On Elastic Mechanical Stratified Slabs

(frictional or no brake-lining)

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Summary. — In that paper are analitically definied the geometrical-structuristic index of the agglomerated materials — artificially builded — when they to the remarkable stresses are subject. That results to attain the best realizations of the thermo-piezo-mix, with a great yielding, can be gained.

Riassunto. — Vengono qui definite analiticamente le incidenze geometrico-strutturistiche sulle ripercussioni meccaniche di resistenza dei materiali composti (artificialmente realizzati con mix-piezotermici), quando sono sollecitate da adeguate forze.

Da qui l'opportunità d'avvalersi di questi risultati al fine di risalire, (per dati tipi standard di sollecitazioni meccaniche), a prassi d'agglomerazione costruttive di maggior efficienza, ferme restando le componenti chimico-fisiche dei compositi isotropi.

Résumé. — On a ici défini analytiquement les indices géometrique-structuristiques des materiaux agglomérés artificiellement construits, lorsque ils sont assojettis à remarquables contraints.

Il s'ensuit qu'on peut utiliser ces résultats, à fin d'arriver aux meilleurs possibles realisations de thermo-piezo-mix, à fort rendement.

1. – The lining compounded with dry or wet fibre, granules, organic and inorganic dustes (various components, more or less accidentally oriented, mutually strenghtened, interweaved), appear isotrop with regard to their mechanical properties, the frictional ones included.

The friction from the tip to the root μ_2 is greater than the friction μ_1 from the root to the tip. Is defined the " directional coefficient "

$\delta = (\mu_2 - \mu_1)/(\mu_2 + \mu_1)$, profoundly affected by the presence of liquids pH (acid and alkaline solutions). Increase δ is a corresponding change in the "felting", greater in acids and alkali than in neutral solutions.

It is necessary to point out, though on the whole, specially in view of the making of brake-linings, deformations, performances, and some of the most important anisotropometrical aspects. It is valid the Hooke law (though partially), for the elastic fibres isorientated in the "post-cure" mixtures.

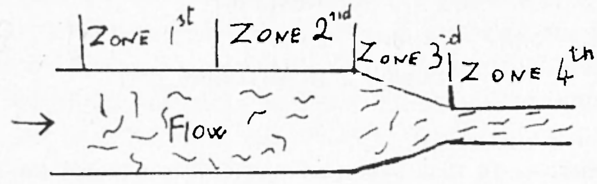


Fig. 1

Effect of reduction in feed dimensions on orientation of fibres. "A material with fibres heterogeneously distribued is made to flow through a straight channel, the tendency will be to straighten out or orientate the fibres parallel with the direction of flow".

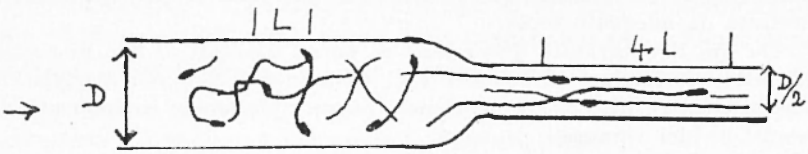


Fig. 1a

Impact Low	Impact High
Flexibility Low	Flexibility High
Unoriented	Oriented
Unstretched	Stretched

Orientation by mechanical working of molecule chains of polymerized vinylidene cloride.

It is valid the elastadesive property among elastically different ingredients, and are valid the rheological laws for components "polymers", organic and inorganic. All that requires complex distributions of stress in the manufactured material, according to the static load and those fluctuating applied, and the physic-chemical-mechanical geometrical structures of support.

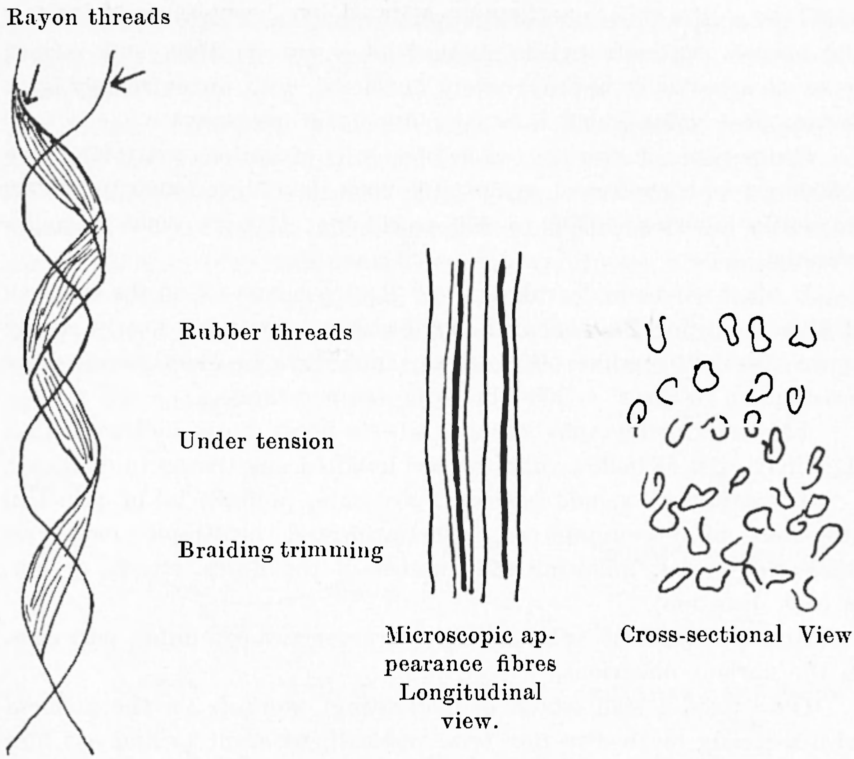


Fig. 1*b*

An external load P, working on a vehicle brake-blocks, will place itself among the single members of the lining, proportionally to the elastic modules, to the bearing sections, with transferts of partial loads due to adhesivity: $A_{d,e}$ elastic, in subordinate relation to the quantities:

