

The Problem of Magnitude Calibrating Functions for Body Waves (*)

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1. - INTRODUCTION

A detailed knowledge of the behaviour of the dynamic properties of seismic waves during their propagation through the Earth's body is one of the principal suppositions for improving the Gutenberg-Richter magnitude scale intensively used for the classification of earthquakes. The amplitude and period variations with epicentral distance are of special interest for magnitude determination, this very important variation being expressed in the magnitude equation by magnitude calibrating functions. The aim of this communication is to derive reliable empirical calibrating functions for *P*-, *PP*- and *S*-waves. It will be shown that in this way the precision of determining the magnitude from individual body waves can be increased three times.

2. - DEFINITION

By earthquake magnitude we understand the quantity M defined for all kinds of waves by the equation (1)

$$M = \log A/T + \beta(\Delta) + \Sigma \delta M, \quad [1]$$

where A = maximum ground amplitude in μ } of the corresponding
 T = period in seconds } wave,
 β = calibrating function }

Δ = epicentral distance in degrees,
 $\Sigma \delta M$ = corrections.

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In this communication we are especially interested in the calibrating functions for body waves. The existing calibrating functions (A -functions of GUTENBERG (2), improved Q -functions of GUTENBERG-RICHTER (3)) were derived by semi-empirical methods on the basis of data from many stations distributed all over the Earth's surface. It seems, however, that the structure of the Earth's upper mantle is of a regional character (4,5) so that the use of mean calibrating functions can lead to large errors in the magnitude values, especially in some distance intervals and various regions.

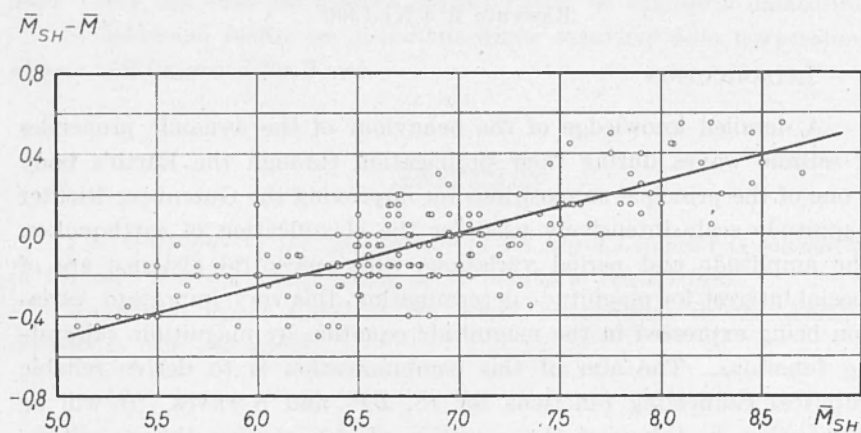


Fig. 1. - Dependence of the differences $\bar{M}_{SH} - \bar{M}$ on the magnitude \bar{M}_{SH} .

We therefore tried to derive independent calibrating functions for P, PP and S applicable to European conditions. We used a method of successive approximation, the main stages of which are described below.

3. - MAGNITUDE CALIBRATING FUNCTIONS FOR BODY WAVES

a) METHOD.

If observations from a network of near stations and only shallow shocks are used, the only term of $\Sigma \delta M$ in [1] is the station correction S including the local effects on amplitude and periods of body waves. Then the calibrating function

$$\beta = M - \log A/T - S \quad [2]$$

can be determined by means of amplitude and period observations of the stations in question for a sufficient set of earthquakes with known M .

In our case the calibrating functions for body waves were determined in three stages: The 1st approximation was derived from the observations of one station on the basis of magnitudes from surface waves (⁶).

