SOME CONSEQUENCES OF THE EXISTENCE OF LOW-VELOCITY LAYERS

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Introduction.

The velocities of elastic waves (P and S) generally increase with depth in the earth. If at some depth this increase is replaced by a decrease over an interval of depth, again followed by an increase at some greater depth, we have, what we call a low-velocity layer, provided the numerical value of the velocity decrease with depth in at least a part of the layer surpasses the critical value v/r (v = velocity, r = radius; see Gutenberg, 1954 b, and Bullen, 1954, pp. 87-89). The most marked low-velocity layer (for P waves) exists on the inner side of the outer core. This low-velocity layer has already been recognized by all seismologists long ago. If a low-velocity layer exists also at the boundary of the inner core, is not yet certain. According to Jeffreys there is one, whereas Gutenberg does not find sufficient observational support for it.

The existence of several low-velocity layers also in the crust and the mantle of the earth (lithosphere resp. asthenosphere) is a fairly recent discovery, mainly by Professor Gutenberg. Several seismologists still do not believe in their existence, and the whole problem is much discussed. Personally I believe in their existence for reasons which will be explained below. It is true that the lithosphere and asthenosphere low-velocity layers are by no means so pronounced as the one on the inner side of the outer core. For this latter low-velocity layer we know the corresponding shadow zone on the surface of the earth and the ray paths in the earth etc. fairly well. The experience gained from these studies is very helpful in studying the other less pronounced low-velocity layers.

Supporting facts and explanations of low-velocity layers.

With regard to the details of the various facts which support the existence of these low-velocity layers and with regard to their explanation I refer to papers by Gutenberg. The purpose of this paper is only to emphasize a series of problems, for the solution of which low-velocity layers are of the utmost importance.

The supporting facts may be summarized as follows.

1. Discrepancies between observations of near-by earthquakes and blasts concerning the velocity of P (or Pg) could be reconciled by assuming a low-velocity layer in the crust. Discrepancies between origin times computed from longitudinal and transverse waves led to the same result (Gutenberg, 1951 b).

2. The observed variation with distance of the amplitudes of the direct longitudinal waves (p) through the crust agrees best with the assumption of a lithosphere low-velocity layer (Gutenberg, 1951 b, fig. 2, p. 148).

3. Observations of shadow zones have given good evidence especially of the asthenosphere low-velocity layer.

4. The asthenosphere low-velocity layers have got direct support from velocity determinations for P and S waves (Gutenberg, 1953).

5. The existence of channel waves which propagate over large distances with little loss of energy is a strong support for low-velocity layers, acting as wave-guides.

6. The observed dispersion of Rg waves is best explained by the assumption of a lithosphere low-velocity layer (Båth, 1954 b, pp. 305-309).

Concerning the explanation of the low-velocity layers, nothing definite is known. Both the pressure and the temperature increase downward; increased pressure entails increased velocity, whereas increased temperature causes decreased velocity. In a depth interval where the temperature effect dominates over the pressure effect, a low-velocity layer can result. In this way quite complicated velocity-depth relations may be possible. However, more direct proofs of this hypothesis from laboratory and other data are mostly lacking (Gutenberg, 1955 a). Earlier it was believed that phase changes similar to the transition from α- to β-quartz could give an explanation for the lithosphere low-velocity layer (Gutenberg, 1951 a). The transition from α- to β-quartz occurs at a certain temperature (see Gutenberg, 1951 a, p. 432) and it could therefore at most explain one low-velocity layer. As a general cause it is insufficient, and several data (especially the observations of channel waves) require more than one low-velocity layer.

