STATISTICAL METHOD IN GEOPHYSICAL PROSPECTING

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§ 1. Let us consider (see figure 1) a very deep horizontal layer of the earth's crust. The lower line signifies a limit more or less defined. The simplest possibility which can be supposed is that on this limit the shear components of the stress-tensor (the components which produce the angular distortion of an element of the body) are very small here or even disappear. This can take place either if in the neighbourhood of the limit the temperature approaches a degree where the distortion effects disappear and only the pressure components of the stress-tensor exist, or if the nearest lower consists of a loose body. But these two examples do not exhaust all possibilities. In the first approximation the limit surface of *every* two layers has the supposed property. Finally, if we wish to reach a higher degree of approximation, we can always introduce some more complex boundary conditions (see also § 4).

It is important to emphasize that the theory proposed on this paper (and the proposed method of geophysical prospecting too) is not limited on the acception of this very simple boundary condition. We employed here this simplification only in order to establish the simpliest formulas.

The line at the top of the figure 1 represents the upper parts of the considered layer where the material properties and the state of vibrations are sufficiently homogeneous and where the measurements characterize the general rock properties of the layer and the general state of vibrations. This line can be situated at a depth of some meters (for a loose rock) and even at a depth of some centimeters (for a hard rock). In no circumstance can this line coincide exactly with the earth's surface.

Let us suppose that this layer is being moved irregularly as the sea after a storm. The causes of this state of being agitated can be numerous: the action of a distant earth-quake, the post-phenomena after an explosion, the dispersion of the tremous and important pulses, the vibration accompanied the slipping of the layer in the earth-interior a.s.o. Let us also suppose that the considered layer is homogeneously « filled » with these *microseisms*. The chaotic state of a layer filled by wandering microseisms is one of the best subjects of the theory of probability. But the stringent application of the theory of probability will come a little later. We begin from the simple mechanical considerations, which can give us results qualitatively not different from the most stringent results of the probability-rules. Naturally these two kind of results are numerically not the sames, see 3e.

The simple mechanical considerations the results of which we wish to emphasize, are illustrated by the figures 1 and 2. On the figure 1 we compare two cases of the layers of the same depth but of different density ρ ; the layers being differently agitated have nevertheless the same values of the stress. The symbol v_y and the arrows near it indicate

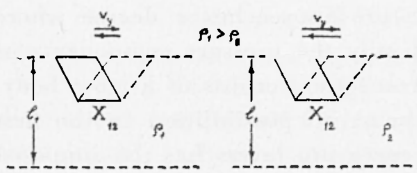

Fig. 1

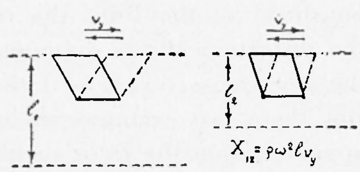

Fig. 2

the y-component of the velocity-vector. The directions of y and z coincide with the horizontal plane, the x-axis is directed vertically. The parallelograms symbolize the variation of the shear components of the stress-tensor; the degree of the obliquity of the parallelograms correspond to the intensity. The letter $\dot{X}_{ij}$ indicates the components of the tensor of the stress variations: $\dot{X}_{11}$, $\dot{X}_{22}$ and $\dot{X}_{33}$ are the pressure variations in the x,- y,- and z- directions; $\dot{X}_{12}$, $\dot{X}_{13}$ and $\dot{X}_{23}$ the shear components of the stress variations. The parallelograms on the figures 1 and 2 are the $\dot{X}_{12}$ components.

Finally let us suppose that the amplitudes of the variables $\dot{X}_{12}$ and v_y are measured in the neighbourhood of the upper line on the figure. Considering this figure we can easily visualize that X_{12} and v_y are proportional. The figures will help us to determine the kind of this proportionality. How is immediately clear from comparison of the left and the right parts of this figure, the strong v_y velocity components correspond to the smaller ρ-values and reciprocally if the value of X_{12} is maintained. It follows from this comparison that the proportionality we look for is $\dot{X}_{12} = \text{const.}\ \rho\ .\ v_y$. It shows the coefficient of proportionality is a linear function of the density.