$$A_{d,e} = \frac{kP \sum (E_{i+1} - E_i)}{(S_i E_i + S_{i+1} \times E_{i+1})}, \qquad [1]$$

with k coefficient, produced by the bearing areas: $S_i \cdot S_{i+1}$, divided by the sum of such products by the relative elastic mod. $\sum S_i E_i$. The eventual " elastic inadhesivities " are made up for by the " chemical adhesivities ", " absorptions ", inner mechanisms " between the phases ". Only one type of organic polymer therefore will be able to alloy differently (bond degree) according to the visco-elastic heterogeneity of the matrix.

" Fibrous materials " increase mechanical strength, stiffness, impact resistance, dimensional stability, and are often one-bi-more-directional properties (direction of the reinforcement). Fibres are flexibility and finesses (ultimate tensile strength of chrysotile fibres and various types of asbestos is approximately 50000 psi, with an extremely large surface area value-which is a very important property).

Comparison of average tensile strengths of various materials, give 155000 psi for carbon steel, against 100-2000 glass fibre, 80000 to 200000 chrysotile asbestos, 10000 to 300 crocidolite, 1000 to 8000 tremolite asbestos.

It must not be undervalued the " Heat resistance " on the strength of fibres, showing the chrysotile a " breaking strength " nearly steady up to 400°-500°, unlike other fibres (amianthus or glass fibres) more resistent up to the $T = 200°$ (breaking temperature).

Electron micrographs of the asbesto-fibres have indicated that they may exist as hollow tubes several hundred angstromes in diameter.

Thereafter we should consider, post-cure, preferencial or principal directions of fibre-reinforced, substratificated ortotropic structures (balanced or not, following the number of the fibres, steady or not, in each direction).

Anyway the elastic-plastic-resistent properties can differ, post-cure, in the various directions.

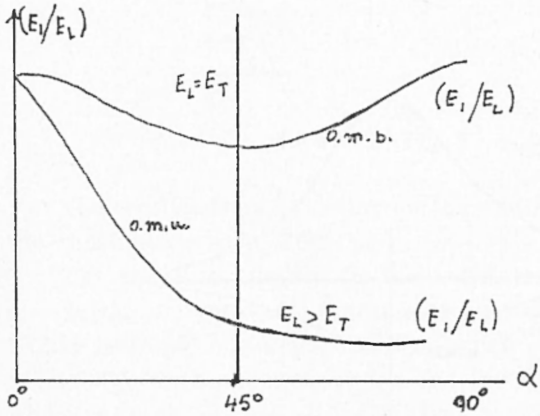
Once fixed a load (static of fluctuating), working for the moment with a steady mean direction (viz. vertical), we want to find out how the mechanical reactions place themself on the slabs, isotrop and not, subject to stresses (fibrous anisotropy).

The mod. of elasticity E, the shearing modulus G (in isotropic bodies), remain steady in any direction, and so " the stress σ, the shearing stress τ, caused constants strain ε and γ ($\varepsilon = \sigma/E$, $\gamma = \tau/G$), and constant $\varepsilon_T = \nu_E$ ($\nu =$ Poissons's ratio).

With transverse strains (contraction or dilation), in anisotropic e.g. " orthotropic materials ", occurs to reconstruct stress-strain diagrams, hence to find the elastic modulus in the longitudinal L in the trasversal T directions (E_L, E_T), the shearing modulus G_{LT} associated with these directions; the Poissons's ratio ν_{LT} caused by transverse strain a stress in L direction, ν_{LT} idem in the T direction. So that the modulus at any intermediate angle is E_1 (in function of a, E_L, E_T, G_{LT}, ν_{LT}), and if σ_1 is a stress applied in the 1-direction at an angle a with $\sigma_1 - L \to 0°$; $T \to 90°$ —, we have the most simple case of anisotropy. This is a particularly important point: the fibrous layer is not undamaged, when it is subejected to a strain, by its mean orientation with regard to a fixed direction (viz. the direction of a static or fluctuating agent load), it influences in fact the quantities relative to the mechanical deformations, though the sperimental data E_L, E_T, G_{LT} remain unchanged.

As the " cure piezo-termics " affect the frictiograms, on the coefficient μ and other mechanical (with the same combination of the packing's ingredients), thus the " cure " mix, extrusive, to which the fibrous layers are bound, affect the anisotropic pecularities of the lining, sometimes with remarkable incidences, as we can see afterwards.

We relate in the meantime, as a proof of what we stated, some comparative graphics of the functional quantities: E_1/E_L; G_{12}/G_{LT} (functions of the angle between direction of stress and the longitudinal axis of the material).



(balanced orthotropic material)

$\nu_{LT} = \nu_{TL}$

(unbalanced orthotropic material)

$\nu_{LT} > \nu_{TL}$

Fig. 2

The mod. E_L, E_T are known, and therefore we have as a result of the graphic in Fig. 2, the unkown E_1 (a, E_L); likewise, when the mod. G_{LT}, ν_{LT}, ν_{TL2} are measured in the directions L (longitudinal) and T (transversal), we have as a result G_{12} etc...

The ratio E_1/E_L is made up of the sum of three terms:

$\cos^4 a$	(directional term),
$E_L \cdot \mathrm{sen}^4\, a$	(mixed geometric-mechanical term),
$1/4\ (E_L/E_T - 2\,\nu_{LT})\, \mathrm{sen}^2\, 2a$	(mixed term).

