Rheology of the Tectonosphere in the short time range

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SUMMARY. — The evidence bearing upon the rheology of the "tectonically significant layers" of the Earth ("tectonosphere") is analyzed for the "short" time range (up to 4 hours). The evidence is based upon seismic wave transmission and damping, and on laboratory studies. For small stresses, the tectonosphere responds essentially elastically. With regard to damping of seismic waves, it is noted that, of the *rheological* models available in the literature, only some nonlinear (logarithmic creep) mechanism does not contradict the data. In addition, a strength limit is present; when it is exceeded, the material responds *phenomenologically* in accordance with the Coulomb-Mohr fracture theory.

RIASSUNTO. — Nel presente lavoro si analizza quanto fin qui studiato in relazione alla reologia degli "strati tettonicamente significativi" della Terra (tectonosfera) anche per una gamma di intervalli brevi, cioè fino a 4 ore. Gli studi suddetti si basano sulla trasmissione e l'estinzione delle onde sismiche e su prove di laboratorio. Alle piccole sollecitazioni la tectonosfera risponde essenzialmente in modo elastico. Per quanto riguarda l'estinzione delle onde sismiche si nota che, fra i modelli *reologici* disponibili nella letteratura, soltanto qualche meccanismo non lineare (scorrimento logaritmico) non contraddice i dati esposti. Esiste, inoltre, un limite di forza; quando questo viene superato il materiale risponde *fenomenologicamente* in conformità alla teoria di fratturazione di Coulomb-Mohr.

1. – INTRODUCTION.

The writer has recently begun a re-evaluation of the rheology of the tectonically significant layers of the Earth ("tectonosphere"). Thus far, the "intermediate" and the "long" time ranges have been

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investigated ([36,37]). It is the aim of the present paper to conclude this series of studies with an investigation of the theology of the tectonosphere in the " short " time range. By " short " are meant periods of stress-cycles of the order of up to 4 hours.

It has been well known for a long time that, in a first approximation in the short time range for stress cycles whose amplitude remains below a critical limit, the Earth behaves elastically. In this instance, the conditions described by the writer ([34]) in the original paper on the rheology of the Earth some 12 years ago have not changed. It remains to give a summary of the most recent data on the elastic properties of the layers of the Earth in question.

Much new data, however, have been accumulated recently on the " attenuation range ", i.e. on the range of stress frequencies and amplitudes where energy is dissipated and rocks no longer behave perfectly elastically. Such data are commonly expressed in terms of a " quality " Q– factor in seismic wave transmission and in the free oscillations of the Earth. The new evidence will be reviewed.

Finally, when the strength limit is exceeded, the pertinent layers of the Earth exhibit the phenomenon of fracture. The latest information on the subject will be reviewed.

The main result of the present study is that the imperfections of elasticity found in the damping of seismic waves and free oscillations point toward the same type of rheological logarithmic-creep equation as has been found in the intermediate and long time ranges. Contrary to earlier concepts, it appears now, therefore, that the deviations (other than macroscopic fracture) from elasticity of the " tectonosphere " correspond to logarithmic creep in all time ranges, and cannot be described by linear models.

2. – The Elastic Range.

A. *Theory.*

It is well known that one needs two constants in order to describe the elastic properties of isotropic materials. As these constants, one may, for instance, take Lamé's constants λ and μ which are defined by the equation

$$\tau_{ik} = \lambda\, \delta_{ik}\, \varepsilon_{jj} + 2\, \mu\, \varepsilon_{ik} \qquad [1]$$

where τ_{ik} is the stress, and ε_{ik} the strain-tensor. The quantity μ is also often called "shear modulus". Instead of the above two constants, others have been introduced as follows: Young's modulus E

$$E = \frac{\mu\,(3\lambda + 2\mu)}{\lambda + \mu} . \qquad [2]$$

Poisson's ratio m

$$m = \frac{\lambda}{2\,(\lambda + \mu)} \qquad [3]$$

and the incompressibility k (bulk modulus)

$$k = \lambda + \frac{2}{3}\,\mu . \qquad [4]$$

Any two of the above-defined constants may be used to describe an isotropic elastic body. For an anisotropic body, the number of required elastic constants is greater; For a material with no axes of symmetry, this number is 21.

