers, metodo ottico e telemetrico, metodo orbitale). L'opera termina con un capitolo di meccanica celeste: il problema dei due corpi, la teoria delle perturbazioni, sviluppo della funzione perturbatrice, teoria analitica del movimento d'un satellite artificiale, determinazione delle orbite (determinazione preliminare, miglioramento di un'orbita, osservazioni di posizioni apparenti del satellite, osservazioni di distanza, effetto Doppler di un satellite, ecc.), determinazione delle orbite per integrazione numerica, la geodesia dinamica per satellite e i due problemi che la interessano (determinazione del campo di gravitazione della Terra, determinazione della posizione geocentrica delle stazioni d'osservazione).

Nel complesso, il trattato di geodesia generale qui recensito, costituisce un'opera d'indubbio valore, che riassume, in modo chiaro e rigoroso, quanto è stato fatto in geodesia fino ai nostri tempi. Di particolare interesse il quarto volume, dedicato alla geodesia dinamica a mezzo di satelliti, ramo nuovissimo delle indagini geodetiche, che già preziosi contributi ha dato alla conoscenza del potenziale terrestre e della forma della Terra.

P. C.