Therefore E_1/E_L engages all the elastic terms relative to the fibres E_L, E_T, G_{LT}, ν_{LT}, as well as the joined " directionalities ".

Angle $a = 0^o$ is longitudinal direction, angle $a - 90^o$ is transverse direction, aE_1/E_L, read on the graphic of Fig. 2, allows to obtain the mod. E, mod. of elasticity in any reference direction.

The ratio E_1/E_L is symmetrical with regard to the ordinate crossing angle $a = 45^o$ (Fig. 2), with " balanced orthotropic material o.m.b. ", while for " unbalanced orthotropic material ", and for $a = 90^o$, E_1/E_L $a \rightarrow$ to asymptotic minimum (evident dissymmetry).

Thus G_{12}/G_{LT} (G_{LT} is the shearing modulus associated with L and T directions), is symmetrical to 45^o, unlike E_1/E_L in the cases " o.m.b. " and " o.m.u. " (v. Fig. 2a).

Two factores m_1, m_2 (caused by shear and direct stresses respectively) showing symmetrical but different behaviours, have been introduced.

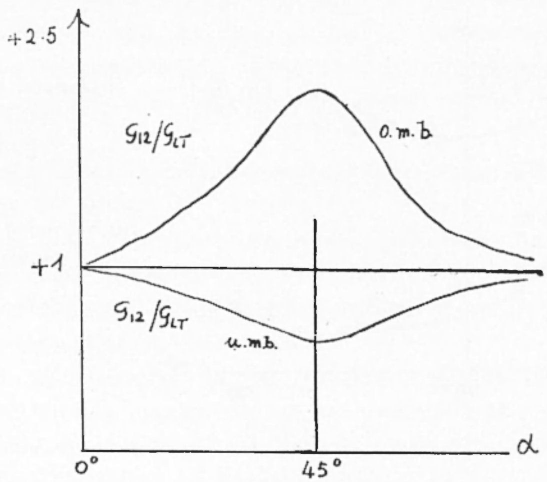


Fig. 2a

The ratio $E_L\ /E_1$ depends on a, E, G, ν_{LT}, as well as G_{LT}/G_{12}, ν_{12} depend on a, E, E, ν_{LT}, all quantities which can be measured in advance.

In these considerations we have voluntarily left out " plastic states " of the material, so that the bending moment of the structure comes out to be higher even of the 50% with regard to the elastic limit moment:

$$My\ (Mp = kMy)\ ;\qquad k = 1{,}5\qquad \text{(for rectangular sections).}$$

In a laminar solid, Fibreglass-Ru-Re-Fillers, $E_L/E_T = 10$, where only Ru, E_T (according to the hardness) can go down to 120-1300 psi; being $E_L = 5.106$, $E_T = 0{,}5.10^6$, $G_{LT} = 0{,}55.10^6$ psi, $\nu_{LT} = \nu_o = 0{,}45$, $\nu_{TL} = \nu_{90^o} = 0{,}045$.

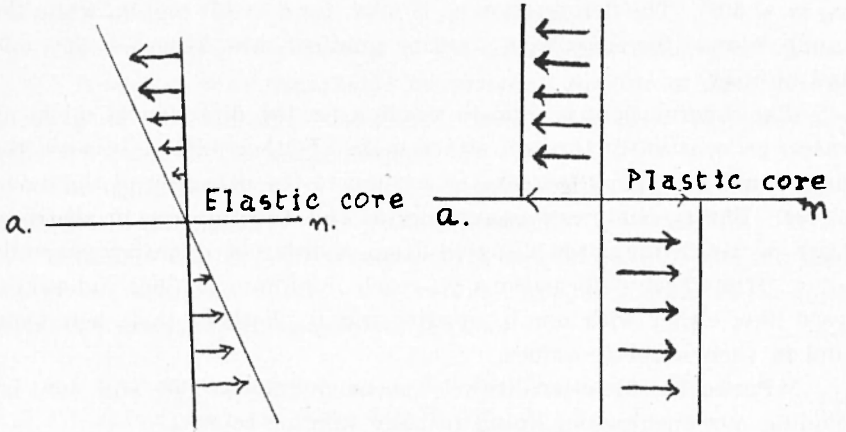


Fig. 3

The tensile stress σ_1, acting on the small plate at the top, is 10^4 psi; the shear stress $\tau_{12} = 4000$ psi.

The stress σ_1, applied in the 1-direction (Fig. 9) causes a strain $\varepsilon_1 = \sigma_1/E_1$. A transverse strain ε_2 is caused by σ_1, where $\varepsilon_2 = -\nu_{12}\,\varepsilon$. Unlike isotropic materials, stress σ_1, when applied at any angle axcept 0° and 90°, causes shear distortion and the shear strain $\gamma_{12} = -m_1\sigma_1/E_L$, where $m_1 = m_1$ (a, ν_{LT}, E_L, E_T, G_{LT}).

A shearing stress τ_{12} applied in 1-2 directions (v. Fig. 7) causes a shear strain $\gamma_{12} = \tau_{12}/G_{12}$, where $G_{12} = G_{12}$ (a, G_{LT}, ν_{LT}, E_L, E_T, ν_{LT}).

Consequently, strain caused by σ_1 are ε_1, ε_2, γ_{12}; strain caused by τ_{12} are ε_1', ε_2', γ_{12}'; total strain being $\varepsilon_1 + \varepsilon_1'$, $\varepsilon_2 + \varepsilon_2'$, $\gamma_2 + \gamma_2'$.