The same is true about the depth. It is seen from the figure 2.

Here two cases of the layers are represented where the density is the same but the depth l is different. It is clear that in order to call for the same velocity-agitation in the case of the small depth we need the very small value of the stress variation $\dot{X}_{12}$. The density is supposed constant for the left and right parts of figure 2). Therefore we can write: $\dot{X}_{12} =$ const. ρ el v_y.

It is not difficult to imagine a pair of figures illustrating the third possibility: we mean the dependency between the coefficient in the considered proportionality formula an the frequency ω.

It is evident that the considered coefficient is also a linear function of ω^2. Therefore we are allowed [see also (1)] to write finally the proportionality formula as follows:

$$\text{(I)} \quad \dot{X}_{12} = \rho\, \omega^2\, l.\, v_y.$$

The same results are easily obtained from the theory of « dimension ». But neither the mechanical consideration of the kind here given nor the « dimensional » considerations could give us the value of the numerical coefficient in the final elementary formula (I). We have accepted in (I) that this numerical coefficient is equal to 1, but we know it only from the stringent calculation which will be given in the following paragraph. This calculation will show that (I) is true but only as a first approximation.

§ 2. Examining the simple formula (I) an idea comes involontarily that we can take advantage of it for the purposes of Geophysical Prospecting. Indeed, if we shall measure the amplitudes of $\dot{X}_{12}$ and v_y (velocities and variations of stress) corresponding to the same frequency ω, we shall be in possession of the methods to calculate the depth l (supposing that the density is known). The expression « to calculate » is perhaps to excessive for the simple division we need in order to obtain the depth l from the formula (I). We get accustomed to more complex calculations in geophysical practice.

Just the determination of the depth of the layers composing a horizontal layer-system is one of the most difficult problems of Geophysics. The possibility of the applications of (I) to the geophysical purposes being assumed, we see that formula is not a pretty Christmas plaything, but an instrument of practical meaning and importance.

Now we show how the formula (I) can be approved by the mathematical calculation. We begin from the well-known formulas representing the dynamical equilibrium in a continuous material system:

$$\frac{\partial X_{11}}{\partial x}+\frac{\partial X_{12}}{\partial y}+\frac{\partial X_{13}}{\partial y}=-\rho\frac{\partial v_x}{\partial t};\ \frac{\partial X_{12}}{\partial x}+\frac{\partial X_{22}}{\partial y}+\frac{\partial X_{23}}{\partial z}=-\rho\frac{\partial v_y}{\partial t};$$

$$\frac{\partial X_{13}}{\partial x}+\frac{\partial X_{23}}{\partial y}+\frac{\partial X_{33}}{\partial z}=-\rho\frac{\partial v_z}{\partial t}. \qquad \text{[A]}$$

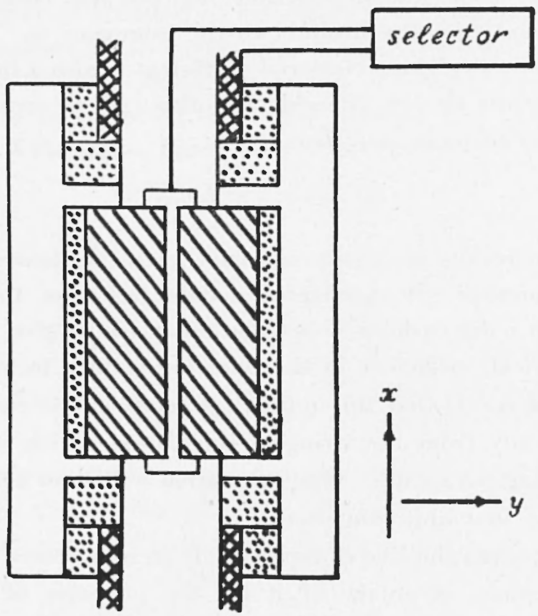

Fig. 3

Here X_{ij} are the components of the stress tensor. The x-axis is vertical.