Gutenberg (1955 a) has recently presented two slightly different velocity-depth curves for P and S waves. In fig. 1 I have given those curves of Gutenberg which I find to be in best accord with observational

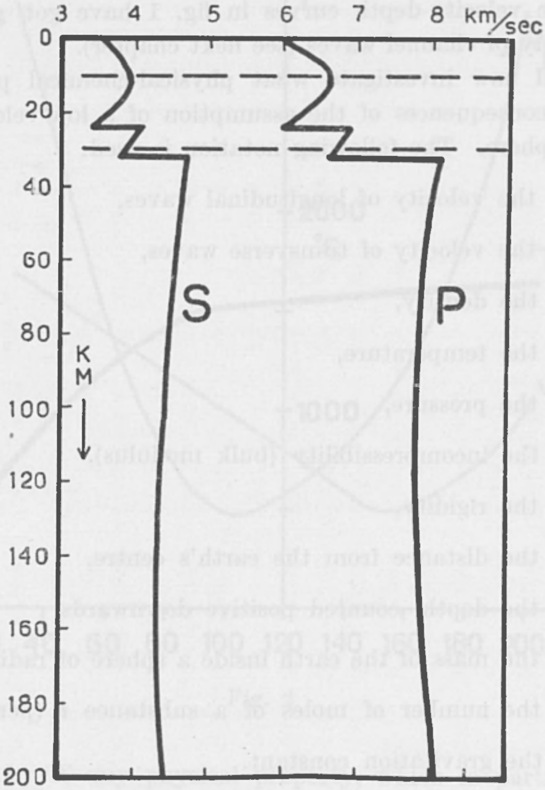

Fig. 1

data at present. In fig. 1 it is assumed that the temperature effect on the velocities dominates over the pressure effect in the depth intervals 10-100 km for P waves and 10-150 km for S waves. In addition there are superposed two discontinnities (with rapid increases of the velocities) at the Conrad and at the Mohorovicic boundaries. It is generally believed that these two boundaries are chemical discontinuities. At each boundary there is a certain pressure p and a certain temperature T. In two chemically distinct materials with in general different densities and different elastic properties, it is then more likely that at the pressure p and the temperature T the elastic velocities will be different than being equal, i. e. it is more natural to expect a jump in the velocities than not. If instead the boundaries were due to phase changes in the same material, the same reasoning holds, as different phases also in general have different densities and different elastic properties. The velocity-depth curves in fig. 1 have got good support from the study of channel waves (see next chapter).

We shall now investigate what physical-chemical properties are obtained as consequences of the assumption of a low-velocity layer in the asthenosphere. The following notation is used:

v_P = the velocity of longitudinal waves,

v_S = the velocity of transverse waves,

ϱ = the density,

T = the temperature,

p = the pressure,

k = the incompressibility (bulk modulus),

μ = the rigidity,

r = the distance from the earth's centre,

z = the depth, counted positive downwards,

m = the mass of the earth inside a sphere of radius r,

n_i = the number of moles of a substance i (per unit mass),

γ = the gravitation constant.

The following assumptions are made for the layer between 30 and 200 km depth:

1. Gutenberg's (1953) velocities v_P and v_S, including low-velocity layers. See fig. 2, where the velocities have been extrapolated up to 30 km, i. e. up to the Mohorovicic discontinuity.

2. Gutenberg's (1951 c) temperature curve (fig. 2).

3. Constant chemical composition.

4. Hydrostatic equilibrium.

5. $\frac{\partial \varrho}{\partial T} = 0$ in agreement with Bullen (1947, p. 213).

6. $\varrho = 3.320$ g/cm³ at $z = 30$ km (immediately below Mohorovicic discontinuity).