The diagrams showed above (Fig. 1 and $2a$), which join these mechanical quantities to the angle $a$mu st be prepared each time according to the materials (E_L, E_T, G_{LT}, ν_{LT}).

Thanks to them we obtain the "strains" caused by σ_1, and the "strains" caused by τ_{12}, as we can see here following:

$$\begin{cases} \sigma_1 \to \varepsilon_1 = \sigma_1/E_1\,, \quad \varepsilon_2 = -\nu_{12}\,\varepsilon_1\,, \quad \gamma_{12} = -m_1\,\sigma_1/E_L \\ \tau_{12} \to \varepsilon_1' = -m_1\,\tau_{12}/E_L\,, \quad \varepsilon_2' = -m_2\,\tau_{12}/E_L\,, \quad \gamma_{12}' = \tau_{12}/G_{12} \end{cases} \qquad [2]$$

The quantities $E_L^{-1}\,E_1(a)$, $G_{LT}^{-1}\,G_{12}(a)$, $\nu_{12}(a)$, for small variations of the "stiffness factor EI", "section modulus" for outermost fibre,

" bending and shear stress ", can lead to great differences in the results. As for the structures o.m.b. or u.m.b., the variations in the same stresses σ_1, τ_{12} ($\Delta\varepsilon_1$, $\Delta\varepsilon_2$, $\Delta\gamma_{12}$) are remarkably different. In both we can note negative minimum for ε_2 ($\alpha \sim 30^o$), and positive minimum for γ_{12} ($\alpha \sim 30^o$). The deformation ε_1, is max. for $\alpha = 45^o$ o.m.b., while the u.m.b. always increases with a strong gradient, also after $\alpha = 30^o$, and has no max.

The deformations ε_1, due to traction (in the direction of σ_1) in all cases, go constantly through either max. or other values, because the directional anisotropy increases according to the direction of the agent stress. The ε_2, compressives (usually at the direction σ_1), in all cases, they no reach max. too different from a order of quantity generally $< \varepsilon_1$. The shear deformations γ_{12} reach minimum at 30^o, and appear very like, either with o.m.b. or with u.m.b., both in their behaviour and in their absolute values.

" Particular directionalities " appear mainly at 30^o and 45^o, including our openings of lining (usually inferior to 80^o).

Superposing several thin plates, at crossed directions, those effect of fibrousity will be attenuated (pseudo-isotropy).

2. – Fibrous glass or asbestos reinforced plastics may be " composite and non-isotropic ".

Composite structure: various layers may be oriented at different angles with respect to each other, in order to provide the best combination to resist loading condition. Outside loads applied result in " internal stresses ", different individual layers (meeting to the layer's fibrousity). Strains and stresses are induced in each layer: external stresses may result not only in internal stresses, but in internal shear stresses. External shear stresses may result in internal stresses as well as internal shear stresses. The " composite structure ", the most elementary, is the bi-layer. Internal stresses σ_{1a}, σ_{1b}, τ_{12a}, τ_{12b}, can be found in order that the sums of the internal stresses in the 1 and 2 directions must equal the external stresses in these directions, and the strains must be the same in all layers. As the layers are firmly bonded together, the strains are the same in a and b layers, and are equal to the strains n the whole structure. Therefore the fundamental conditions (in which the hypothesis is traslated) are:

$$\left\{\begin{array}{lll} \varepsilon_{1a} = \varepsilon_{1b} = \varepsilon_1 \; ; & \varepsilon_{2a} = \varepsilon_{2b} = \varepsilon_2 \; ; & \gamma_{12a} = \gamma_{12b} = \gamma_{12} \\ \sigma_{1a}\, t_a + \sigma_{1b}\, t_b = \sigma_1\, t \; ; & \sigma_{2a}\, t_a + \sigma_{2b}\, t_b = \sigma_2\, t & \\ \tau_{12a}\, t_a + \tau_{12b}\, t_b = \tau_{12}\, t & & \end{array}\right. \qquad [3]$$

where t = total thickness of the bi-layer = sum of t_a and t_b, thicknesses of the horizontal superposed a, b, of different orthotropic materials oriented at arbitrary angles α and β, with respect to applied stresses σ_1, σ_2, τ_{12} (σ_1 acting in direction 1; σ_2 in the lower layer — acting in direction 2).

The " sandwich " is realized in the linings also under the " piezocure " (specially with remarkable thicknesses), because of the " plastic working of non-metallic materials ", which decreases exponentially with the depth in the bonded layer.

The compressive stress between punch and die distorts the sheet and gives a stratigraphy (slip-plane movements).

Materials a and b have respectively longitudinal L_a and L_b, and transverse directions T_a, T_b. The rational hypotesis done on the kind which we are studying (sandwich) impose:

$$\begin{aligned} \varepsilon_1 &= \varepsilon_{1a} = \varepsilon_{1b} = f_1 (\sigma_{1a}, \sigma_{2a}, E_{1a}, E_{2a}, E_{La}, \nu_{21a}, \tau_{12a}, m_{1a}) \;, \\ \varepsilon_2 &= \varepsilon_{2a} = \varepsilon_{2b} = f_2 (\sigma_{1a}, \sigma_{2a}, E_{1a}, E_{2a}, E_{Lb}, \nu_{12a}, \nu_{12b}, m_{2b}) \;. \end{aligned} \tag{4}$$

Solutions of the foregoing " linear equations ", leads the three simultaneous equations, give the unknown: σ_{1a}, σ_{2a}, τ_{12a}.

