On Elastic Mechanical Stratified Slabs
(friictional or no brake-lining)

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SUMMARY. — In that paper are analytically defined the geometrical-structuristic index of the agglomerated materials — artificially builded — when they to the remarkable stresses are subject. That results to attain the best realizations of the thermo-piezo-mix, with a great yielding, can be gained.

RIASSUNTO. — Vengono qui definite analiticamente le incidenze geometrico-strutturistiche sulle ripercussioni meccaniche di resistenza dei materiali composti (artificialmente realizzati con mix-piezotermici), quando sono sollecitati da adeguate forze.

Da qui l'opportunità d'avvalersi di questi risultati al fine di arrivare, (per dati tipi standard di sollecitazioni meccaniche), a prassi d'agglomerazione costruttive di maggior efficienza, ferme restando le componenti chimico-fisiche dei composti isotopi.

RÉSUMÉ. — On a ici défini analytiquement les indices géométrique-structuristiques des matériaux agglomérés artificiellement construits, lorsque ils sont soumis à remarquables contraintes.

Il s'éssait qu'on peut utiliser ces résultats, à fin d'arriver aux meilleurs possible réalisations de thermo-piezo-mix, à fort réellement.

1. The lining compounded with dry or wet fibre, granules, organic and inorganic dustes (various components, more or less accidentally oriented, mutually strenghtened, interweaved), appear isotrop with regard to their mechanical properties, the frictional ones included.

The friction from the tip to the roof \( \mu_2 \) is greater than the friction \( \mu_1 \) from the root to the tip. Is defined the "directional coefficient"
\[ \delta = (\mu_1 - \mu_2)/(\mu_1 + \mu_2), \] profoundly affected by the presence of liquids pH (acid and alkaline solutions). Increase \( \delta \) is a corresponding change in the "felting", greater in acids and alkali than in neutral solutions.

It is necessary to point out, though on the whole, specially in view of the making of brake-linings, deformations, performances, and some of the most important anisotropometrical aspects. It is valid the Hook law (though partially), for the elastic fibres is orientated in the "post-cure" mixtures.

\[ \text{Fig. 1} \]

Effect of reduction in feed dimensions on orientation of fibres.

"A material with fibres heterogeneously distributed is made to flow through a straight channel, the tendency will be to straighten out or orientate the fibres parallel with the direction of flow."

\[ \text{Fig. 1a} \]

Orientation by mechanical working of molecule chains of polymerized vinylidene chloride.

It is valid the elastadhesive property among elastically different ingredients, and are valid the rheological laws for components "polymers", organic and inorganic. All that requires complex distributions of stress in the manufactured material, according to the static load and those fluctuating applied, and the physic-chemical-mechanical geometrical structures of support.
An external load $P$, working on a vehicle brake-blocks, will place itself among the single members of the lining, proportionally to the elastic modules, to the bearing sections, with transfers of partial loads due to adhesivity: $A_{ee}$ elastic, in subordinate relation to the quantities:

$$A_{ee} = \frac{kP \sum (E_{i+1} - E_i)}{(S_i E_i + S_{i+1} E_{i+1})},$$

with $k$ coefficient, produced by the bearing areas: $S_i, S_{i+1}$, divided by the sum of such products by the relative elastic mod. $\sum S_i E_i$. The eventual "elastic inadhesivities" are made up for by the "chemical adhesivities", "absorptions", inner mechanisms "between the phases". Only one type of organic polymer therefore will be able to alloy differently (bond degree) according to the visco-elastic heterogeneity of the matrix.

"Fibrous materials" increase mechanical strength, stiffness, impact resistance, dimensional stability, and are often one-bi-more-direct-
Fibres have flexibility and finesses (ultimate tensile strength of chrysotile fibres and various types of asbestos is approximately 50000 psi, with an extremely large surface area value—which is a very important property).

Comparison of average tensile strengths of various materials, give 155000 psi for carbon steel, against 100-200 glass fibre, 80000 to 200000 chrysotile asbestos, 10000 to 300 crocidolite, 1000 to 8000 tremolite asbestos.