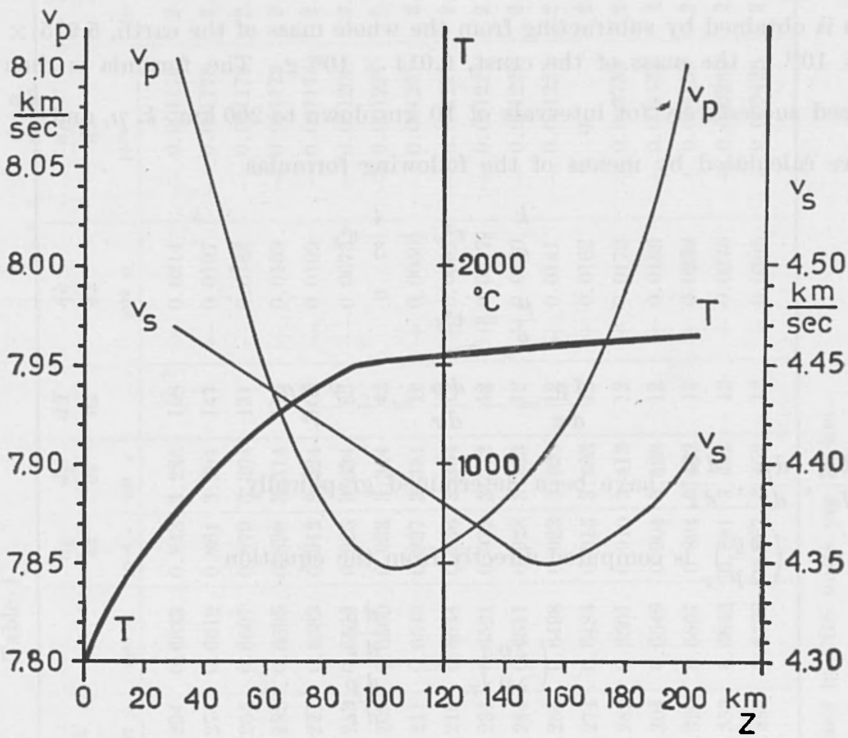

Fig. 2

Denoting by X any physical property, which in particular could be ϱ, k, μ, we have in general

$$\frac{dX}{dz} = \left(\frac{\partial X}{\partial p}\right)_{T,\,n_i} \frac{dp}{dz} + \left(\frac{\partial X}{\partial T}\right)_{p,\,n_i} \frac{dT}{dz} + \sum_i \left(\frac{\partial X}{\partial n_i}\right)_{T,\,p,\,n_j} \frac{dn_i}{dz}$$

with $i \neq j$. In computing the density gradient only the pressure term (first term) is taken into account:

$$\frac{d\varrho}{dz} = -\frac{d\varrho}{dr} = + \frac{\gamma\, m\, \varrho}{r^2 \left(v_P^2 - \frac{4}{3} v_S^2\right)}$$

Whereas the temperature can be ignored in the density determination, its influence on elasticity (k, μ) cannot be neglected. This is evident from the explanation of low-velocity layers outlined above. The formula for $\frac{d\varrho}{dz}$ is first applied for $z = 30$ km ($r = 6340$ km); the corresponding m is obtained by subtracting from the whole mass of the earth, $5.975 \times \times 10^{27}$ g, the mass of the crust, 0.014×10^{27} g. The formula is then used successively for intervals of 10 km down to 200 km $\cdot$ k, μ, and $\frac{dp}{dz}$ are calculated by means of the following formulas

$$\frac{k}{\varrho} = v_P^2 - \frac{4}{3} v_S^2$$

$$\frac{\mu}{\varrho} = v_S^2$$

$$\frac{dp}{dz} = -\frac{dp}{dr} = +\frac{\gamma m \varrho}{r^2}$$

$\frac{dk}{dz}$, $\frac{d\mu}{dz}$, $\frac{dT}{dz}$ have been determined graphically.

$\left(\frac{\partial \varrho}{\partial p}\right)_T$ is computed directly from the equation

$$\left(\frac{\partial \varrho}{\partial p}\right)_T = \frac{\frac{d\varrho}{dz}}{\frac{dp}{dz}}$$

All values are given in Table 1, where CGS units have been used, unless otherwise mentioned.

We note that it is necessary for both k and μ to pass through minima, whereas ϱ is very little influenced by the assumptions of low-velocity layers (Fig. 3). Bullen has recently introduced a discontinuous density increase of 0.5 g/cm³ at a depth of 80 km (see Lambert and Darling, 1951, pp. 359-360). This could, however, not be taken into account here as it then also seems necessary to make corresponding assumptions for μ.