The linear equations with three unknown σ_{1a}, σ_{2a}, τ_{12a}, can be more easily employed, also without solution:

$$\begin{cases} t_a\, t_b/t\, (A_{11}\, \sigma_{1a} + A_{12}\, \sigma_{2a} + A_{13}\, \tau_{12a}) = \sigma_1/E_{1b} - \nu_{21}\, \sigma_2/E_{2b} - m_{1b}\, \tau_{12}/E_{Lb} \\ t_a\, t_b/t\, (A_{21}\, \sigma_{1a} + A_{22}\, \sigma_{2a} + A_{23}\, \tau_{12a}) = -\gamma_{12b}\, \sigma_1/E_{13} + \sigma_2/E_{2b} - m_{2b}\, \tau_{12}/E_{Lb} \\ t_a\, t_b/t\, (A_{31}\, \sigma_{1a} + A_{32}\, \sigma_{2a} + A_{33}\, \tau_{12a}) = -m_{1b}\, \sigma_1/E_{13} - m_{23}\, \sigma_2/E_{Lb} + \tau_{12}/G_{12b} \end{cases}$$

$$\begin{aligned} & A_{11} = 1/E_{1a}\, t_a + 1/E_{1b}\, t_b;\; A_{22} = 1/E_{2a}\, t_a + 1/E_{2b}\, t_b;\; A_{33} = 1/G_{12a}\, t_a + 1/G_{12b}\, t_b; \\ & A_{21} = -\nu_{21a}/E_{2a}\, t_a - \nu_{21b}/E_{2b}\, t_b \;;\quad A_{13} = A_{31} = -m_{1a}/E_{La}\, t_a - m_{1b}/E_{Lb}\, t_b \;; \\ & A_{23} = A_{32} = -m_{2a}/E_{La}\, t_a - m_{2b}/E_{Lb}\, t_b \;. \end{aligned} \tag{5}$$

In fact the coefficient A_{ii}, A_{ji} been known E_{1a}, E_{1b}, E_{2a}, E_{2b}, E_{La}, E_{Lb}, G_{1a}, G_{2b} can be reduced, and moreover:

$$A_{21} = A_{21} \;;\qquad A_{31} = A_{13} \;,\qquad A_{32} = A_{23} \;.$$

We must use the diagrams of Fig. 2 and 2a etc... likewise we work over a layer, of course been known E_{1a}, E_{1b}, E_{2a}, E_{2b} which may, f. e., be equal to each other, and equal to E_L ($E_L = E_T$) at the conditions $\alpha = 0^o$, hence the relations $E_1/E_L = 1$, or they are equal to each other at the conditions

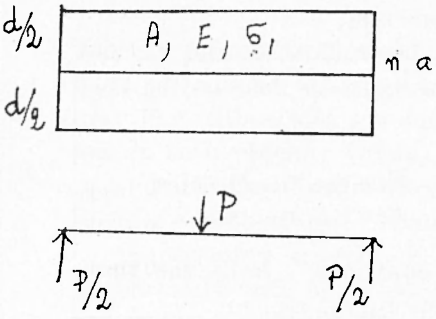
$$E_{60^o},\; E_{30^o}\; (E_{1a} = E_{1b} = E_{30^o});\; E_{2a} = E_{2b} = E_{60^o};\; \nu_{LT}/\nu_{TL} = E_L/E_T \;.$$

The other three equations for $\sigma_{1b}, \sigma_{2b}, \tau_{12b}$, come from:

$$\sigma_{1b}\, t_b = \sigma_1\, t - \sigma_{1a}\, t_a \;;\quad \sigma_{2b}\, t_b = \sigma_2\, t - \sigma_{2a}\, t_a \;;\quad \tau_{12b}\, t_b = \tau_{12}\, t - \tau_{12a} \qquad [6]$$

3. – For computing stresses, stiffness, for the "composite structures", some modifications of the standard formulas, which below we relate:

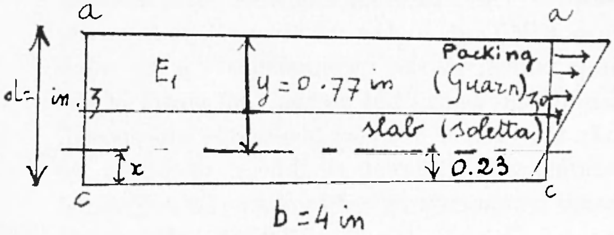
ISOTROPIC MATERIALS

Moment of Inertia	Stiffness Factor	Bending Stress	Shear Stress
$J = b\, d^3 / 12$  Fig. 4	$E\, I\, \Sigma A_i E_i$ $n.a.x. = \dfrac{\Sigma A_i E_i x_i}{\Sigma A_i E_i}$	$\sigma = 6\, M / b\, d^2$ for outermost fibre	$\tau = V\, Q / b\, J$ for max. shear at the neutral axis. $= \dfrac{3}{2}\; V / b\, d$

NON-ISOTROPIC MATERIALS

Moment of Inertia	Stiffness Factor	Bending Stress	Shear Stress
	$E_i,\ A_i,\ x_i$		
Cross section composite beam $I_i = b\, d_i{}^3 / 12$	are mod. of el., cross sectional area $bdi = Ai$ and distance from the bottom of cross section, to the centre of gravity of any particular layer.	E_y = mod. el. for the layer at y point. The max. bending stress not necessarily occur at the outermost (bottom) fibre, as it does in isotropic materials.	v is the total shear on the cross section, τ = shear stress intensity along some horizontal plane, Q' is the weighted statical moment $E_i\, A_i\, y'$ about the n.a. of the portions of the c.s. between the horizontal plane and the bottom of the c.s.