It must not be undervalued the “Heat resistance” on the strength of fibres, showing the chrysotile a “breaking strength” nearly steady up to 600-500°, unlike other fibres (amianthus or glass fibres) more resistant up to the T ≈ 200° (breaking temperature).

Electron micrographs of the asbestos-fibres have indicated that they may exist as hollow tubes several hundred angstromes in diameter.

Anyway the elastic-plastic-resistant properties can differ, post-cure, in the various directions.

Once fixed a load (static or fluctuating), working for the moment with a steady mean direction (viz. vertical), we want to find out how the mechanical reactions place themselves on the slabs, isotrop and not, subject to stresses (fibrous anisotropy).

The mod. of elasticity $E$, the shearing modulus $G$ (in isotropic bodies), remain steady in any direction, and so the stress $a$, the shearing stress $\gamma$, caused constants strain $\varepsilon$ and $\gamma$ ($\varepsilon = \sigma/E, \gamma = \tau/G$), and constant $\tau_L = \tau_2$ ($\nu =$ Poisson's ratio).

With transverse strains (contraction or dilation), in anisotropic e.g. “orthotropic materials”, occurs to reconstruct stress-strain diagrams, hence to find the elastic modulus in the longitudinal $L$ in the transversal $T$ directions ($E_L, E_T$), the shearing modulii $G_{LT}$ associated with these directions; the Poisson's ratio $\nu_{LT}$ caused by transverse strain a stress in $L$ direction, $\tau_{LT}$ idem in the $T$ direction. So that the modulus at any intermediate angle is $E_1$ (in function of $a, E_{LL}, E_{TT}, G_{LT}, \nu_{LT}$) and if $a$ is a stress applied in the 1-direction at an angle $\alpha$ with $\alpha = -L - 0°; T - 50°$, we have the most simple case of anisotropy. This is a particularly important point: the fibrous layer is not undamaged, when it is subjected to a strain, by its mean orientation with regard to a fixed direction (viz. the direction of a static or fluctuating
agent load), it influences in fact the quantities relative to the mechanical deformations, though the experimental data $E_L$, $E_T$, $G_{LT}$ remain unchanged.

As the "cure piezo-termics" affect the frietograms, on the coefficient $\mu$ and other mechanical (with the same combination of the packing's ingredients), thus the "cure" mix, extrinsic, to which the fibrous layers are bound, affect the anisotropic peculiarities of the lining, sometimes with remarkable incidences, as we can see afterwards.

We relate in the meantime, as a proof of what we stated, some comparative graphics of the functional quantities: $E_E/E_L$, $G_{LT}/G_{LT}$ (functions of the angle between direction of stress and the longitudinal axis of the material).

The mod. $E_L$, $E_T$ are known, and therefore we have as a result of the graphic in Fig. 2, the unknown $E_1$ ($a$, $E_1$); likewise, when the mod. $G_{LT}$, $v_{LT}$, $v_{TL}$ are measured in the directions $L$ (longitudinal) and $T$ (transversal), we have as a result $G_{LT}$ etc...

The ratio $E_E/E_L$ is made up of the sum of three terms:

\[
\cos^2 a \quad \text{(directional term)}
\]

\[
E_L \sin^2 a \quad \text{(mixed geometric-mechanical term)}
\]

\[
1/4 \left( \frac{E_L}{E_L} - 2v_{LT} \right) \sin^2 2a \quad \text{(mixed term)}
\]

Therefore $E_E/E_L$ engages all the elastic terms relative to the fibres $E_L$, $E_T$, $G_{LT}$, $v_{LT}$, as well as the joined "directionalities".
Angle \( \alpha = 0^\circ \) is longitudinal direction, angle \( \alpha = 90^\circ \) is transverse direction, \( aE_r/E_L \) read on the graphic of Fig. 2, allows to obtain the mod. \( E \), mod. of elasticity in any reference direction.