The meaning of employed symbols (ρ, v_x, v_y, v_z) is already given in the preceding paragraph. Moreover we can suppose that the stresses are linear functions of the deformation and of the velocity of deformation. This linearity expresses the fact that the considered vibrations have small amplitudes; we consider only the microseisms.

This generalised law of Hooke we see in the formula:

$$X_{12}=\mu_e\left(\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right)+\mu_v\left(\frac{\partial v_x}{\partial y}+\frac{\partial v_y}{\partial x}\right) \qquad \text{[B]}$$

Here μ_e and μ_v are the material constants, the first represents the elastic property of rocks, the second their viscosity. In order to simplify the calculation we suppose that the vibratory state on and near the upper horizontal line where one measure the values of X_{12} and v_y (or of X_{13} and v_z), is sufficiently homogeneous. We employ the word « homogeneous » in the sense that all variables characterizing the vibratory state depend practically only on x, but not on y and z. This very general assumption permits us to find the equations which determine X_{12} and v_y:

$$\mu_e \frac{\partial^2 v_y}{\partial x^2} + \mu_v \frac{\partial^3 v_y}{\partial x^2 \partial t} = -\rho \frac{\partial^2 v_y}{\partial t^2} \; ; \; \mu_e \frac{\partial^2 X_{12}}{\partial x^2} + \mu_v \frac{\partial^3 X_{12}}{\partial x^2 \partial t} = -\rho \frac{\partial^2 X_{12}}{\partial t^2} \qquad [C]$$

On the same way the equations for the variables X_{13} and v_z can be found too. The solution of these equations can be given in the form:

$$X_{12} = Mod. \left(X_{12}^{(\omega)} \frac{\sinh \frac{l-x}{a} \sqrt{\frac{\omega^2}{1 \pm b^2 i \omega}}}{\sinh \frac{l}{a} \sqrt{\frac{\omega^2}{1 + i \omega b^2}}} \right) \cos(\omega t + \varphi) \qquad [D]$$

if we accept the elementary boundary condition mentioned in the § 1. The introduced abbreviations a and b signify:

$$a = \sqrt{\frac{\mu_e}{\rho}} \; , \; b = \sqrt{\frac{\mu_v}{\mu_e}} \; .$$

Finally for the low frequencies and for the viscous rocks ($\mu_v >> \mu_e$) we have the formula:

$$|X_{12}|^{(\omega)} = \rho \, \omega^2 \, i \, | v_y |^{(\omega)} \qquad [E]$$

which coincide with the elementary formula (I); except that we have introduced in (E) the supplementary indices (ω) for the amplitudes of X_{12} and v_y in order to emphasize that these amplitudes belong to the same frequency.

How we have remarked on the § 1 the formula (E) is not the unique formula, but only the sempliest one. But if instead of the simple boundary condition of the § 1 we introduced the more complicated conditions, we do not alter the general form of the formula (E), we shall get:

$$|X_{12}|^{(\omega)} = const. \; l \, | v_y |^{(\omega)} \qquad [EE]$$

Only the coefficient (*const*) will have other value, see § 4.

The necessity to lock for the correspondance to the same frequency is a weak point of the proposed method. Another difficulty arises from the necessity of measuring the X_{12}. The measurement of the kinematical quantities is well known, especially the measurement of the acceleration. The velocity we need, can be measured immediately or calculated from the measurement of the acceleration $\dot{v}$; the second possibility belong to the case of the sinusoidal vibrations, where the amplitude of $\dot{v}$ differs from v only by the multiplier ω. On the contrary, the measurement of the stresses is very difficult and rarely undertaken. Nevertheless it is not impossible, since we need to know the *variations* of the stress (that is $\dot{X}_{12}$), but not the stresses themselves (that is X_{12}). For instance if we imagine working with a piezoelectric body the measurement of the absolute values of X_{12} demands such a complex laboratory apparatus, that its accomplishment can not be realized in the conditions we have in practical. The absolute values of X_{12} are too little and the quantity of electricity created on the surfaces of the piezoelectric body can be measured only with the most precise instruments. Naturally an amplifier can not be used because there are only the constant quantities of electricity. However they are not the stresses we need, but the variations of them. Here we have not a constant quantity but a speedely altered state. Thus the amplification is possible throughout and the instrument that strictly speaking makes the measurement need not be especially precise.