A plane packing with "adherent and composite slab":



$$\Sigma E_i S_i = E_1 S_1 + E_2 S_2 = EI$$
$$E_1/E_2 = 10 ;$$
$$n.a.x. = 0{,}23 \text{ in} .$$

Fig. 5

For a unit bending moment $M = 1$ in-lb:

Plane	y	E_y (noti)	$\sigma_y = y\ E_y / EI$ in-lb	σ / σ_y in-lib	M
(a–a)	0,77″	10.10⁶ psi	7,7/0,45 = 1,7	(40000 psi) / 1,7 =	2400 in-lb
(b–b)	0,27″	1.10⁶ psi	0,27/0,45 = 0,6	(20000 psi) / 0,6 =	33000 in-lb
(c–c)	0,23″	1.10⁶ psi	0,23/0,45 = 0.5	(20000 psi) / 0,5 =	40000 in-lb

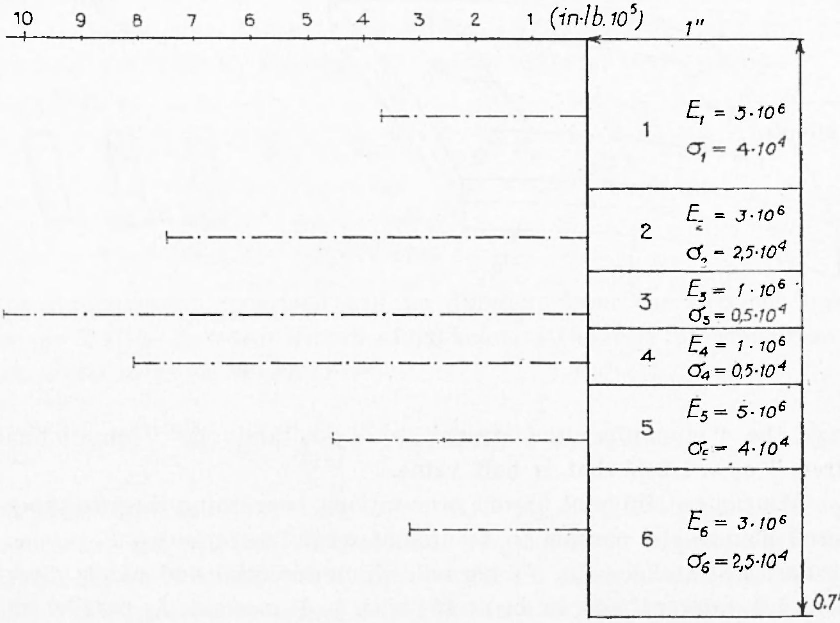


Fig. 6

Here appear a " critical plane " not " interstratified " but the bottom of the " bi-composite ". The " internal moment " or " moment of resistance " $M = EI/R = E/R \sum ay^2$, in the " isotrops ", is inversily proportional to the bending radius, in the " composites " is no more proportional " to the square of the depth, but to the 1.89 power of the depth ". In Fig. 6, we study the case of 6 layers horizontal superposed, with different elastic modulus, and different thickness, of which we have calculated the resistence's moments $\sigma y = F_y\, y/EI$; $EI = \sum E_i\, I_i$: (Fig. 6).

Internal Moment.

The length of the horizontal line (dash and point) gives the value of M in in-lb (max. 10.200 inl-b, min. 3500 in-lb) and we can note how the material is differently stresses inside.

4. – Let us consider at last the situation of a plane superficial element of " lining " (Fig. 7), a real shape of which, with a given strain p, we

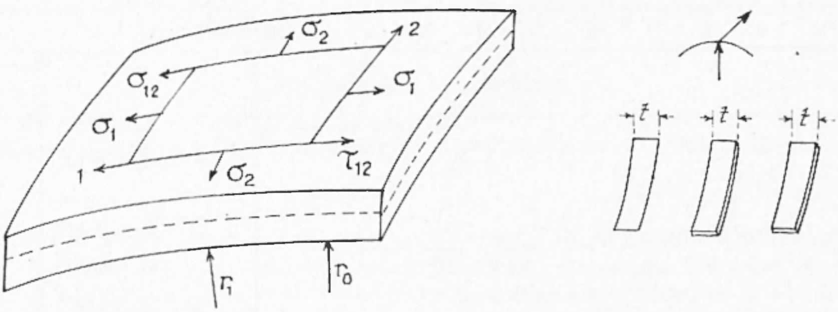


Fig. 7

have the " circumferential stress " $\sigma_1 = p\, r_o$ and the " longitudinal stress " $\sigma_2 = 1/2\, \sigma_1$ that is half value.

We can see different fibrous orientations, concerning the two superposed planes; the parallel to 1 (circumferential direction) —T_a perpendicular, L_b parallel to 2, T_b normal, circumferential and axials directions 1,2 (intertwisted); or L_a at 45° with 1, T_a normal, L_b parallel underlying to T_a, and T_b parallel to L_a; being $t_a \gtreqless t_b$ in any case. The angle α (L_{a1}) can be anyone, and the angle β $(L_b, 1)$ $(\alpha = \beta, \alpha \neq \beta)$ too. The calculus become easier as:

$$\begin{aligned}
&E_{1a}=E_{1b}=E_{2a}=E_{2b} \quad ; \nu_{12a}=\nu_{21a}=\nu_{12b}=\nu_{21b} \; ; \; m_{1a}=m_{1b}=m_{2a}=m_{2b}=0 \\
&A_{12}=A_{21} \quad ; A_{13}=A_{31}=A_{32}=A_{23}=0 \\
&A_{11}=1/E_{1a}\, t_a + 1/E_{1b}\, t_b \; ; \; A_{11}=A_{22} \\
&A_{22}=1/E_{2a}\, t_a + 1/E_{2b}\, t_b
\end{aligned} \qquad [7]$$