The ratio \( E_r/E_L \) is symmetrical with regard to the ordinate crossing angle \( \alpha = 45^\circ \) (Fig. 2), with "balanced orthotropic material o.m.b.", while for "unbalanced orthotropic material", and for \( \alpha = 90^\circ \), \( E_r/E_L \) \( \alpha \to \) to asymptotic minimum (evident dissymmetry).

Thus \( \theta_{LT}/\theta_{LT} \) (\( \theta_{LT} \) is the shearing modulus associated with \( L \) and \( T \) directions), is symmetrical to 45°, unlike \( E_r/E_L \) in the cases "o.m.b." and "o.m.u." (v. Fig. 2a).

Two factors \( m_1, m_2 \) (caused by shear and direct stresses respectively) showing symmetrical but different behaviours, have been introduced.

\[ \text{fig. 2a} \]

The ratio \( E_L/E_1 \) depends on \( \alpha, E, G, v_{LT} \), as well as \( \theta_{LT}/\theta_{LT} \) \( v_m \) depend on \( \alpha, E, G, v_{LT} \), all quantities which can be measured in advance.

In these considerations we have voluntarily left out "plastic states" of the material, so that the bending moment of the structure comes out to be higher even of the 50% with regard to the elastic limit moment:

\[ M_y \ (M_p = kM_y) \ ; \ \ k = 1.5 \] (for rectangular sections).
In a laminar solid, Fibreglass-Reinforced-Fillers, \( E_L/E_T = 10 \), where only \( E_L \), \( E_T \) (according to the hardness) can go down to 120-1300 psi; being \( E_L = 5.10^6 \), \( E_T = 0.5.10^6 \), \( G_{LT} = 0.55.10^6 \) psi, \( \nu_{LT} = \nu_L = 0.45 \), \( \nu_{LT} = \nu_{LT} = 0.015 \).

The tensile stress \( \sigma_a \), acting on the small plate at the top, is 10 psi; the shear stress \( \tau_{12} = 4000 \) psi.

The stress \( \sigma_a \), applied in the 1-direction (Fig. 9) causes a strain \( \varepsilon_1 = \sigma_a/E_L \). A transverse strain \( \varepsilon_2 \) is caused by \( \sigma_L \), where \( \varepsilon_2 = -R_a \varepsilon \).

Unlike isotropic materials, stress \( \sigma_a \), when applied at any angle except 0° and 90°, causes shear distortion and the shear strain \( \gamma_1 = \gamma_{12} / E_L \), where \( m_1 = m_1(\varepsilon, \nu_{LT}, E_L, E_T, G_{LT}) \).

A shearing stress \( \tau_{12} \) applied in 1-2 directions (v. Fig. 7) causes a shear strain \( \gamma_{12} = \tau_{12}/G_{12} \), where \( G_{12} = G_{12}(\varepsilon, G_{LT}, \nu_{LT}, E_L, E_T, V_{LT}) \).

Consequently, strain caused by \( \sigma_a \) are \( \varepsilon_1, \varepsilon_2, \gamma_{12} \); strain caused by \( \tau_{12} \) are \( \varepsilon_1, \varepsilon_2, \gamma_{12} \); total strain being \( \varepsilon_1 + \varepsilon_2 + \varepsilon_1 = \gamma_{12} \). The diagrams showed above (Fig. 1 and 2a), which join these mechanical quantities to the angle must be prepared each time according to the materials (\( E_L, E_T, G_{LT}, \nu_{LT} \)).