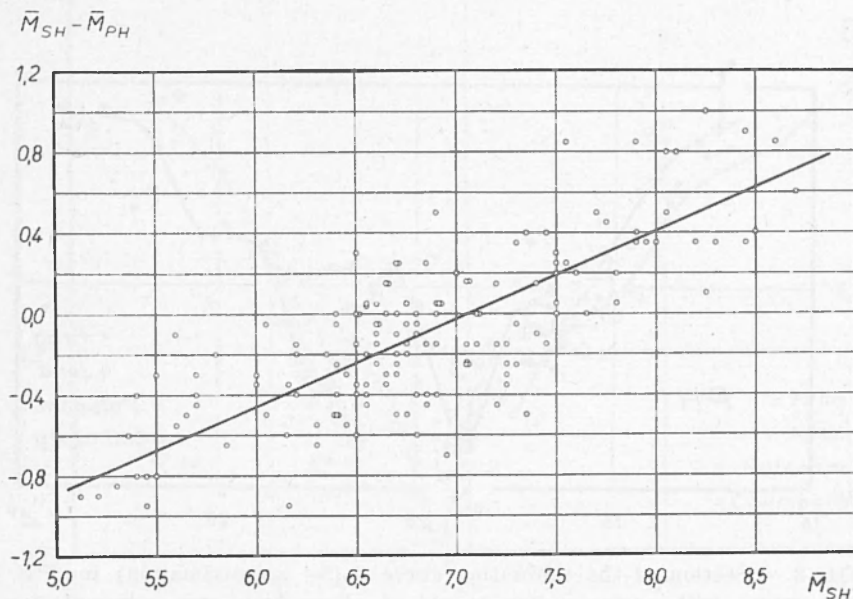


Fig. 2. — Dependence of the differences $\bar{M}_{SH} - \bar{M}_{PH}$ on the magnitude \bar{M}_{SH} .

The 2nd approximation was deduced from the observations of four Middle European stations on the basis of magnitudes determined by means of the first approximation (⁷). The 3rd approximation was then obtained from the observations of four Middle European stations on the basis of magnitudes determined by means of the second approximation. The details are given in the following sub-sections.

aa) 1st Approximation

The first approximation of calibrating functions for body waves was deduced by means of the observations from one station (Prague) (B -functions in paper (⁶)). The values determined from the surface waves observed in Prague (⁶) were used as the basic magnitudes in [2].

The number of observations was as follows: 89 for PH , 88 for PV (both in the range 10° - 100°), 56 for PPH (distance range 20° - 160°), and 96 for SH (distance range 10° - 90°). Due to the relatively small number of observations only the marked increase of amplitudes of P and S near 20° could be considered and the calibrating curves were obtained as polynomials in Δ , the coefficients of which were adjusted

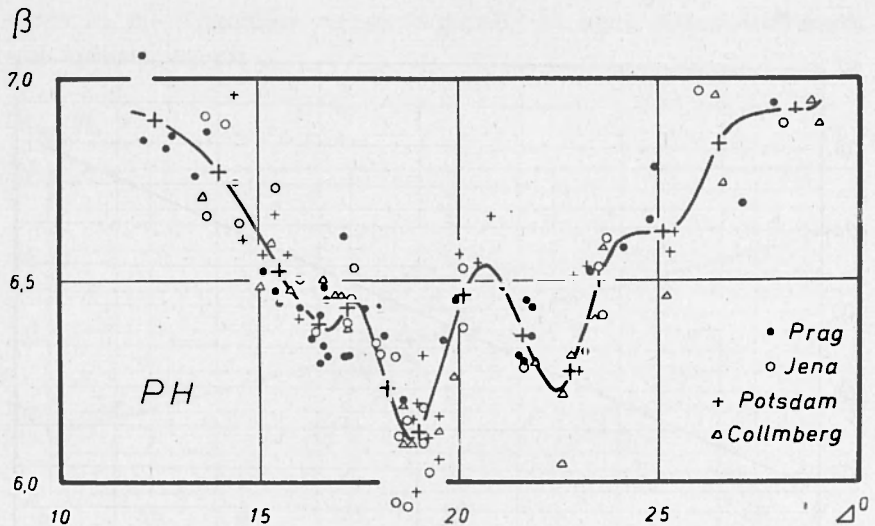


Fig. 3. - Section of the calibrating curve β (3rd approximation) for PH -waves with corresponding points of observation and representative points of natural intervals (large crosses).

by the method of least squares. According to a slightly different definition the functions $B(\Delta)$ were later transformed to the system of equation [1] (β -functions in paper (9)).

ab) 2nd Approximation

By deducing the station corrections for individual body waves for seismological stations Jena, Collmberg and Potsdam (^{10, 11, 12}), related to Prague, we obtained a sufficient set of homogeneous observations to derive the second approximation of β -functions. The number of observations was as follows: 499 for PH , 166 for PV , 250 for PPH and 520 for SH . The magnitudes from body waves determined by means of the first approximation of calibrating functions were used in Equ. [2].