values which must be introduced in the three fundamental equations hence these become:

$$\begin{cases} A_{11}\,\sigma_{1a} + A_{12}\,\sigma_{2a} = \dfrac{t}{t_a\, t_b}\,(\sigma_1/E_{1b} - \nu_{12}\,\sigma_2/E_{2b}) \\[2ex] A_{21}\,\sigma_{1a} + A_{22}\,\sigma_{2a} = \dfrac{t}{t_a\, t_b}\,(-\nu_{12b}\,\sigma_1/E_{1b} + \sigma_2/E_{2b}) \; ; \; A_{33}\,\tau_{12} = 0 \end{cases} \qquad [8]$$

from where we get our unknowns:

$$\sigma_{1a} = \sigma_{1b} = \sigma_1 \; ; \quad \sigma_{2a} = \sigma_{2b} = \sigma_2 \; ; \quad \tau_{12a} = \tau_{12b} = 0$$

internal stresses, which become equal to the external strains, with no shear internal stresses, also for shear strains different from zero.

The symmetry of the orientations of the fibres with regard to the main directions of the stresses 1-2, cause the internal stresses σ_{1a}, σ_{1b}, σ_{2a}, σ_{2b} to be equal the external stresses imposed σ_1, σ_2 and annul the internal stress shear, although we impose external stresses shear.

For:

$$\begin{aligned}
&E_{1a} = E_{1b} = E_{300} = E_{600} = E_{2a} = E_{2b} \\
&G_{12a} = G_{12b} \; ; \quad \nu_{12a} = \nu_{12b} = \nu_{300} = \nu_{600} = \nu_{21a} = \nu_{21b} \\
&m_{1a} = m_{300} \; ; \quad m_{1b} = -m_{1a} \\
&m_{2a} = m_{600} \; ; \quad m_{2b} = -m_{2b}
\end{aligned} \qquad [9]$$

the fundamental equations will be different from the previous only in the 3td (the first two remain same, showing that the internal stresses are equal to those imposed)

$$A_{33}\,\tau_{12a} = \frac{t}{t_a\, t_b}\,(-m_{1b}\,\sigma_1/E_{1b} - m_{2b}\,\sigma_2/E_{1b})\,. \qquad [10]$$

Therefore being $\sigma_{1a} = \sigma_{1b} = \sigma_1$; $\sigma_{2a} = \sigma_{2b} = \sigma_2$, from the third equation we get $\tau_{12a} \neq 0$. Remarkable stresses shear appear when the layers are so oriented " even if there are no external shear stresses ". The shear stresses in layers b are oriented in the opposite direction to the shear stresses in layers a.

5. – We determine at this point the position of the max. pressure and the distribution of the pressure on the lining (rigid shoe calculation).

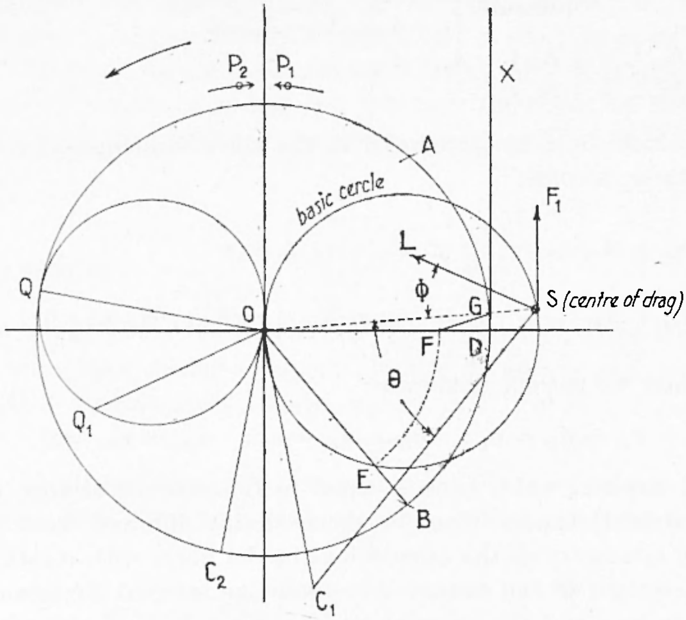


Fig. 8

In Fig. 8, there is the " centre of drag ", point where act the resultant of the forces of the shoe on the drum *s*. The centre of the drum

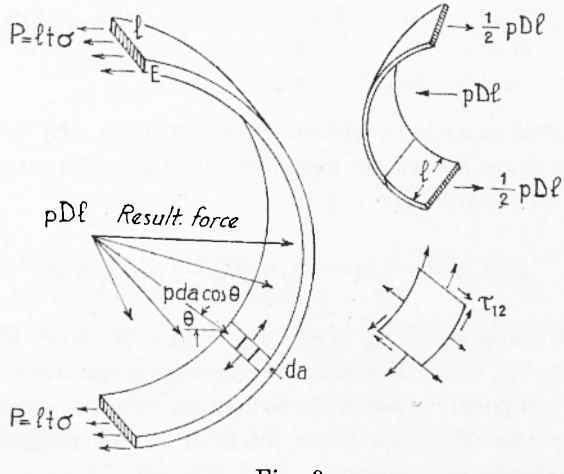


Fig. 9

with radius r is remarked by 0; $\overset{\frown}{AB}$ is the arch of contact of the shoe-*packing;* C_1 is the centre point of the shoe pivot.

We can also the max. pressure $\overline{O}Q_1$ " pressure distribution ", and resultant line of pressure $\overline{OG_1}$; the angle of friction Φ, the brake force for shoe F_1.