The elasticity of a material has the effect that waves may be propagated through it. There are two types of waves possible, longitudinal (P) and transverse (S) waves. The connections between the corresponding wave velocities v_P and v_S with the elastic constants are

$$v_P^2 = (\lambda + 2\mu)/\varrho \qquad [5]$$

$$v_S^2 = \mu/\varrho \qquad [6]$$

where ϱ is the density of the material. As is seen, the wave velocity is not determined solely by the elastic constants, but also by the density of the material. Thus, a knowledge of v_P and v_S is not sufficient to deduce λ and μ if the density is not known.

B. *Laboratory Measurements*

Laboratory measurements of the elastic constants can be performed by various means. The incompressibility of a rock sample can be measured directly by compression tests: more common, however, are ultrasonic measurements whereby the P and S wave velocities are found. Since the density of a rock sample can be easily measured, the elastic constants can then be deduced from Eqs. [5] and [6]. The latter method is also suitable for use at high confining pressures. Data for the highest pressures are obtained by observations of shock waves produced by explosions.

The number of investigations along these lines is very large. Pertinent results have been collected by Wuerker ([41]) and more recently by Birch (7). The values of the elastic constants vary of course widely for various rocks, but are, for E and μ, generally of the order of 10^{11}-10^{12} dynes/cm^2, the value of E being for any one rock usually about double that of μ. Poisson's ratio m is commonly around 0.2.

C. *Indirect Measurements*

Of almost greater importance for tectonophysical purposes than direct measurements of the elastic constants of rock samples are indirect inferences regarding the "tectonosphere" that can be drawn from seismological investigations. Here, the work of Bullen (9) is of greatest importance. In essence, from the seismic travel-time curves one can obtain the wave velocities v_P and v_S as a function of depth. But of course, as is evident from Eqs. [5] and [6], this is not sufficient to calculate the elastic constants of the material in question, inasmuch as the density of the material must also be known. The latter can only be estimated from general relations on compressibility of materials and from the moment of inertia of the Earth as a boundary condition. Modern satellite observations forced a recent revision of the value of the moment of inertia, so that Bullen (10) made new estimates of the elastic parameters of the Earth in 1969. These values are shown here in Table 1.

D. *Relation of Elastic Properties to Fabric and Composition*

It is evident that the elastic properties of rocks depend heavily on fabric and composition, particularly in the upper regions of the Earth.

Let us note first the problem of fabric. Here, it is to be mentioned particularly that the porosity of the specimen influences the elastic constants. In fact, to account properly for the porosity-effect, it has to be noted that the porosity changes with pressure. For spherical pores, Mackenzie (26) found theoretically the following equation

$$\frac{1}{k_o} = \frac{1}{k}\left\{1 + \frac{3}{2}\left(\frac{1-m_o}{1-2\,m_o}\right)\left(\frac{\varnothing}{1-\varnothing}\right)\right\}^{-1} \qquad [7]$$

where k is the incompressibility, m Poisson's ratio and $\varnothing$ the porosity. The subscript $_o$ refers to zero porosity. For the relationship between pressure p and porosity $\varnothing$, Walsh (38) gave the following relation for spherical pores:

$$\frac{d\varnothing}{dp} = \frac{3}{2\,k_o}\left(\frac{1-m_o}{1-2\,m_o}\right)\left(\frac{\varnothing}{1-\varnothing}\right). \qquad [8]$$

Table I. – VALUES OF DENSITY AND ELASTIC CONSTANTS IN THE EARTH, AFTER BULLEN [10]. DENSITY IN G/CM^3, STRESS VALUES IN MEGABAR (10^{12} DYNE/CM^2).