Thanks to them we obtain the "strains" caused by \( \sigma_a \) and the "strains" caused by \( \tau_{12} \), as we can see here following:

\[
\begin{align*}
\sigma_a & \rightarrow \varepsilon_1 = \sigma_a/E_L, \quad \varepsilon_2 = -R_a \varepsilon, \quad \gamma_{12} = -m_1 \sigma_a/E_L \\
\tau_{12} & \rightarrow \varepsilon_1 = -m_1 \tau_{12}/E_L, \quad \nu_{12} = -m_1 \tau_{12}/E_L, \quad \gamma_{12} = \tau_{12}/G_{12}
\end{align*}
\]

The quantities \( E_L(E(a), G_{LT}G_{LT}(a), \nu_L(a)) \), for small variations of the "stiffness factor \( EI \)", "section modulus" for outermost fibre,
"Bending and shear stress" can lead to great differences in the results. As for the structures o.m.b. or u.m.b., the variations in the same stresses $\sigma_1, \sigma_2 (M_{\alpha}, T_{\alpha}, J_{\alpha})$ are remarkably different. In both we can note negative minimum for $\gamma_1$ ($\alpha \sim 30^\circ$), and positive minimum for $\gamma_2$ ($\alpha \sim 30^\circ$). The deformation $\varepsilon_{\alpha}$ is max. for $\alpha = 45^\circ$ o.m.b., while the u.m.b. always increases with a strong gradient, also after $\alpha = 30^\circ$, and has no max.

The deformations $\varepsilon_{\alpha}$ due to traction (in the direction of $\sigma_1$) in all cases, go constantly through either max. or other values, because the directional anisotropy increases according to the direction of the agent stress. The $\varepsilon_{\alpha}$ compressive (usually at the direction $\sigma_1$) in all cases, they reach max. too different from a order of quantity generally $< \varepsilon_{\alpha}$. The shear deformations $\gamma_{12}$ reach minimum at $30^\circ$, and appear very like, either with o.m.b. or with u.m.b., both in their behaviour and in their absolute values.

"Particular directionalities" appear mainly at $30^\circ$ and $45^\circ$, including our openings of lining (usually inferior to $80^\circ$).

Superposing several thin plates, at crossed directions, those effect of fibromancy will be attenuated (pseudo-isotropy).

2. Fibrous glass or asbestos reinforced plastics may be "composite and non-isotropic".

Composite structure: various layers may be oriented at different angles with respect to each other, in order to provide the best combination to resist loading condition. Outside loads applied result in "internal stresses", different individual layers (meeting to the layer’s fibromancy). Strains and stresses are induced in each layer: external stresses may result not only in internal stresses, but in internal shear stresses. External shear stresses may result in internal stresses as well as internal shear stresses. The "composite structure", the most elementary, is the bi-layer. Internal stresses $\sigma_{\alpha \beta}, \sigma_{\alpha \beta}^2, \tau_{\alpha \beta}$ can be found in order that the sums of the internal stresses in the 1 and 2 directions must equal the external stresses in these directions, and the strains must be the same in all layers. As the layers are firmly bonded together, the strains are the same in a and $b$ layers, and are equal to the strains in the whole structure. Therefore the fundamental conditions (in which the hypothesis is translated) are:

$$
\begin{align*}
\sigma_{1a} &= \sigma_{1b} = \varepsilon_1 ; \\
\sigma_{2a} &= \sigma_{2b} = \varepsilon_2 ; \\
\gamma_{12a} &= \gamma_{12b} = \gamma_{12} \\
\sigma_{1b} \ell_a + \sigma_{2b} \ell_b &= \sigma_1 \ell \\
\sigma_{1a} \ell_b + \sigma_{2a} \ell_a &= \sigma_2 \ell \\
\tau_{12a} \ell_a + \tau_{12b} \ell_b &= \tau_{12} \ell
\end{align*}
$$
where \( t \) = total thickness of the bi-layer = sum of \( t_1 \) and \( t_2 \), thicknesses of the horizontal superposed \( a, b \), of different orthotropic materials oriented at arbitrary angles \( \alpha \) and \( \beta \), with respect to applied stresses \( \sigma \), \( \tau \), \( \sigma_1 \), \( \sigma_2 \), \( \tau_{12} \) (\( \sigma_1 \) acting in direction 1; \( \sigma_2 \) in the lower layer — acting in direction 2).