With a piezoelectric body they usually measure the pressure- components of stress. Let us give therefore some details of the measurement of the shear components, for example X_{12}.

The principal scheme of such a measurement is represented by fig. 3 and 4. How usually we have two piezoelectric slabs. They are cut out from a piezoelectric body in such a manner that the maximal piezoelectric effect is called forth by action of the shear components of stress. In order to suppress any other possible action (even a little on) the partition pieces of almost soft material are put between and near to the metallic pieces which transmit the stress action from the rock to the piezoelectric body. They are put so that all components of stress except one are eliminated. For instance in the figures 3 and 4 the piezoelectric slabs and other parts of the apparatus are so disposed that only the $\dot{X}_{12}$ — component give the altered quantities of electricity on the slabs and the electrical current in the circuit.

§ 3. Having supposed that the values of variables $\dot{X}_{12}$ and v_y (or $\dot{X}_{13}$

and v_z) are measured by one way or other, we can turn to the interpretation. As was also observed in the 2e, the theoretical difficulty of the proposed method is the choice of the amplitudes corresponding to the particular vibrations of v_y and of X_{12} having the same frequency. In order to resolve this problem two methods can be conceived.

The first possibility which can be proposed is the consideration of the average values of v_y and X_{12}, but not the momentary values. The simple average values commonly employed are here not applicable. Only the tensorial average values introduced by us [see (2) and (3)] can be usefull.

Let us introduce for these average values the following letters: M_{ij} the tensor of the average values of the velocity (v_x, v_y, v_z) and M_{ijk} the tensor of the average values of the variations of stress (X_{ij}). With these symbols the basic equations of Statistical Seismology can be written in the following form:

$$\left.\begin{aligned}
\frac{\partial M_{111}}{\partial x}+\frac{\partial M_{112}}{\partial y}+\frac{\partial M_{113}}{\partial z}&=-\rho\frac{\partial M_{11}}{\partial t},\\
\frac{\partial M_{112}}{\partial x}+\frac{\partial M_{122}}{\partial y}+\frac{\partial M_{123}}{\partial z}&=-\rho\frac{\partial M_{12}}{\partial t},\\
\frac{\partial M_{122}}{\partial x}+\frac{\partial M_{222}}{\partial y}+\frac{\partial M_{223}}{\partial z}&=-\rho\frac{\partial M_{22}}{\partial t},\\
\frac{\partial M_{113}}{\partial x}+\frac{\partial M_{123}}{\partial y}+\frac{\partial M_{133}}{\partial z}&=-\rho\frac{\partial M_{13}}{\partial t},\\
\frac{\partial M_{123}}{\partial x}+\frac{\partial M_{223}}{\partial y}+\frac{\partial M_{233}}{\partial z}&=-\rho\frac{\partial M_{23}}{\partial t},\\
\frac{\partial M_{133}}{\partial x}+\frac{\partial M_{233}}{\partial y}+\frac{\partial M_{333}}{\partial z}&=-\rho\frac{\partial M_{33}}{\partial t}.
\end{aligned}\right\}\quad\text{(II)}$$

[see also our article (4)]. The elementary case which we consider corresponds to:

$$\frac{\partial M_{122}}{\partial x}=-\rho\frac{\partial M_{22}}{\partial t}$$

$$\frac{\partial M_{123}}{\partial x}=-\rho\frac{\partial M_{23}}{\partial t}$$

$$\frac{\partial M_{133}}{\partial x} = -\rho \frac{\partial M_{33}}{\partial t}$$

From the purely mathematical point of view the difficulty of resolving them is not greater than the one of resolving the common equations