Depth (km)	Density	Pressure	Incompressi-bility	Modulus
33	3.32	0.009	1.14	0.62
200	3.36	0.064	1.33	0.71
400	3.41	0.132	1.59	0.82
600	4.01	0.206	2.47	1.27
1000	4.66	0.383	3.55	1.87
1400	4.90	0.57	4.14	2.13
1800	5.12	0.77	4.81	2.37
2200	5.33	0.98	5.51	2.60
2600	5.54	1.20	6.19	2.86
2700	5.59	1.26	6.35	2.92
2883	5.68	1.37	6.47	3.01
2883	9.79	1.37	6.49	0
3000	9.97	1.49	6.80	
3500	10.65	1.99	8.53	
4000	11.19	2.45	10.23	
4500	11.60	2.84	11.66	
4982	11.89	3.15		
6371	12.22	3.55		

Using the above relations, Anderson et al. [4] calculated tables of the porosity-corrections that have to be expected in common minerals.

Studies of other types of pores, notably elliptical pores and narrow cracks have been reviewed by Walsh and Brace [40] who calculated the relationships applicable for the calculation of effective bulk-elastic constants from a knowledge of the rock properties and the prevailing holes. Brace [8] also made some theoretical calculations of the moduli of elasticity of polycrystalline aggregates from the crystalline fabric of such materials. A comprehensive study of this problem has also been reported by Anderson and Liebermann [3]. For geodynamic applications, such studies give interesting means for a comparison of the seismically inferred elastic constants with the hypothesised composition and thermal state of the " tectonosphere ".

Finally, the question of the effect of composition on the elastic constants is also of significance. With regard to geodynamic questions, it will be sufficient to mention those investigations that bear upon the upper mantle. In this connection, it has been noted by Kanamori and Mizutani [21] that increasing serpentinization of dunites and eclogites reduces the P-wave velocity therein. Similarly, Christensen [14] found that Young's modulus, Lamé's constants and the bulk modulus of periodotites and dunites decrease rapidly with serpentinazation. Again, this has a bearing upon the explanation of the various *observed* properties of the tectonosphere.

3. – Attenuation Range

A. *General Theory*

In the short time-range, the rheology of the Earth can primarily be investigated by means of a study of its wave-transmission properties. In this connection, it has been observed that seismic waves are *damped*. A damped harmonic wave can be described as follows.

$$A = A\,(\vec{x}, t) \exp i\,(\vec{k}\,\vec{x} - \omega\, t) \qquad [9]$$

where A is a decreasing function of x (position) and t (time), ω is the circular frequency, k the wave number-vector. The function $A(x, t)$ depends on the damping-mechanism, i.e. on the rheology of the material. In spherical elastic waves (distance r from source) without damping, the amplitude $A(r,t)$ decreases, because of the geometry, like $1/r$. Phenomenologically, it is often found that $A(r,t)$ can then be represented as follows.

$$A\,(r, t) = A_o\, e^{-\alpha r}/r \qquad [10]$$

where a is called attenuation constant. Instead of a, one often introduces the quality factor Q, defined as follows [23]:

$$Q = \omega/2\, v\, a \qquad [11]$$

where v is the phase velocity of the wave. In a damped standing wave, the decay of the amplitude goes with the factor $\exp(-\gamma t)$, where

$$\gamma = \omega/2\, Q\,. \qquad [12]$$

In a damped harmonic elastic wave, one has further

$$2\pi/Q = \Delta E/E \qquad [13]$$

where ΔE is the amount of energy dissipated per cycle in a given volume and E the peak elastic energy in that volume.

As with the function $A(x, t)$, the behavior of Q (particularly as a function of frequency) is characteristic for the rheology of the material in question.

B. *The Data*

The most important result of recent investigations on the damping of elastic waves in the upper mantle and crust of the Earth is that Q is frequency-independent with values in the upper mantle mostly around 100-500; the values change with depth. Although there are still some difficulties with some of the interpretations of the available data, the above fact of frequency-independence of Q seems fairly well established (see Knopoff (23) or Jackson and Anderson (19) for a summary) for periods from those corresponding to seismic waves to those corresponding to the free oscillations of the Earth. A similar frequency-independence of Q has also been found in laboratory-experiments with rocks (23). Any deductions regarding the rheology of the tectonosphere must take cognizance of this fact of Q-constancy.