Each of the following formulas

$$\frac{dk}{dz} = \left(\frac{\partial k}{\partial p}\right)_T \frac{dp}{dz} + \left(\frac{\partial k}{\partial T}\right)_p \frac{dT}{dz}$$

$$\frac{d\mu}{dz} = \left(\frac{\partial \mu}{\partial p}\right)_T \frac{dp}{dz} + \left(\frac{\partial \mu}{\partial T}\right)_p \frac{dT}{dz}$$

Table 1

z	v_P	v_S	m	$\frac{k}{\varrho}$	$\frac{\mu}{\varrho}$	ϱ	k	μ	$\frac{d\varrho}{dz}$	$\frac{dp}{dz}$	$\frac{dT}{dz}$	$\frac{dk}{dz}$	$\frac{d\mu}{dz}$	$\frac{\partial\varrho}{\partial p}$
km	km/sec	km/sec	$10^{27}\times$	$10^{11}\times$	$10^{11}\times$		$10^{12}\times$	$10^{12}\times$	$10^{-2}\times$	$10^{9}\times$		$10^{12}\times$	$10^{12}\times$	$10^{-12}\times$
30	8.10	4.47	5.961	3.897	1.998	3.320	1.294	0.6633	0.843	3.285	158	— 0.0214	— 0.001178	2.566
40	8.05	4.46	5.944	3.828	1.989	3.328	1.274	0.6619	0.861	3.294	147	— 0.0197	— 0.001178	2.614
50	8.00	4.45	5.927	3.760	1.980	3.337	1.255	0.6607	0.879	3.304	131	— 0.0188	— 0.001178	2.660
60	7.95	4.44	5.910	3.692	1.971	3.346	1.235	0.6595	0.898	3.314	116	— 0.0163	— 0.001178	2.710
70	7.91	4.43	5.893	3.640	1.962	3.355	1.221	0.6583	0.913	3.324	101	— 0.0109	— 0.001178	2.747
80	7.88	4.42	5.876	3.605	1.954	3.364	1.213	0.6573	0.925	3.334	83	— 0.0073	— 0.001200	2.774
90	7.86	4.41	5.859	3.585	1.945	3.373	1.209	0.6560	0.933	3.344	42	0	— 0.001227	2.790
100	7.85	4.40	5.842	3.581	1.936	3.382	1.211	0.6548	0.937	3.354	12	+ 0.0050	— 0.001227	2.794
110	7.85	4.39	5.825	3.593	1.927	3.391	1.218	0.6534	0.936	3.364	12	+ 0.0092	— 0.001227	2.782
120	7.86	4.38	5.808	3.620	1.918	3.400	1.231	0.6521	0.932	3.373	12	+ 0.0117	— 0.001227	2.763
130	7.87	4.37	5.791	3.648	1.910	3.409	1.244	0.6511	0.928	3.383	12	+ 0.0130	— 0.001227	2.743
140	7.88	4.36	5.774	3.675	1.901	3.418	1.256	0.6498	0.923	3.393	12	+ 0.0141	— 0.001227	2.720
150	7.90	4.35	5.757	3.718	1.892	3.427	1.274	0.6484	0.915	3.403	12	+ 0.0162	0	2.689
160	7.92	4.35	5.740	3.750	1.892	3.436	1.289	0.6501	0.910	3.413	12	+ 0.0173	+ 0.003733	2.666
170	7.95	4.36	5.723	3.786	1.901	3.445	1.304	0.6549	0.904	3.423	12	+ 0.0189	+ 0.004433	2.641
180	7.99	4.37	5.706	3.838	1.910	3.454	1.326	0.6597	0.894	3.432	12	+ 0.0239	+ 0.004833	2.605
190	8.04	4.38	5.689	3.906	1.918	3.463	1.353	0.6642	0.881	3.442	12	+ 0.0275	+ 0.006260	2.560
200	8.10	4.40	5.672	3.980	1.936	3.472	1.382	0.6722	0.867	3.452	12	+ 0.0308	+ 0.009429	2.512

The derivatives with respect to z are expressed in CGS units per 10 km.

have then been applied to three depth intervals, 30-80 km, 90-140 km, and 150-200 km, each interval comprising 6 observations of each quan-