The magnitude M of an earthquake could be determined on the basis of a number of observations at p different stations k for n diffe-

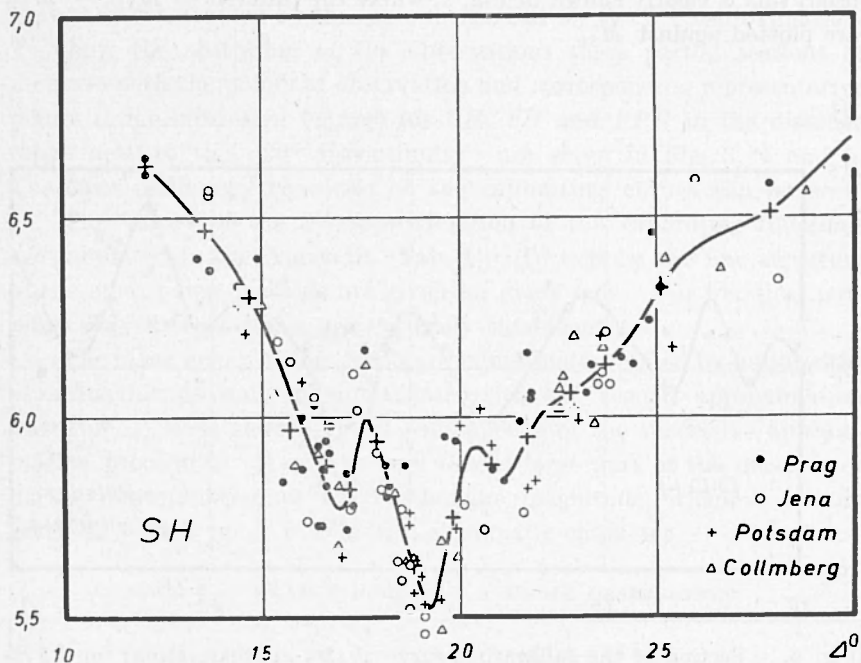


Fig. 4. - Section of the calibrating curve β (3rd approximation) for SH-waves.

rent waves j (maximum 14 observations). It seemed correct to use for M the mean value of the individual magnitude values M_j^k , i. e.

$$\bar{M} = \frac{1}{n + p} \sum_j p \bar{M}_j, \tag{3}$$

where

$$\bar{M}_j = \frac{1}{p} \sum_k M_j^k; \tag{4}$$

$j = PH, PV, PPH, SH, k =$ Prague, Jena, Collmberg, Potsdam, $p =$ number of stations, $n =$ number of wave types.

It was found, however, that the magnitudes from different body waves had different scale units, similarly as was observed for magnitudes

from body and surface waves. Plotting e. g. the differences $\overline{M}_{SH} - \overline{M}$ as a function of \overline{M}_{SH} , we get a similar dependence as when comparing body and surface wave magnitudes (Fig. 1). This divergence occurs especially between \overline{M}_{SH} and the magnitudes from other body waves used; this is clearly shown in Fig. 2, where the differences $\overline{M}_{SH} - \overline{M}_{PH}$ are plotted against \overline{M}_{SH} .

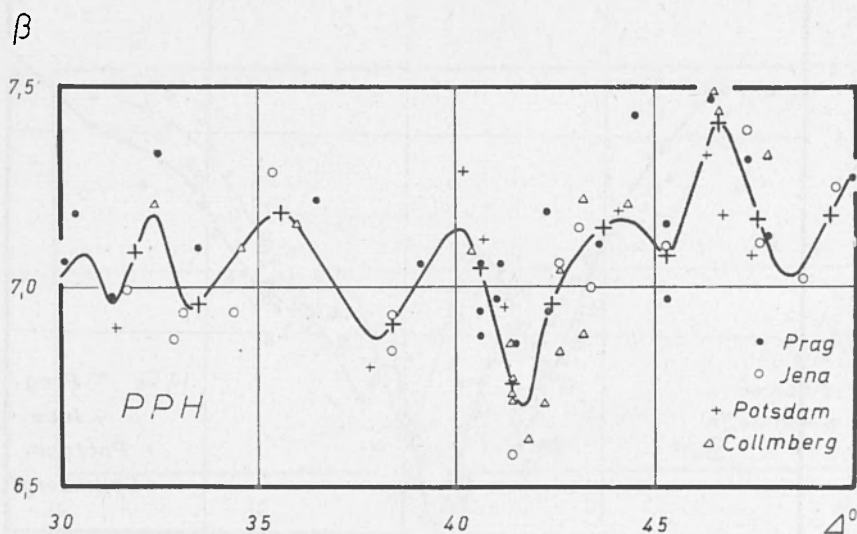


Fig. 5. - Section of the calibrating curve β (3rd approximation) for PPH-waves.