The "sandwich" is realized in the linings also under the "piercings" (special with remarkable thicknesses), because of the "plastic working of non-metallic materials", which decreases exponentially with the depth in the bonded layer.

The compressive stress between punch and die distorts the sheet and gives a stratigraphy (slip-plane movements).

Materials \( a \) and \( b \) have respectively longitudinal \( E_a \) and \( E_b \), and transverse directions \( T_a, T_b \). The rational hypothesis done on the kind which we are studying (sandwich) impose:

\[
\begin{align*}
\epsilon_1 = \epsilon_{1} & = \epsilon_1 = f_1 (t, \sigma, \tau, E_a, E_b, E_{12}, E_{33}, T_a, T_b, T_{12} ; t_{\text{m}}) \ , \\
\epsilon_2 = \epsilon_{2} & = \epsilon_2 = f_1 (t, \sigma, \tau, E_a, E_b, E_{12}, E_{33}, T_a, T_b, T_{12} ; t_{\text{m}}) .
\end{align*}
\]

Solutions of the foregoing "linear equations", lead the three simultaneous equations, give the unknown: \( \sigma_{1a}, \sigma_{3a}, \tau_{12a} \).

The linear equations with three unknown \( \sigma_{1a}, \sigma_{3a}, \tau_{12a} \) can be more easily employed, also without solution:

\[
\begin{cases}
1_{1a} 1_{2a} / (A_{1a} \sigma_{1a} + A_{12a} \sigma_{3a} + A_{13a} \tau_{12a}) = \sigma_1 / E_{1a} - \tau_{12a} \sigma_3 / E_{12a} - m_{1a} \tau_{12a} / E_{13a} , \\
1_{1a} 1_{3a} / (A_{1a} \sigma_{1a} + A_{12a} \sigma_{3a} + A_{13a} \tau_{12a}) = - \tau_{12a} \sigma_3 / E_{12a} + \sigma_3 / E_{12a} - m_{1a} \tau_{12a} / E_{13a} , \\
1_{1a} 1_{3a} / (A_{1a} \sigma_{1a} + A_{12a} \sigma_{3a} + A_{13a} \tau_{12a}) = - m_{3a} \sigma_1 / E_{3a} - m_3 \sigma_1 / E_{3a} + \tau_{12a} / E_{13a} , \\
A_{1a} = - \tau_{12a} / E_{12a} \sigma_3 - m_{3a} \tau_{12a} / E_{13a} , \quad A_{2a} = \sigma_1 / E_{1a} - \tau_{12a} \sigma_3 / E_{12a} - m_{1a} \tau_{12a} / E_{13a} , \quad A_{3a} = - \tau_{12a} / E_{12a} \sigma_3 - m_{3a} \tau_{12a} / E_{13a} .
\end{cases}
\]

In fact the coefficient \( A_{11}, A_{12}, A_{13} \) have been known \( E_a, E_{1a}, E_{3a}, E_{12}, E_{33}, \theta_3, \theta_{12} \) can be reduced, and moreover:

\[
A_{11} = A_{11} ; \quad A_{22} = A_{13} ; \quad A_{33} = A_{13} .
\]

We must use the diagrams of Fig. 2 and 2a etc... likewise we work over a layer, of course been known \( E_{1a}, E_{3a}, E_{12}, E_{33} \) which may, f. e., be equal to each other, and equal to \( E_1 (E_2 = E) \) at the conditions \( \alpha = 0 \), hence the relations \( E_1/E_2 = 1 \), or they are equal to each other at the conditions

\[
E_{1a}, E_{3a} (E_{1a} = E_{3a}); E_{12} = E_{12a}; E_{33} = E_{33a}; \tau_{12} = \tau_{12a}; E_1(E) .
\]
The other three equations for $\sigma_{12}, \sigma_{21}, \tau_{12}$, come from:

$$\sigma_{12} - \sigma_1 - \sigma_{22} = 0$$
$$\sigma_{21} - \sigma_1 - \sigma_{22} = 0$$
$$\tau_{12} - \tau_{11} - \tau_{22} = 0$$ [6]