C. *Rheological Implications*

The most commonly assumed imperfections of elasticity are modifications of the elasticity equations by adding a viscosity term. This can, in fact, be done in two ways so as to yield either a " Kelvin solid" or a " Maxwell liquid " (36,37).

The damping of an elastic wave in a Kelvin-type of material can be calculated (23). One finds, in approximation, that the absorption coefficient γ is proportional to the square of the frequency;

$$\gamma = \text{const}\,\omega^2 \qquad [14]$$

With Eq. [12] this yields for Q

$$Q = \omega/2\gamma = \text{const}/\omega\,. \qquad [15]$$

In a Maxwell material, one finds [23]

$$Q = \text{const}\,\omega\,. \tag{16}$$

As is evident, none of the above two linear models yields the observed frequency-independence of Q. Knopoff [23] has shown that no combination of Maxwell and Kelvin properties (or: "dashpots" and "springs") can give a frequency-independent Q. It is possible to achieve this, by means of linear models, only if higher time-derivatives are used in the stress-strain relation and if the material is no longer homogeneous. A series of such possibilities has been discussed by Caputo [12,13].

From simple macro-rheological models, a constant Q can therefore only be obtained by some sort of nonlinear rheological equation. Such a model is the non-linear (logarithmic) creep model, for which the (approximate) frequency-independence of Q was deduced for the first time by Becker [6] in 1925. In connection with geology, the nonlinear logarithmic creep law is usually given in the form of Lomnitz [25]

$$\varepsilon(t) = \frac{\tau_o}{M}\,[1 + q\,\text{lognat}\,(1 + Bt)] \tag{17}$$

where ε is the strain, τ_o the stress, M a modulus of elasticity (rigidity). Using the above law as a stress-strain relation containing an explicit time dependence, and employing a principle of superposition, Lomnitz showed that Q becomes practically frequency-independent for the above law and that

$$1/Q \cong q\,\pi/2\,. \tag{18}$$

Although the logarithmic law is, in effect, the integral of a nonlinear stress-strain relation [35] so that the validity of the superposition principle is somewhat doubtful, the above results can nevertheless be taken as an indication of the prevailing conditions.

Many speculations have been made regarding the microscopic mechanisms causing the constancy of Q (and causing logarithmic creep). Orowan [30] has given an able summary of the most current ideas on the subject. However, for a discussion of the *macroscopic rheology* of the tectonosphere, this is of little direct significance.

4. – Large Stresses

A. *Phenomenology*

When we consider the discussion thus far, we note that it refers to stresses that do not cause permanent displacements in the media in question. However, when a certain stress-limit, commonly called strength-limit, is exceeded, the material is deformed permanently, be it by actual fracture or by ductile failure. Thus it is of importance to describe, with regard to an understanding of the rheology of the tectonosphere, what is known about the strengh-limit of rocks and their failure patterns. Some of our knowledge has come from high-pressure research (see Katz ([22]) for a recent state of-the-art report), but most important are those inferences that can be drawn directly from natural phenomena like the occurrence of earthquakes.

B. *Fracture*

The most commonly observed permanent-type deformation of rocks in the laboratory in the short time range is sudden ("brittle") fracture. The physics of fracture is to this day a somewhat involved and imperfectly understood discipline, as evidenced, for instance, by the seven(!)–volume treatise on the subject edited by Liebowitz ([24]). Regarding the fracture of rocks, a short description ([33]) is worthy of note. Of the more phenomenological approaches to the subject, the fracture criterion of Coulomb-Mohr is still most satisfactory. A recent account of this criterion and a historical discussion has been given by Handin ([17]). As is well known, Mohr stated that, in a triaxial stress state, two glide planes form when the yield stress is exceeded which contain the intermediate principal stress direction and which form similar angles $\varphi \leqslant 45^\circ$ with the maximum pressure direction. This criterion seems to fit fairly well the observed phenomenology of fault patterns ([2]), earthquake foci (where "fault plane solutions" can be interpreted in terms of Mohr glide-planes) and rock failure in the laboratory [the number of papers on this subject is legion: a brief review was given by Handin in 1969 ([17])]. Neglecting the effect of the intermediate principal stress on rock failure does not seem to be entirely justified ([18, 27]), but the effect is certainly minor. Further modifications of the Coulomb-Mohr theory have been made by Walsh and Brace (39) for anisotropic rock and by Adler (1) to allow for the existence of planes of weakness.