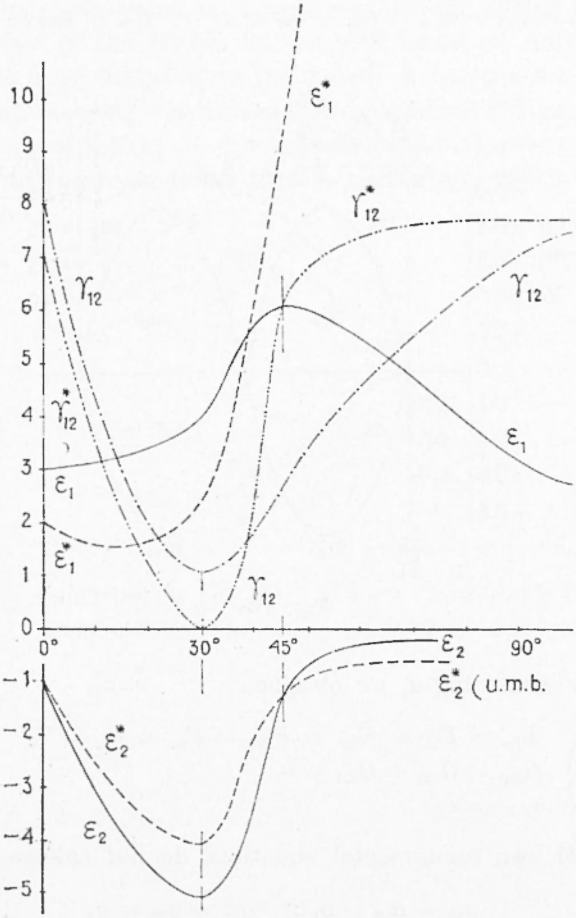


Fig. 10

More types of construction for " composites ", can be easily studied: layers a and b with the L and T direction laid in the circumferential and axial directions; the layers are laid at $\alpha = 10°$, $20°$, $30°$, $45°$, to the axis of the cylinder, e.g. alternate 30° angles in left-hand and right-hand spirals. For the position $\alpha = 0°$, 45°, 90°, we have:

$$\left\{ \begin{array}{llll} E_{1a}/E_L = 1\ ; & E_{1b}/E_L = 1\ ; & E_{2a}/E_L = 1\ ; & E_{2b}/E_L = 1 \\ m_{1a} = m_{1b} = m_{2a} = m_{2b} = 0\ ; & & \nu_{12a} = \nu_{21a} = \nu_{12b} = \nu_{21b} = \nu_{LT}\ , & \end{array} \right. \quad [11]$$

and therefore:

$$\sigma_{1a} = \sigma_{1b} = \sigma_1\ , \quad \sigma_{2a} = \sigma_{2b} = \sigma_2\ ; \quad \tau_{12a} = \tau_{12b} = 0\ . \quad [12]$$

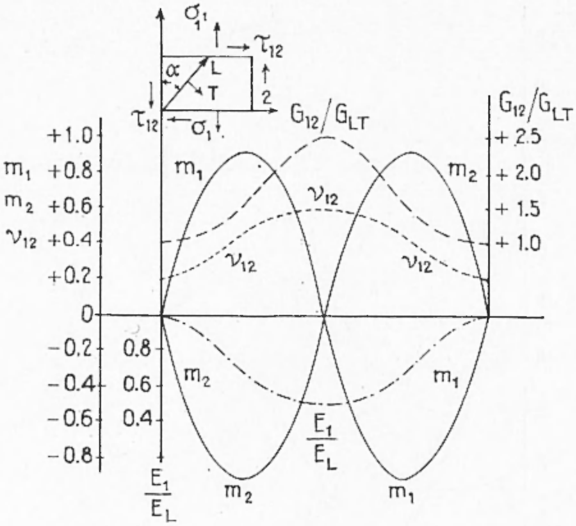


Fig. 10*a*

For the remaining α, we obtaine:

$$\left\{ \begin{array}{l} E_{1a} = E_{1b} = E_{30°} = E_{60°} = E_{2a} = E_{2b} = E_L \\ G_{12a} = G_{12b} = G_{LT} \\ \nu_{LT} = \nu_{TL}\ . \end{array} \right. \quad [13]$$

The first two fundamental equations do not change, however:

$$\sigma_{1a} = \sigma_{1b} = \sigma_1\ ; \quad \sigma_{2a} = \sigma_{2b} = \sigma_2 \quad [14]$$

while the third becomes:

$$\tau_{12a}\,(1/G_{12a}\,t_b + 1/G_{12b}\,t_b) = \frac{t_a}{t_a\,t_b}\,(-\,m_{1b}\,\sigma_1/E_{Lb} - m_{2b}\,\sigma_2/E_{Lb})\ , \quad [15]$$

from which

$$\tau_{12a} \neq 0\ . \quad [16]$$

Therefore being $\alpha = 0^o$, $\alpha = 45^o$, $\alpha = 90^o$, the shear deformations disappear, while such shear deformations appear again being $\alpha = 10^o$, 20^o, 30^o, 40^o, 60^o, 70^o, 80^o, 90^o, with max. values for $\alpha = 25^o$ and $\alpha = 65^o$.

While reckoning the stresses internal to the fibrous or no packing, we must, of course, keep into consideration the variation of the loads (Fig. 9) which give the σ_1 and τ_{12} measurable or obtainable, and therefore, going away from the zone of max. pressure, also the joining affects possible fibrousities of the fibrous lining, with mono or multi-structure, will fade. We have studied then the main principal possible incidences (Figg. 10-10a), keeping unchanged the constitutive formulas (qualitative and quantitative) of the ingredients and load-stress, on the deformations of the brake-lining (isotrop, anisotrop, fibrous and stratiform structures).