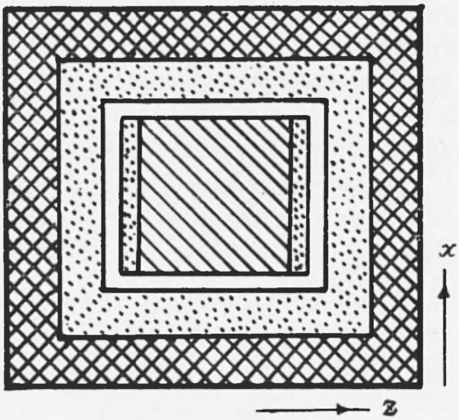

Fig. 4

of elasticity. In our case the same mathematical transformations and operations will lead us to the formula

$$\dot{M}_{1jk} = \rho\, l\, \omega^2 M_{ik} \qquad \text{[F]}$$

which is very similar to the one (E) given earlier. In the formula (F) the M_{22} and M_{33} correspond to v_y and v_z. But in order to calculate them we are obliged to know not only one of values v_y (or v_z) and not many value of v_y only (or v_z only) but necessarily many values of v_y and v_z together. The same is true for the M_{122}, M_{123}, M_{133} corresponding to X_{12} and X_{13}.

Concerning the obtained formule (F) the same remarke if true as the one which is done in the § 2 concerning the formula (E). On the hasis of the same reasoning as these we can introduced also here the generalised formule

$$\dot{M}_{1jk} = const.\ l\ M_{ik} \qquad \text{[FF]}$$

where the value of *const.* belong the chosen boundary condition on the limit of the layers (i, $k = 2, 3$).

Naturally the values of M are not directly measured; it is as always the values of v and X which we look for. Having the values of v and X obtained experimentally, the values of M can be calculated numerically or with a graphical mechanism.

But this theoretical way is not the only one. The correspondence between the values of v_y and X_{12} [see formula (E)] can be found with

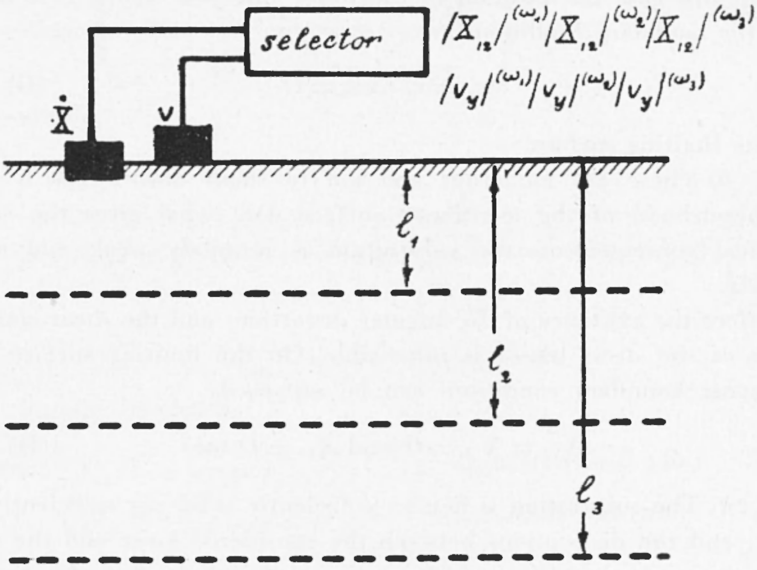

Fig. 5

a selector. The selector can be constructed in very numerous ways. There exist already the patents for these selectors. Thus we have the following simple scheme for the geophysical prospecting methods using the discovered formula (E) (see figure 5).

Two elements characterizing the microseisms are measured by two instruments. The first is a common seismograph, the second an instrument of the kind described in 2e. The currents furnished by these instruments are fed to the selector which shows the corresponding pairs of values v and X. Employing the formula (E) or the similar formula if we wish to introduce boundary conditions other than those of le we calculate one or a series of layers which are deeply below our instruments. As we have said the formula (E) is only the first approximation. If we shall employ the following approximations [see (4)] we shall be able to calculate the physical constants of the layers too.

§ 4. How we have already remarked the formulae (E) and (F) are not the unique formulae, but only simpliest of their kind.

Generally three kind of boundary conditions can be imagined for the surface below the considered layer:

a) The substratum is infinitely solid and firm; there are no dislocations in the neighbourhood of the limiting surface.