To determine the second approximation of the calibrating functions we therefore used the values of \overline{M}_i defined by [4] as M in [2].

The whole set of observations for an individual wave was divided by means of minimum frequency points of inflexion of the cumulative frequency graph into natural intervals, which were represented by their centres of gravity. The β -curve was then drawn through these representative points allowing for the tendency of detailed distribution of the observations. Thus the fine structure of the calibrating functions could be investigated with a considerable degree of certainty.

The detailed results are described in paper (?). The mean error of the second approximation decreased on an average three times in comparison with those of the first approximation (see Tab. 1).

ac) 3rd Approximation

The same procedure as for the second approximation was repeated once more. Here the magnitudes M^k were determined on the basis of the second approximation of the calibrating functions. The β -curves were then constructed by the same method as for the 2nd approximation. To show the scattering of the observations three partial sections of β -curves with the points of observation and corresponding representative points (large crosses in figures) for PH , SH and PPH in the distance range near to the «20° discontinuity» are given in Fig. 3, 4 and 5. The clear oscillatory character of the calibrating curves can be seen.

The values of the 3rd approximation of the calibrating functions are tabulated in the Appendix (Tab. I). To express the fine structure of the curves two decimals are given for every 0.5°. For practical purposes only first decimals are naturally significant.

The mean errors of the third approximation decreased by hundredths of a magnitude unit in comparison with the second approximation (see Tab. 1), thus showing good convergence of the successive approximation procedure. It can be seen that a large part of the mean error in the determination of the earthquake magnitude, which is usually given as ± 0.3 to ± 0.4 , is of a systematic character.

Tab. 1 - MEAN ERRORS σ OF A SINGLE OBSERVATION

Wave	Approximation		
	1st	2nd	3rd
PH	± 0.39	$\pm 0.12_5$	$\pm 0.11_4$
PV	± 0.33	$\pm 0.14_6$	$\pm 0.12_3$
PPH	± 0.36	$\pm 0.13_7$	$\pm 0.10_9$
SH	± 0.38	$\pm 0.13_8$	$\pm 0.12_8$

b) COMPARISON OF β -FUNCTIONS WITH GUTENBERG-RICHTER'S Q -FUNCTIONS.

A detailed comparison of our β -functions with Gutenberg-Richter's Q -functions (3) for individual waves would be very interesting. Unfortunately this comparison can not be done in absolute values of the cali-

brating functions. There is a systematic difference between the β -functions and the corresponding Q -functions given by the station corrections