The Mohr criterion is entirely phenomenological. Many attempts have been made to explain it in terms of basic physical principles. Most of these attempts start in some way from the theory of Griffith (16) who introduced the idea that small cracks are present in a solid at all times and that it is the stress concentrations at the tips of these cracks which become critical in failure. These various theories are interesting in their own right, but of little importance for the understanding of the rheology of the tectonosphere. We refer the interested reader to a bibliography by Riecker (31).

The above discussion refers only to shear failure. In tensile failure, the fracture surface is normal to the maximum tension (28).

C. *Ductile Failure.*

At high confining pressures, rock has been found to fail in a ductile fashion. Cracks are slowly propagated and the process can be stopped at any time by reducing the stresses. Orowan (29) suggested that the transition from brittle to ductile fracture occurs when the friction across the fracture surface is as great or greater than the shear strength of the rock. The phenomenon of transition from brittle to ductile failure was further investigated by Byerlee (11) who found that the Orowan hypothesis is justified. In ductile failure, the angle which the sliding surfaces form with the compressional axis is close to 30°.

D. *Comparison of Theoretical Failure Patterns with Observations.*

The occurrence of faults and earthquakes indicates that failure-type processes do occur in the " tectonosphere ". Observed fault systems can easily be explained by reference to the Mohr-Coulomb fracture theory as proposed long ago in the well-known theory faulting by Anderson (2). With earthquakes, the problem is much more difficult. " Fault plane solutions " from first onsets of *P*-waves admit an interpretation in terms of " fracture " surfaces at the focus of an earthquake, but it is difficult to envisage brittle fracture as being able to take place in intermediate and deep-focus earthquakes (29). Nevertheless, the phenomenological appearance of eathquakes is that of a " shock " as from a " brittle " fracture.

Brittle fracture certainly fits the behavior of rocks in the laboratory for low confining pressures. How much this behavior is relevant with regard to an understanding of the rheology of the tectonosphere, is uncertain.

Thus, it must be assumed that sudden fracture of some sort (whatever this is physically) occurs in the tectonosphere whenever the strength limit is exceeded. The latter is determined by the Mohr fracture criterion, notably as expressed by Mohr enveloppes. For an Australian sandstone Jaeger (20) found (in bars).

$$\sigma_1 = 612 + 4{,}6\,\sigma_3 \qquad [19]$$

where σ_1 is the largest, and σ_3 the smallest compressive stress. For a South African quartzite, Cook and Hodgson (15) found similarly

$$\sigma_1 = 2500 + 6\,\sigma_3\,. \qquad [20]$$

As a general remark, it can be said that the more compressible a rock, the lower is its fracture strength (5).

Of great interest is the shearing strength of rocks at high confining pressures. Riecker and Seifert (32) have made a particularly noteworthy set of determinations of the shearing strengths of upper-mantle mineral analogs; some of their results are reproduced here in Table 2. They were able to reach about 50 kb confining pressure which, depending on the density law chosen, corresponds to some 170 km depth (cf. Table 1).

Table 2. — SHEARING STRENGTHS OF SOME MINERALS AT HIGH CONFINING PRESSURES, AFTER RIECKER AND SEIFERT (32). ALL STRESS VALUES IN KILOBARS (= 10^9 DYNE/CM^2).

Av. press.	Olivine	Enstatite	Diopside	Labradorite
5.5	1.77		2.21	3.03
9.9	3.03	4.58	3.29	
15.2	5.43			
19.3	7.58	7.74	6.32	6.32
24.8	7.60			8.21
30.3	12.60	10.11	9.23	10.39
39.9	14.15	11.38	12.38	12.14
49.7	15.17	14.90	14.03	13.15
55.1		16.10		

5. – Conclusion

In conclusion, we may summarize briefly the results of the present investigations of the short-time behavior of the tectonosphere.