In this case the position of the lower line (see figura 1) is fixed and the boundary conditions are:

$$v_x = v_y = v_z = O \qquad \text{(G)}$$

on the limiting surface.

b) There are numerous and not to small dislocations in the neighbourhood of the mentioned surface. Or (what gives the same physical consequences) the substratum is infinitely weak and even liquid.

Here the existence of the angular distortions and the shear components of the stress tensor is impossible. On the limiting surface the following boundary conditions can be supposed:

$$X_{12} = X_{13} = O \text{(and } X_{11} = O \text{ too)} \qquad \text{(H)}$$

c) The substratum is neither sufficiently solid nor sufficiently liquid; and the dislocations between the considered layer and the substratum are present in quantity not great enough in order to determine (H).

The only conditions we can suppose on the limiting surface in this general case are:

$$\left.\begin{array}{l} X_{ij} \text{ in the substratum} = X_{ij} \text{ in the considered loyer} \\ V_i \text{ in the substratum} = V_i \text{ in the considered layer} \end{array}\right\} \qquad \text{(J)}$$

For the statistical applications the conditions (G), (H), (J) can be written:

$$M_{ij} = O \qquad (G_s)$$

$$\dot{M}_{ij} = O \qquad (H_s)$$

$$\left.\begin{array}{l} M_{ij} \text{ in the substratum} = M_{ij} \text{ in the considered layer} \\ \dot{M}_{ijk} \text{ in the substratum} = \dot{M}_{ijk} \text{ in the considered layer} \end{array}\right\} \qquad (J_s)$$

on the limiting surface (the lower line on the figure 1).

The case b) / conditions (H) or (H_s) / giving the simpliest formula are already considered in the §§2 and 4. Let us now take in consideration the cases a) and c).

For the first case instead of the formula (D) we have:

$$v_y = Mod. \left(v_y^{(\omega)} \frac{\sin h \frac{l-x}{a} \sqrt{\frac{\omega^2}{1+i\omega b^2}}}{\sin h \frac{l}{a} \sqrt{\frac{\omega^2}{1\pm i\omega b^2}}} \right) \cos(\omega t + \varphi), \qquad \text{(K)}$$

v_z — simile

or:

$$M_{22} = Mod. \left(M_{22}^{(\omega)} \frac{\sin h \frac{l-x}{al} \sqrt{\frac{\omega^2}{1\pm i\omega b^2}}}{\sin h \frac{1}{a} \sqrt{\frac{\omega^2}{1\pm i\omega b^2}}} \right) \cos(\omega t + \varphi), \qquad (K_s)$$

M_{23} — simile, M_{33} — simile

Finally we obtain:

either: $|X_{12}|^{(\omega)} = const.\ l\, |v_y|^{(\omega)} \ldots etc.$ (non statistical case) (L)

or: $|\dot{M}_{122}|^{(\omega)} = const.\ l\, |\dot{M}_{22}|^{(\omega)} \ldots etc.$ (statistical case) (L_s)

In the general non-statistical case we have not only one equation (C) but two equations:

$$\mu_{es} \frac{\partial^2 X_{12}^{(s)}}{\partial x^2} + \mu_{vs} \frac{\partial^3 X_{12}^{(s)}}{\partial x^2 \partial t} = -\rho s \frac{\partial^2 X_{12}^{(s)}}{\partial t^2}, \qquad \text{(M)}$$

for the substratum and

$$\mu_{ec} \frac{\partial^2 X^{(c)}}{\partial z^2} + \mu_{vc} \frac{\partial^3 X^{(c)}}{\partial x^2 \partial t} = -\rho c \frac{\partial^2 X^{(c)}}{\partial t^2}, \qquad \text{(P)}$$

for the considered layer.