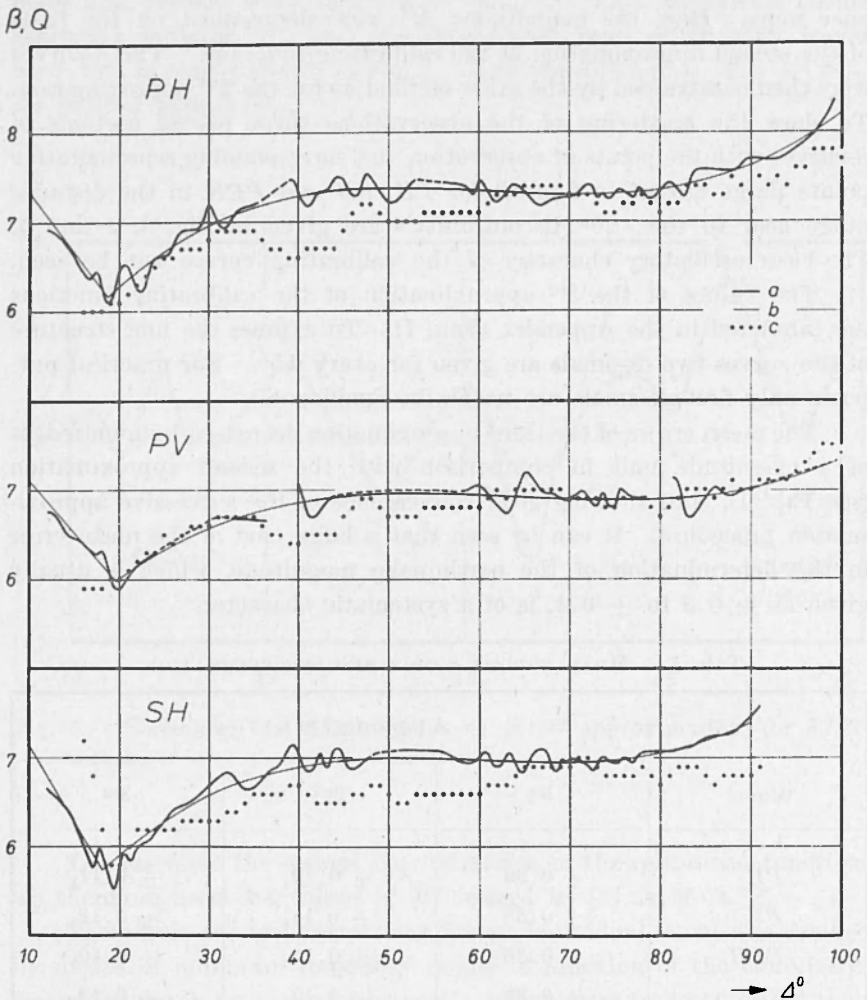


Fig. 6. — Comparison of β -functions with Gutenberg-Richter's Q -functions for *PH*, *PV* and *SH* (*a* — 3rd approximation of β -functions, *b* — 1st approximation of β -functions, *c* — Q -functions).

of Prague relative to Gutenberg-Richter's system of observations. It is not easy to determine these corrections, because the system of observations for Q -functions has not yet been published. Therefore we can compare only the course of both functions. This is done in Fig. 6

for P and S and in Fig. 7 for PP (curve a — β -functions, curve c — Q -functions). These figures also give the 1st approximation of β -functions (curve b). Although the general course of all the curves is similar, large differences between β - and Q -functions occur in several distance

β, Q

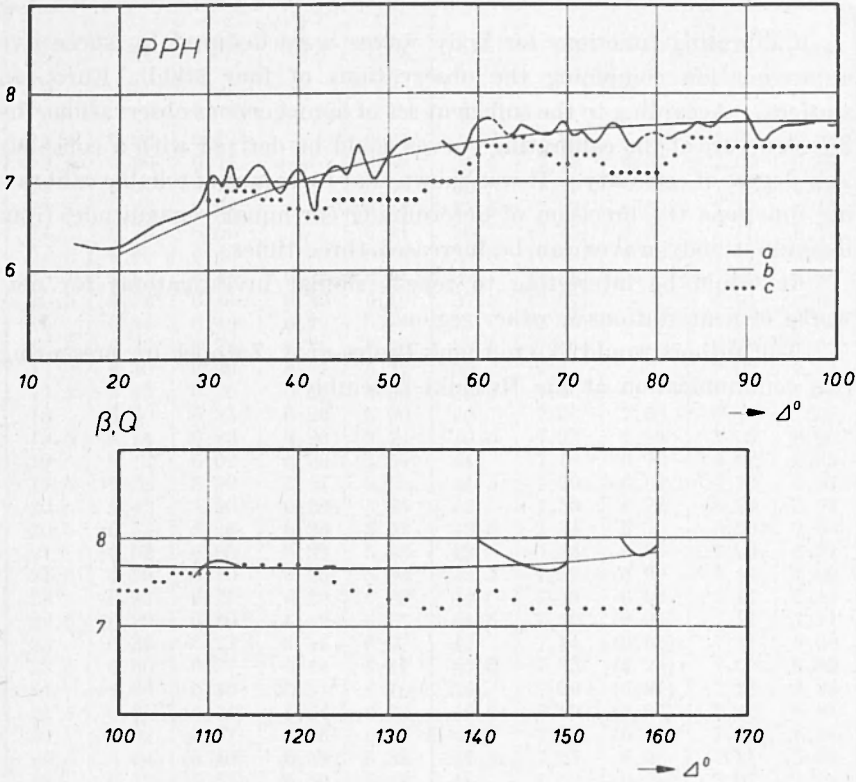


Fig. 7. — Comparison of β -functions with Gutenberg-Richter's Q -functions for PPH (a — 3rd approximation of β -functions, b — 1st approximation of β -functions, c — Q -functions).

ranges, e. g. near 40° for PH (maximum difference 0.85) or near 50° for SH (maximum difference 0.6). Large differences in the distance range $\Delta < 20^\circ$ for P and S are evidently caused by regional influences.