3. For computing stresses, stiffness, for the "composite structures", some modifications of the standard formulas, which below we relate:

**Isotropic Materials**

<table>
<thead>
<tr>
<th>Moment of Inertia</th>
<th>Stiffness Factor</th>
<th>Bending Stress</th>
<th>Shear Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J = \frac{bd^3}{12}$</td>
<td>$EI \sum_i A_i$</td>
<td>$\sigma = \frac{M}{b d^3}$ for outermost fibre</td>
<td>$\tau = \frac{Q}{b J}$ for max. shear at the neutral axis $-\frac{1}{2} \frac{Y}{bd}$</td>
</tr>
</tbody>
</table>

**Non-Isotropic Materials**

<table>
<thead>
<tr>
<th>Moment of Inertia</th>
<th>Stiffness Factor</th>
<th>Bending Stress</th>
<th>Shear Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross section composite beam $I_t = \frac{bd^4}{12}$</td>
<td>$E_i \sum_i A_i x_i$</td>
<td>$\sigma_y = \text{mod. el. for the layer at } y$</td>
<td>$\tau = \text{total shear on the cross section}$</td>
</tr>
<tr>
<td>are med. of el. cross sectional area $bd - \Delta t$ and distance from the bottom of cross section, to the centre of gravity of any particular layer.</td>
<td></td>
<td>$\text{bending stress at } y$</td>
<td>$\text{stress intensity along some horizontal plane}$ $Q'$</td>
</tr>
</tbody>
</table>
| $R_y$ = shear stress of the layer at $y$ | $\text{not necessarily occur at the outermost (bottom) fibre}$ | $\text{is the weighted statical moment}$ $R_y A_i y'$ about the n.a. of the portions of the c.s. between the horizontal plane and the bottom of the c.s.
ON ELASTIC MECHANICAL STRATIFIED SLABS

A plane packing with "adherent and composite slab":

\[ \sum E_i S_i = E_i S_i + E_i S_i = EI \]

For a unit bending moment \( M = 1 \) in-lb:

<table>
<thead>
<tr>
<th>Plane</th>
<th>( y )</th>
<th>( E_y ) (noti)</th>
<th>( a_y = y E_y / EI )</th>
<th>( a_y ) in-lb</th>
<th>( M )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a-a)</td>
<td>0.77&quot;</td>
<td>10.10^4 psi</td>
<td>7.7/0.45 = 17.7</td>
<td>(49000 psi)/1.7 = 29000 in-lb</td>
<td></td>
</tr>
<tr>
<td>(b-b)</td>
<td>0.37&quot;</td>
<td>1.10^4 psi</td>
<td>0.37/0.45 = 0.8</td>
<td>(20000 psi)/0.8 = 33000 in-lb</td>
<td></td>
</tr>
<tr>
<td>(c-c)</td>
<td>0.23&quot;</td>
<td>1.10^4 psi</td>
<td>0.23/0.45 = 0.5</td>
<td>(20000 psi)/0.5 = 40000 in-lb</td>
<td></td>
</tr>
</tbody>
</table>

\[ \sum E_i S_i = E_i S_i + E_i S_i = EI \]

Fig. 6
Here appear a “critical plane” not “interstratified” but the bottom of the “bi-composite”. The “internal moment” or “moment of resistance” \( M = EI/R = E/J \sum y^2 \), in the “isotropes”, is inversely proportional to the bending radius, in the “composites” is no more proportional “to the square of the depth, but to the 1.89 power of the depth.” In Fig. 6, we study the case of 6 layers horizontal superposed, with different elastic modulus, and different thickness, of which we have calculated the resistance’s moments \( cy = F_y/EI; EI = \sum E_i I_i \) (Fig. 6).

**Internal Moment.**

The length of the horizontal line (dash and point) gives the value of \( M \) in in-lb (max. 10,000 in-lb, min. 3,500 in-lb) and we can note how the material is differently stresses inside.