The solution can be chosen in the form:

$$X^{(s)} = X_{12}^{(s.\omega)} \cdot e^{-\frac{\omega x}{a_s \sqrt{1+i\omega b_s^2}}} \cdot e^{\pm i\omega t}$$

$$X_{12}^{(c)} = \left[X_{12}^{(1,c\,\omega)} \sin h \frac{\omega x}{a_c \sqrt{1\pm\omega b_c^2}} + X_{12}^{(2,c,\omega)} \cos h \frac{\omega x}{a_c \sqrt{1\pm i\omega b_c^2}} \right] \cdot e^{\pm i\omega t}$$

(R)

Here the conditions (J) can be written as following

$$X_{12}^{(s,\omega)} e^{-\frac{\omega l}{a_c\sqrt{1\pm i\omega b_c^2}}} = X_{12}^{(1,c,\omega)} \sin h\left(\frac{\omega l}{a_s\sqrt{1\pm i\omega b_s^2}}\right) +$$

$$+ X_{12}^{(2,c,\omega)} \cos h\left(\frac{\omega l}{a_s\sqrt{1\pm i\omega b_s^2}}\right), \qquad \text{(S)}$$

$$-\frac{a_c\sqrt{1\pm i\omega b_c^2}}{a_s\sqrt{1\pm i\omega b_s^2}} X_{12}^{(s,\omega)} e^{-\frac{\omega l}{a_c\sqrt{1\pm i\omega b_c^2}}} = -$$

$$-X_{12}^{(1,c,\omega)} \cos h\left(\frac{\omega l}{a_s\sqrt{1+i\omega b_s^2}}\right) + X_{12}^{(2,c,\omega)} \sin h\left(\frac{\omega l}{a_s\sqrt{1\pm i\omega b_s^2}}\right).$$

Finally, instead of (D), we have

$$X_{12}^{(c)} = Mod.\left(X_{12}^{(2,\omega,c)} \cdot e^{\pm i\omega t} \cdot \frac{a_c\sqrt{1\pm i\omega b_c^2} \cdot \sin h\left(\frac{(l-x)\omega}{a_c\sqrt{1\pm i\omega b_c^2}}\right) +}{a_c\sqrt{1\pm i\omega b_c^2} \cdot \sin h\left(\frac{l\omega}{a_c\sqrt{1\pm i\omega b_c^2}}\right) +} \right. \qquad \text{(T)}$$

$$\left. \frac{+ a_s\sqrt{1\pm i\omega b_s^2} \cdot \cos h\left(\frac{(l+x)\omega}{a_s\sqrt{1\pm i\omega b_s^2}}\right)}{+ a_s\sqrt{1+i\omega b_s^2} \cdot \cos h\left(\frac{l\omega}{a_s\sqrt{1\pm i\omega b_s^2}}\right)} \right)$$

and the formula (EE) (if $\mu_v >> \mu_e$).

In the general statistical case the equations (II) must be remplaced by

$$\frac{\partial M_{122}^{(s)}}{\partial x} = -\rho_s \frac{\partial M_{22}^{(s)}}{\partial t}, \quad \frac{\partial M_{123}^{(s)}}{\partial x} = -\rho_s \frac{\partial M_{23}^{(s)}}{\partial t}, \quad \frac{\partial M_{133}^{(s)}}{\partial x} = -\rho_s \frac{\partial M_{33}^{(s)}}{\partial t} \qquad (M_s)$$

for the substratum, and

$$\frac{\partial M_{122}^{(c)}}{\partial x} = -\rho_c \frac{\partial M_{22}^{(c)}}{\partial t}, \quad \frac{\partial M_{123}^{(c)}}{\partial x} = -\rho_c \frac{\partial M_{23}^{(c)}}{\partial t}, \quad \frac{\partial M_{133}^{(c)}}{\partial x} = -\rho_c \frac{\partial M_{33}^{(c)}}{\partial t} \qquad (P_s)$$

for the considered layer.

Further we can calculate:

$$\left.\begin{aligned}
&M_{122}^{(s)} = M_{122}^{(s,\omega)} \cdot e^{-\frac{\omega x}{a_s\sqrt{1\pm i\omega b_s^2}}} \cdot e^{\pm i\omega t}\\
&M_{122}^{(c)} = \Big[M_{122}^{(1,c,\omega)} \sinh\frac{\omega x}{a_c\sqrt{1\pm i\omega b_c^2}} +\\
&+ M_{122}^{(2,c,\omega)} \cosh\frac{\omega x}{a_c\sqrt{1\pm i\omega b_c^2}}\Big] \cdot e^{+i\omega t}\\
&M_{123} \text{ and } M_{133} \text{ — simile}
\end{aligned}\right\} \quad (R_s)$$