A comparison of the β -functions for individual waves shows very good agreement; e. g. perfect agreement in every detail can be observed for P - and S -waves in the range 10° - 48° . The general course and the

interesting oscillatory character of the calibrating functions is connected with the structure of the Earth's mantle. This question will not be discussed here.

4. - CONCLUSION

Calibrating functions for body waves were deduced by successive approximation combining the observations of four Middle European stations. According to the sufficient set of homogeneous observations the fine structure of the calibrating curves could be derived with a considerable degree of certainty. It was shown that by means of reliable calibrating functions the precision of determining earthquake magnitudes from individual body waves can be increased three times.

It would be interesting to repeat similar investigations for networks of near stations in other regions.

The authors would like to thank Professor A. Zátopek for presenting this communication at the Helsinki Assembly.

A P P E N D I X

Tab. I. - β -FUNCTIONS FOR BODY WAVES
(3rd approximation)

Δ°	β_{PH}	β_{PV}	β_{PPH}	β_{SH}	Δ°	β_{PH}	β_{PV}	β_{PPH}	β_{SH}
12	6,92	6,77	—	6,64	34	7,17	6,67	7,03	6,65
12,5	6,90	6,75	—	6,59	34,5	7,18	6,67	7,09	6,66
13	6,87	6,70	—	6,54	35	7,19	6,67	7,16	6,68
13,5	6,82	6,62	—	6,48	35,5	7,22	6,66	7,19	6,72
14	6,76	6,52	—	6,41	36	(7,25)	6,66	7,16	6,77
14,5	6,69	6,48	—	6,34	36,5	(7,29)	6,66	7,08	6,84
15	6,61	6,46	6,31	6,24	37	(7,33)	—	7,00	6,88
15,5	6,52	6,44	6,30	6,12	37,5	(7,37)	—	6,91	6,90
16	6,44	6,40	6,29	5,99	38	(7,40)	—	6,87	6,94
16,5	6,39	6,35	6,28	5,86	38,5	(7,44)	—	6,93	7,05
17	6,40	6,30	6,27	5,77	39	7,47	—	7,03	7,14
17,5	6,45	6,26	6,27	5,98	39,5	7,46	—	7,11	7,15
18	6,30	6,33	6,26	5,90	40	7,36	7,06	7,14	7,05
18,5	6,14	6,22	6,26	5,74	40,5	7,23	6,88	7,09	6,90
19	6,12	6,02	6,26	5,57	41	7,18	6,73	6,92	6,85
19,5	6,25	5,90	6,27	5,59	41,5	7,20	6,69	6,73	6,87
20	6,42	5,90	6,28	5,85	42	7,30	6,72	6,74	6,98
20,5	6,53	5,95	6,29	5,92	42,5	7,34	6,79	6,99	7,05
21	6,52	6,03	6,32	5,85	43	7,35	6,85	7,09	6,91
21,5	6,39	6,10	6,35	5,90	43,5	7,39	6,90	7,14	7,00
22	6,26	6,15	6,39	6,00	44	7,48	6,92	7,17	7,11
22,5	6,22	6,20	6,42	6,05	44,5	7,50	(6,94)	7,17	7,11
23	6,32	6,23	6,45	6,07	45	7,44	(6,95)	7,11	7,00
23,5	6,50	6,27	6,48	6,11	45,5	7,35	(6,96)	7,09	6,89
24	6,60	6,30	6,51	6,16	46	7,29	(6,97)	7,26	6,88
24,5	6,61	(6,34)	6,54	6,23	46,5	7,29	(6,97)	7,40	6,91
25	6,62	(6,37)	6,56	6,31	47	7,40	(6,98)	7,35	6,94
25,5	6,64	(6,40)	6,58	6,38	47,5	7,55	6,98	7,21	6,99
26	6,72	(6,43)	6,59	6,42	48	7,52	(6,98)	7,07	7,03
26,5	6,84	(6,46)	6,61	6,45	48,5	7,44	(6,98)	7,03	(7,06)
27	6,90	(6,49)	6,63	6,47	49	7,35	(6,99)	7,08	(7,07)
27,5	6,92	(6,52)	6,65	6,49	49,5	7,27	(6,99)	7,20	(7,08)
28	6,92	(6,55)	6,67	6,52	50	7,22	(6,99)	7,29	(7,08)
28,5	6,92	(6,57)	6,70	6,56	50,5	7,21	(6,99)	7,34	(7,08)
29	6,94	(6,60)	6,77	6,61	51	7,23	(7,00)	7,40	(7,09)
29,5	7,05	(6,63)	6,89	6,66	51,5	7,29	(7,00)	7,44	(7,09)
30	7,18	(6,67)	7,02	6,70	52	7,36	7,00	7,47	(7,09)
30,5	7,14	(6,69)	7,08	6,75	52,5	7,44	7,00	7,48	7,09
31	7,00	(6,70)	7,03	6,80	53	7,47	7,00	—	7,09
31,5	6,93	(6,70)	6,95	6,84	53,5	(7,47)	(7,00)	—	7,09
32	6,95	(6,69)	7,12	6,85	54	(7,46)	(6,99)	—	7,09
32,5	7,01	(6,69)	7,19	6,83	54,5	(7,46)	(6,99)	—	7,09
33	7,10	6,68	7,02	6,77	55	(7,46)	(6,99)	—	7,08
33,5	7,15	6,68	6,96	6,68	55,5	(7,45)	(6,99)	—	7,08