For small stresses, the tectonosphere-materials behave essentially elastically, with elastic constants as given in Table 1. There is, however, an energy-dissipation mechanism active which, in macro-rheological terms, can be represented only by the assumption of the occurrence of logarithmic creep. A Kelvin-type or Maxwell-type of energy dissipation contradicts the observed behavior of Q. It may be noted that one finds thus the same type of deviation from linear rheology as has been found in the intermediate and long time ranges. In comparison with earlier ideas ([34]), this represents a considerable simplification, inasmuch as the same rheological equation (the logarithmic creep law) can account for seismic damping, the seismic aftershock sequences, the uplift of Fennoscandia and many other phenomena.

Furthermore, when a certain stress limit is exceeded fracture-type phenomena occur. The exact physical explanation of seismic faulting is not yet entirely clear; however, heuristically, the Coulomb-Mohr theory yields an entirely adequate description of the phenomena. The shearing strengths would then be thought to correspond to those that have been measured in the laboratory (cf. Table 2).

REFERENCES

([1]) Adler L., *Failure in geologic material containing planes of weakness.* « Trans. Soc. Min. Eng. AIME » March, p. 88-94, 1963.

([2]) Anderson E. M., *The Dynamics of Faulting and Dyke Formation.* Oliver and Boyd, London, 1942.

([3]) Anderson O. L. and Liebermann R. C., *Equations for the elastic constants and their pressure derivatives for three cubic lattices and some geophysical applications.* « Phys. Earth & Planet. Int. » **3**, p. 61-85, 1970.

([4]) Anderson O. L., Schreiber E. and Liebermann R. C., *Some elastic constant data on minerals relevant to geophysics.* « Revs. Geophysics » **6**, p. 491-524, 1968.

([5]) Balakrishna S., *Experimental study of fracture mechanism in sandstones.* « Geophys. J. Roy. Astron. Soc. » **14**, p. 119-128, 1967.

([6]) Becker R., *Elastische Nachwirkung und Plastizität.* « Zeitschr. f. Physik » **33**, p. 185-213, 1925.

([7]) BIRCH F., *Compressibility: Elastic constants*, in Handbook of Physical Costants, Revised Edition. « Geolog. Soc. Amer. Memoir » **97**, p. 97-173, 1966.

([8]) BRACE W. F., *Relation of elastic properties of rocks to fabric*. « J. Geophys. Res. » **70**, p. 5657-5667, 1965.

([9]) BULLEN K. E., *Introduction to the Theory of Seismology*. 3d edition. London: Cambridge University Press, 1965.

([10]) BULLEN K. E., *Seismic and related evidence on compressibility in the Earth*, in: « The Applications of Modern Physics to the Earth and Planetary Interiors ». New York: Wiley-Interscience, p. 287-297, 1969.

([11]) BYERLEE J. D., *Brittle-ductile transitions in rocks*. « J. Geophys. Res. » **73**, p. 4741-4750, 1968.

([12]) CAPUTO M., *Linear models of dissipation whose Q is almost frequency-independent*. « Annali di Geofisica » **XIX**, p. 383-393, 1966.

([13]) CAPUTO M., *Linear models of dissipation whose Q is almost frequency-independent*. « Geophys. J. Roy. Astron. Soc. » **13**, p. 529-539, 1967.

([14]) CHRISTENSEN N. I., *Elasticity of ultrabasic rocks*. « J. Geophys. Res. » **71**, p. 5921-5931, 1966.

([15]) COOK N. G. W. and HODGSON K., *Some detailed stress-strain curves for rock*. « J. Geophys. Res. » **70**, p. 2883-2888, 1965.

([16]) GRIFFITH A. A., *The phenomena of rupture and flow in solids*. « Phil. Trans. Roy. Soc. London » **221**, p. 163-198, 1921.

([17]) HANDIN J., *On the Coulomb-Mohr failure criterion*. « J. Geophys. Res. » **74**, p. 5343-5345, 1969.