4. — Let us consider at last the situation of a plane superficial element of “lining” (Fig. 7), a real shape of which, with a given strain \( p \), we have the “circumferential stress” \( \sigma_c = p r \) and the “longitudinal stress” \( \sigma_l = 1/2 \sigma_c \) that is half value.

We can see different fibers orientations, concerning the two superposed planes; the parallel to 1 (circumferential direction) — \( T_1 \) perpendicular, \( L_2 \) parallel to 2, \( T_2 \) normal, circumferential and axial directions 1,2 (intertwisted); or \( L_2 \) at 45° with 1, \( T_2 \) normal, \( L_2 \) parallel underlying to \( T_1 \), and \( T_1 \) parallel to \( L_2 \); being \( t_2 = t_1 \) in any case. The
angle $\alpha$ ($L_a$) can be anyone, and the angle $\beta$ ($L_b$) ($\alpha = \beta, \alpha \neq \beta$) too.

The calculus become easier as:

\begin{align*}
E_{la} &= E_{lb} = E_{2a} = E_{2b} \\
v_{ltt} &= v_{21a} = v_{12b} = v_{21b} \\
m_{1a} &= w_{1a} = w_{2b} = 0 \\
A_{11} &= A_{22} = A_{21} = A_{12} = 0
\end{align*}

\begin{align*}
A_{11} &= L_{ia} t_a + 1/E_{lb} t_b; A_{12} = A_{21} \\
A_{22} &= L_{ia} t_a + 1/E_{lb} t_b
\end{align*}

values which must be introduced in the three fundamental equations hence these become:

\begin{align*}
A_{11} \sigma_{ia} + A_{12} \sigma_{ib} &= \frac{L_{ia}}{E_{lb}} \left( \sigma_{ia} / E_{lb} - \sigma_{ib} / E_{lb} \right) \\
A_{21} \sigma_{ia} + A_{22} \sigma_{ib} &= \frac{L_{ia}}{E_{lb}} \left( \sigma_{ia} / E_{lb} + \sigma_{ib} / E_{lb} \right); A_{22} \tau_{ib} &= 0
\end{align*}

\[ \text{[8]} \]

From where we get our unknowns:

\[ \sigma_{ia} = \sigma_{ib} = \tau_{ia}; \sigma_{ib} - \sigma_{ia} = \tau_{ib} \]

Internal stresses, which become equal to the external stresses, with no shear internal stresses, also for shear strains different from zero.

The symmetry of the orientations of the fibers with regard to the main directions of the stresses $1-2$, cause the internal stresses $\sigma_{ia}, \sigma_{ib}, \tau_{ia}, \tau_{ib}$ to be equal the external stresses imposed $\sigma_i, \sigma_2$ and annul the internal stress shear, although we impose external stresses shear.

For:

\begin{align*}
E_{ia} &= E_{ib} = E_{2ab} = E_{4ab} = E_{2b} = E_{4b} \\
\theta_{2ab} &= \theta_{4ab} \\
v_{12a} &= v_{12b} = v_{42a} = v_{42b} = v_{21a} - v_{21b} \\
m_{2a} &= m_{2b} = -m_{1a} \\
m_{2b} &= m_{2a} = -m_{1b}
\end{align*}

\[ \text{[9]} \]

the fundamental equations will be different from the previous only in the $3^{rd}$ (the first two remain same, showing that the internal stresses are equal to those imposed)

\[ A_{22} \tau_{ib} = \frac{L_{ia}}{E_{lb}} \left( m_{1i} \sigma_{ia} / E_{lb} - m_{2i} \sigma_{ib} / E_{lb} \right). \]

\[ \text{[10]} \]

Therefore being $\sigma_a = \sigma_b = \sigma_i; \sigma_{ab} = \sigma_{2a} = \sigma_{2b}$ from the third equation we get $\tau_{ib} \neq 0$. Remarkable stresses shear appear when the layers are so oriented "even if there are no external shear stresses".