Suite tab. I

Δ°	β_{PH}	β_{PV}	β_{PPH}	β_{SH}	Δ°	β_{PH}	β_{PV}	β_{PPH}	β_{SH}
56	(7,45)	(6,98)	7,45	7,07	81,5	7,42	(7,14)	7,49	7,08
56,5	7,44	6,98	7,41	7,05	82	7,38	7,08	7,52	7,10
57	7,44	6,98	7,37	7,01	82,5	7,28	6,86	7,54	7,11
57,5	7,43	(6,98)	7,33	6,98	83	7,30	6,70	7,55	7,12
58	7,42	(6,98)	7,29	6,96	83,5	7,53	6,70	7,58	7,13
58,5	7,40	6,99	7,34	6,95	84	7,58	6,75	7,64	7,16
59	7,33	7,02	7,49	6,94	84,5	7,58	6,83	7,66	7,20
59,5	7,31	7,04	7,60	6,96	85	7,58	6,90	7,64	7,22
60	7,45	7,03	7,66	7,02	85,5	7,58	6,98	7,61	7,24
60,5	7,43	7,01	7,69	7,07	86	7,58	7,02	7,61	7,24
61	7,34	6,95	7,71	7,03	86,5	7,58	7,00	7,66	7,25
61,5	7,24	6,89	7,70	6,92	87	7,57	6,96	7,72	7,25
62	7,23	6,82	(7,67)	6,90	87,5	7,57	6,93	7,76	7,26
62,5	7,27	6,76	(7,61)	6,94	88	7,57	6,94	7,78	7,28
63	7,33	6,73	(7,55)	7,04	88,5	7,59	6,96	7,79	7,32
63,5	7,40	6,77	7,52	7,04	89	7,63	6,98	7,79	7,37
64	7,42	6,88	7,56	6,93	89,5	7,69	6,99	7,78	7,42
64,5	7,39	7,07	7,62	6,84	90	7,71	7,01	7,75	7,47
65	7,34	7,19	7,62	6,86	90,5	7,70	7,02	7,65	7,52
65,5	7,28	7,19	7,60	6,98	91	7,70	(7,04)	7,53	7,58
66	7,22	7,11	7,57	7,06	91,5	7,69	(7,06)	7,48	—
66,5	7,18	7,03	7,55	7,09	92	7,69	(7,07)	7,47	—
67	7,18	6,98	7,54	7,09	92,5	7,68	(7,09)	7,54	—
67,5	7,21	6,96	7,53	7,06	93	7,69	(7,11)	7,58	—
68	7,24	6,96	7,53	6,90	93,5	7,72	(7,12)	7,60	—
68,5	7,26	6,94	7,57	6,85	94	7,75	(7,14)	7,62	—
69	7,27	6,91	7,64	6,99	94,5	7,78	(7,16)	7,63	—
69,5	7,27	6,88	7,67	6,95	95	7,82	(7,17)	7,63	—
70	7,28	6,89	7,59	6,99	95,5	7,86	(7,19)	7,63	—
70,5	7,29	6,93	7,52	6,98	96	7,90	(7,21)	7,64	—
71	7,33	6,97	7,53	6,97	96,5	7,95	7,22	7,65	—
71,5	7,38	6,99	7,55	6,95	97	7,99	7,24	7,66	—
72	7,42	6,95	7,52	6,92	97,5	8,05	7,25	7,66	—
72,5	7,40	6,84	7,46	6,86	98	8,12	7,27	—	—
73	7,27	6,81	7,43	6,92	98,5	8,21	(7,29)	—	—
73,5	7,30	6,96	7,46	7,01	99	8,31	(7,30)	7,64	—
74	7,28	6,90	7,54	6,99	99,5	—	(7,32)	7,64	—
74,5	7,26	6,81	7,62	6,95	100	—	(7,33)	7,64	—
75	7,32	6,83	7,65	6,94	...	—	—	...	—
75,5	7,36	6,97	7,62	6,97	103	—	—	7,68	—
76	7,34	6,99	7,56	7,01	103,5	—	—	7,68	—
76,5	7,30	6,94	7,46	6,97	104	—	—	7,68	—
77	7,27	6,91	7,39	6,95	104,5	—	—	7,68	—
77,5	7,37	6,91	7,40	7,04	...	—	—	...	—
78	7,36	6,92	7,46	7,07	108,5	—	—	7,57	—
78,5	7,28	6,92	(7,50)	(7,08)	109	—	—	7,62	—
79	7,27	—	(7,53)	(7,08)	109,5	—	—	7,68	—
79,5	7,30	—	(7,54)	(7,08)	110	—	—	7,72	—
80	7,35	—	(7,54)	7,08	110,5	—	—	7,74	—
80,5	7,41	—	(7,51)	7,08	111	—	—	7,74	—
81	7,44	—	7,48	7,07	111,5	—	—	7,73	—