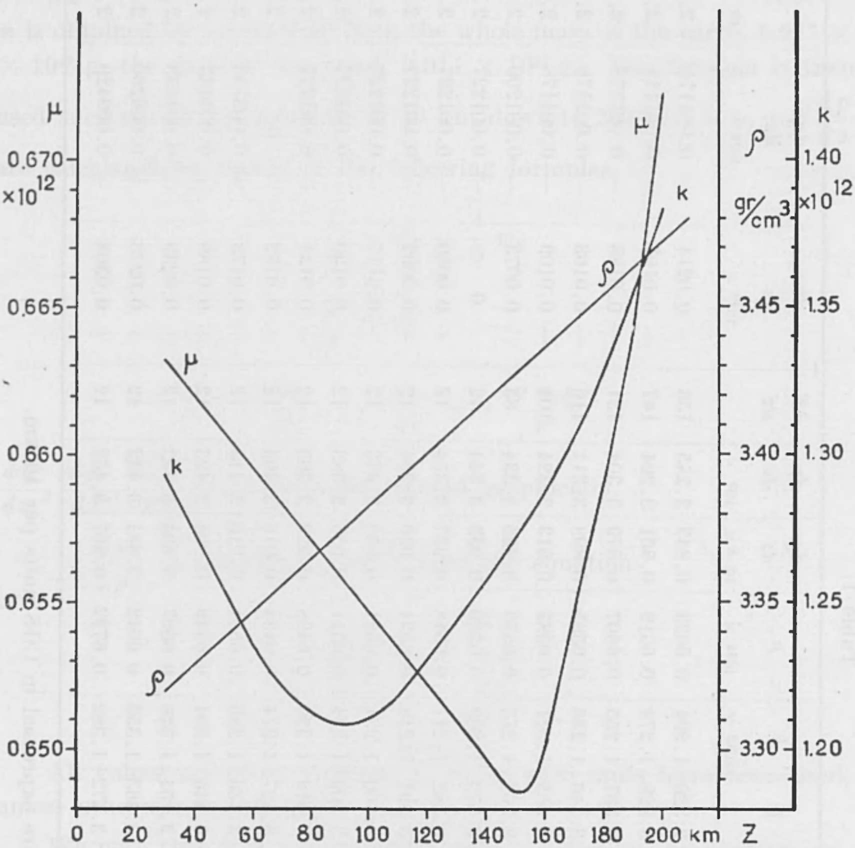

Fig. 3

tity. Least-square solutions have been made, yielding mean values of the unknowns

$$\left(\frac{\partial k}{\partial p}\right)_T , \quad \left(\frac{\partial k}{\partial T}\right)_p , \quad \left(\frac{\partial \mu}{\partial p}\right)_T , \quad \left(\frac{\partial \mu}{\partial T}\right)_p$$

for each of the three intervals. The results of this computation are given in Table 2. Smaller intervals could not easily be used, as too unreliable values would then have been obtained.

The values in Table 2 are direct consequences of our assumptions, especially of the existence of a low-velocity layer. The next step would

Table 2. – CGS UNITS USED

Δz km	$\frac{\partial k}{\partial p}$	$\frac{\partial k}{\partial T}$	$\frac{\partial \mu}{\partial p}$	$\frac{\partial \mu}{\partial T}$
30–80	$+2.247$	-1.888×10^8	$+1.675 \times 10^{-4}$	-0.922×10^7
90–140	$+4.395$	-3.506×10^8	-0.364	-0.805×10^5
150–200	$+3.134 \times 10^2$	-8.765×10^{10}	$+1.968 \times 10^2$	-5.581×10^{10}

be to investigate if these values are reasonable or not. If any of them is unreasonable, this will call for a revision of the assumptions made above. However, laboratory data (see e. g. Adams, 1951) are still too scanty to decide this question. There is general agreement as far as signs are concerned with the exception of $\frac{\partial \mu}{\partial p}$ for the second interval. But μ is practically independent of p, at least for the first layer, and nearly so for the second layer. The values of $\frac{\partial k}{\partial p}$ are in fair agreement with measurements on rock samples. There are still too few laboratory investigations of $\frac{\partial k}{\partial T}$ to allow a comparison.

We note from our results in Table 2 that the effects both of p and T on k and μ are much larger in the third layer than in the first two layers, which in general agree reasonably well with each other. It is too early to say if this obvious change is due to different chemical composition or to physical changes of the material. Some authors have assumed melting to begin at about 80 km depth. This would entail decreased bulk modulus and decreased density, passing from the crystalline to the glassy state.