The shear stresses in layers $b$ are oriented in the opposite direction to the shear stresses in layers $a$. 

5. - We determine at this point the position of the max. pressure and the distribution of the pressure on the lining (rigid shoe calculation).

In Fig. 8, there is the "centre of drag", point where act the resultant of the forces of the shoe on the drum. The centre of the drum
with radius \( r \) is remarked by \( \theta \); \( \overline{AB} \) is the arch of contact of the shoe-packing; \( C \) is the centre point of the shoe pivot.

We can also the max. pressure \( \theta Q_1 \) " pressure distribution ", and resultant line of pressure \( \theta V_1 \); the angle of friction \( \Phi \) the brake force for shoe \( F \).

More types of construction for " composites ", can be easily studied: layers \( a \) and \( b \) with the \( L \) and \( T \) direction laid in the circumferential and axial directions; the layers are laid at \( \alpha = 10^\circ, 20^\circ, 30^\circ, 45^\circ \) to the
axis of the cylinder, e.g. alternate 30° angles in left-hand and right-hand spirals. For the position \( \alpha = 0°, 45°, 90° \), we have:

\[
\begin{align*}
\frac{B_{yL}}{B_L} &= 1 ; \quad \frac{B_{yR}}{B_L} = 1 ; \quad \frac{B_{yH}}{B_L} = 1 ; \\
\frac{m_{y1} - m_{y2}}{m_{y1} - m_{y2}} &= 0 ; \quad \frac{v_{12a} - v_{12b}}{v_{12a} - v_{12b}} = \frac{v_{12a} - v_{12b}}{v_{12a} - v_{12b}} = v_{LT} .
\end{align*}
\]

and therefore:

\[
\sigma_{12a} - \sigma_{12b} = \sigma ; \quad \sigma_{12c} = \sigma_{12d} = \sigma ; \quad \tau_{12a} = \tau_{12b} = 0 .
\]

For the remaining \( \alpha \), we obtain:

\[
\begin{align*}
\frac{B_{yR}}{B_L} &= \frac{B_{yH}}{B_L} = \frac{B_{yH}}{B_L} = \frac{v_{12a} - v_{12b}}{v_{12a} - v_{12b}} = \frac{v_{12a} - v_{12b}}{v_{12a} - v_{12b}} = v_{LT} .
\end{align*}
\]

The first two fundamental equations do not change, however:

\[
\sigma_{12a} = \sigma_{12b} = \sigma_1 ; \quad \sigma_{12c} = \sigma_{12d} = \sigma_2
\]

while the third becomes:

\[
\tau_{12a} \left( \frac{1}{\theta_{12a}} + \frac{1}{\theta_{12b}} \right) = \frac{v_{12a}}{v_{12a}} - \frac{v_{12b}}{v_{12b}} \left( -\frac{m_{y1}}{m_{y1}} \sigma_{12a} - m_{y2} \sigma_{12d} \right) ,
\]

from which

\[
\tau_{12a} \neq 0 .
\]
Therefore being $\alpha = 0^\circ$, $\alpha = 15^\circ$, $\alpha = 90^\circ$, the shear deformations disappear, while such shear deformations appear again being $\alpha = 10^\circ$, $20^\circ$, $30^\circ$, $40^\circ$, $60^\circ$, $70^\circ$, $80^\circ$, $90^\circ$, with max. values for $\alpha = 25^\circ$ and $\alpha = 65^\circ$.

While reckoning the stresses internal to the fibrous or no packing, we must, of course, keep into consideration the variation of the loads (Fig. 9) which give the $\sigma$ and $\tau$ measurable or obtainable, and therefore, going away from the zone of max. pressure, also the joining affects possible fibrosities of the fibrous lining, with mono or multi-structure, will fade. We have studied then the main principal possible incidences (Figg. 10-10e), keeping unchanged the constitutive formulas (qualitative and quantitative) of the ingredients and load-stress, on the deformations of the brake-lining (isotrop, anisotrop, fibrous and stratiform structures).