Suite tab. I.

Δ°	β_{PH}	β_{PV}	β_{PPH}	β_{SH}	Δ°	β_{PH}	β_{PV}	β_{PPH}	β_{SH}
112	—	—	7,72	—	145	—	—	7,69	—
112,5	—	—	7,71	—	146	—	—	7,66	—
113	—	—	7,69	—	147	—	—	7,64	—
113,5	—	—	7,67	—	148	—	—	7,63	—
...	—	—	...	—	149	—	—	7,66	—
123	—	—	7,67	—	150	—	—	7,73	—
124	—	—	7,67	—	...	—	—	...	—
...	—	—	...	—	156	—	—	7,98	—
140	—	—	7,96	—	157	—	—	7,85	—
141	—	—	7,90	—	158	—	—	7,80	—
142	—	—	7,84	—	159	—	—	7,84	—
143	—	—	7,78	—	160	—	—	7,92	—
144	—	—	7,73	—					

ABSTRACT

The calibrating functions $\beta(\Delta)$ for determining the earthquake magnitude M defined by relation [1] were deduced for body waves by means of a relatively large set of homogeneous observations (499 for PH, 166 for PV, 250 for PPH and 520 for SH) from four European stations (Prague, Jena, Collmburg and Potsdam). The combination of data from the latter stations was made possible by estimating the corresponding station corrections of individual waves in relation to Prague. According to divergent scales for body and surface waves only magnitudes from body waves based on the first or second approximation of the calibrating functions (^{6,7}), respectively, were considered. A similar divergence between the scales for P- and S-waves was found. The average magnitudes of the respective wave were therefore used for deducing the third approximation $\beta(\Delta)$ of the calibrating functions (see Appendix, Tab. I). Due to a sufficient number of observations the fine structure of the calibrating functions could be investigated. The clear oscillatory character of the calibrating functions in several distance intervals was observed, this phenomenon being connected to the structure of the Earth's mantle. The mean error of a single observation in β was estimated to be on an average ± 0.12 of the magnitude unit. Using the calibrating functions β deduced, the precision of determining the magnitudes from body waves was thus increased three times in comparison with the original calibrating functions $B(\Delta)$ published in (⁶).