([18]) HANDIN J., HEARD H. C. and MAGOUIRK J. N., *Effects of the intermediate principal stress on the failure of limestone, dolomite and glass at different temperatures and strain rates*. « J. Geophys. Res. » **72**, p. 611-640, 1967.

([19]) JACKSON D. D. and ANDERSON O. L., *Physical mechanisms of seismic wave attenuation*. « Revs. Geophys. » **8**, 1-63, 1970.

([20]) JAEGER J. C., *Rock failures at low confining pressures*. « Engineering » **189**, p. 283-384, 1960.

([21]) KANAMORI H. and MIZUTANI H., *Ultrasonic measurements of elastic constants of rocks under high pressures*. « Bull. Earthq. Res. Inst. » **43**, p. 173-194, 1965.

([22]) KATZ S., *High-pressure research and geophysics, a status report* « Trans. Amer. Geophys. Un. » **47**, p. 265-268, 1966.

([23]) KNOPOFF L. Q., « Revs. Geophys. » **2**, p. 625-660, 1964.

([24]) LIEBOWITZ H. (ed.), *Fracture, and Advanced Treatise in Seven Volumes*. Academic Press; New York, 1969 ff..

([25]) LOMNITZ C., *Linear dissipation in solids*. « J. Appl. Phys. » **28**, p. 201-205, 1957.

([26]) MACKENZIE J. K., *The elastic constants of a solid containing spherical holes*. « Proc. Phys. Soc. Lond. » **63 B**, p. 2-11, 1950.

(27) Mogi K., *Effect of the intermediate principal stress on rock failure.* « J. Geophys. Res. » **72**, p. 5117-5131, 1967.

(28) Murrell S.A.F., *The effects of triaxial stress systems on the fracture of rocks, and on the nature of faulting and earthquakes.* « Geophys. J. Roy. Astron. Soc. » **14**, p. 81-87, 1967.

(29) Orowan E., *Mechanism of seismic faulting,* in *Rock Deformation,* « Geol. Soc. Amer. Mem. » **79**, p. 323-345, 1960.

(30) Orowan E., *Seismic damping and creep in the mantle.* « Geophys. J. Roy. Astron. Soc. » **14**, p. 191-218, 1967.

(31) Riecker R. E., *Bibliography of Rock Deformation.* « Terr. Sci. Lab., Air Force Camb. Res. Lab., Office of Aerospace Res., USAF, Spec. Rep. » No. **5**, 1964.

(32) Riecker R. E. and Seifert K. E., *Shear deformation of upper mantle mineral analogs: Tests to 50 kilobars at 27° C.* « J. Geophys. Res. » **69**, p. 3901-3911, 1964.

(33) Rinehart J. S., *Fracture of rocks.* « Internat. J. Fracture Mech. » **2**, 3, p. 534-551, 1966.

(34) Scheidegger A. E., *Rheology of the Earth: the basic problem in geodynamics.* « Canad. J. Phys. » **35**, p. 383-397, 1957.

(35) Scheidegger A. E., *On the rheology of rock creep.* « Rock Mechanics » **2**, p. 138-145, 1970.

(36) Scheidegger A. E., *The rheology of the Earth in the intermediate time range.* « Annali di Geofisica » **XXIII**, p. 27-43, 1970.

(37) Scheidegger A. E., *The rheology of the Earth in the long time range.* « Annali di Geofisica » **XXIII**, 4, p. 325-346, 1970.

(38) Walsh J. B., *The effect of cracks on the compressibility of rocks.* « J. Geophys. Res. » **70**, p. 381-390, 1965.

(39) Walsh J. B. and Brace W. F., *A fracture criterion for brittle anisotropic rock.* « J. Geophys. Res » **69**, p. 3449-3456, 1964.

(40) Walsh J. B. and Brace W. F., *Elasticity of rock: A review of some recent theoretical studies.* « Rock Mech. and Eng. Geol. » **4**, p. 283-297, 1966.

(41) Wuerker R. G., *Annotated tables of strength and elastic properties of rocks.* « AIME Paper » No. 663-G, 27 pp., 1956.