$$\left.\begin{aligned}
&M_{122}^{(s,\omega)} e^{-\frac{\omega l}{a_s\sqrt{1\pm i\omega b_s^2}}} = M_{122}^{(1,c,\omega)} \sinh\left(\frac{\omega l}{a_c\sqrt{1\pm i\omega b_c^2}}\right) +\\
&\quad + M_{122}^{(2,c,\omega)} \cosh\left(\frac{\omega l}{a_c\sqrt{1\pm i\omega b_c^2}}\right),\\
&\quad -\frac{a_c\sqrt{1\pm i\omega b_c^2}}{a_s\sqrt{1\pm i\omega b_s^2}} M_{122}^{(s,\omega)} e^{-\frac{\omega l}{a_s\sqrt{1\pm i\omega b_s^2}}} = -\\
&- M_{122}^{(1,c,\omega)} \cosh\left(\frac{\omega l}{a_c\sqrt{1\pm i\omega b_c^2}}\right) +\\
&\quad + M_{122}^{(2,c,\omega)} \sinh\left(\frac{\omega l}{a_c\sqrt{1\pm i\omega b_c^2}}\right),\\
&M_{123} \text{ and } M_{133} \text{ — simile}
\end{aligned}\right\} \quad (S_s)$$

$$M_{122}^{(c)} = Mod\left(M_{122}^{(2\omega,c)} \cdot e^{i\omega t} \cdot \frac{a_c\sqrt{1\pm i\omega b_c^2} \cdot \sinh\left(\frac{(l-x)\,\omega}{a_c\sqrt{1\pm i\omega b_c^2}}\right) + a_s\sqrt{1\pm i\omega b_s^2} \cdot \cosh\left(\frac{(l+x)\,\omega}{a_s\sqrt{1\pm i\omega b_s^2}}\right)}{a_c\sqrt{1\pm i\omega b_c^2} \cdot \sinh\left(\frac{l\,\omega}{a_c\sqrt{1\pm i\omega b_c^2}}\right) + a_s\sqrt{1\pm i\omega b_s^2} \cdot \cosh\left(\frac{l\,\omega}{a_s\sqrt{1\pm i\omega b_s^2}}\right)} \right) \quad (T_s)$$

and obtain the formula (FF) (if $\mu_v >> \mu_c$).

The statistical method of the tensor average values applied in this paper has interest not only for the Seismology but also for all branches of Geophysics. The author has also given the applications of this method on the phenomena of the electromagnetic / see (5) and (6) / fields und of the fields of gravitation / see (3) /.

Finally we must mentioned that there are two kinds of statistical methods in Geophysics:

a) Methods of the pure prospecting / see (1) and (6) / and

b) Methods of the estimation of the certitude of the geophysical prospecting / see (7), (8) and (9) /.

SUMMARY

As an example of application of Statistical Methods the author considers the natural vibration of the earth crust or its post-explosion vibrations. He gives a short description of his theoretical results on the statistics of these chaotic waves.

Two important and very simple formulas issued from this general theory are mentioned. They represent the proportionality between certain components of the tension tensor and of the velocity vector measured near or on the surface. The first formula — (« classical » method) — affirms the proportionality between the selected elements of components. The selected elements of the tension and of the velocity should correspond to the same kind of vibrations and to the same frequency. The second formula — (« statistical » method) — introduced the average values of the components. These results can be applied to the determination of the horizontal layers on the basis of measurements on or near the surface. This possibility comes from the fact that the coefficient of proportionality in the mentioned formulas contents the thickness of the layers. In first approximation this coefficient gives only the thickness, in second approximation, the physical properties of the layers can be also calculated.

Each method, « classical » and « statistical », can be realised on two ways. For instance, if we apply the first method, the measurement of the tension and the velocity can be completed by using the selector (the scheme of it is also given). In this case the calculation is not necessary. On the contrary if using the selector appears too expensive, we can limit ourself only on the measurement of the tension and the velocity and carry out the necessary selection by means of calculation. The same is true for the application of the second formula.

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