The corresponding dependence of the elastic velocities on pressure and temperature were then computed from the following formulas

$$\frac{\partial v_P^2}{\partial p} = \frac{1}{\varrho}\left(\frac{\partial k}{\partial p} + \frac{4}{3}\frac{\partial \mu}{\partial p}\right) - \frac{v_P^2}{\varrho}\frac{\partial \varrho}{\partial p}$$

$$\frac{\partial v_P^2}{\partial T} = \frac{1}{\varrho}\left(\frac{\partial k}{\partial T} + \frac{4}{3}\frac{\partial \mu}{\partial T}\right)$$

$$\frac{\partial v_S^2}{\partial p} = \frac{1}{\varrho}\frac{\partial \mu}{\partial p} - \frac{v_S^2}{\varrho}\frac{\partial \varrho}{\partial p}$$

$$\frac{\partial v_S^2}{\partial T} = \frac{1}{\varrho}\frac{\partial \mu}{\partial T}.$$

These computations were made for three different levels, i. e. 60, 120, and 180 km depth. The results are given in Table 3, where also

$$\frac{dv_P}{dz} \quad \text{and} \quad \frac{dv_S}{dz}$$

are given, computed from

$$\frac{dv_{P,S}^2}{dz} = \frac{\partial v_{P,S}^2}{\partial p} \frac{dp}{dz} + \frac{\partial v_{P,S}^2}{\partial T} \frac{dT}{dz} .$$

Table 3

z km	$\frac{\partial v_P^2}{\partial p}$ CGS	$\frac{\partial v_P^2}{\partial T}$ CGS	$\frac{dv_P}{dz}$ km/sec/10 km	$\frac{\partial v_S^2}{\partial p}$ CGS	$\frac{\partial v_S^2}{\partial T}$ CGS	$\frac{dv_S}{dz}$ km/sec/10 km
60	$+0.6715$	-0.6010×10^8	-0.03	$+0.5006 \times 10^{-4}$	-0.2756×10^7	-0.004
120	$+1.1500$	-1.0315×10^8	$+0.02$	-0.1071	-0.2368×10^5	-0.004
180	$+1.6671 \times 10^2$	-4.6920×10^{10}	$+0.06$	$+0.5698 \times 10^2$	-1.6158×10^{10}	$+0.02$

As a check on the calculations we find that the computed $\frac{dv_{P,S}}{dz}$ agree well with the values directly obtained from the given $v_{P,S}$. The remaining small deviations are explained by the fact that in computing $\frac{dv_{P,S}}{dz}$ we used mean values of

$$\frac{\partial k}{\partial p}, \quad \frac{\partial k}{\partial T}, \quad \frac{\partial \mu}{\partial p}, \quad \frac{\partial \mu}{\partial T}$$

for certain intervals.

As a result of these calculations we have found it possible to explain low-velocity layers in the asthenosphere by the combined effect of pressure and temperature on the elastic properties of the material. With a minimum number of reasonable assumptions, stated above, we have calculated the values of

$$\frac{\partial k}{\partial p}, \quad \frac{\partial k}{\partial T}, \quad \frac{\partial \mu}{\partial p}, \quad \frac{\partial \mu}{\partial T},$$

which are necessary to explain the given velocity variation with depth. The values obtained for these quantities cannot yet be fully confirmed by data from other sources, nor can they be denied. The values given will be relevant to further investigations of this layer (30-200 km) in the earth as well as to further laboratory investigations of various rock samples under high pressure and high temperature.

Channel waves

A low-velocity layer in any medium may act as a wave guide. We know this from the oceans and the atmosphere. The question of T phases (P waves propagated along a wave guide at some depth in the oceans) is still discussed, and not all agree on their water-borne nature. If agreement is found in a limited area between the velocity of T over an oceanic path and the velocity of sound through the ocean, this may not be of conclusive importance. But if we compare these two velocities in widely different regions and still find agreement within the error limits between the velocities of T and of sound in each locality, the conclusion is evident. Ewing, Press, and Worzel (1952) studied the T wave propagation in the Pacific, and Båth (1954 a) on the limit of the Arctic Ocean. In the Pacific the velocities of both T and sound were markedly higher than in my case, but in both cases the agreement between the velocities of T and of sound were perfect. The only natural explanation is that T is water-borne; in fact, it seems extremely unlikely that the bottom structure in the two regions should have such differences so as to produce the agreements found. The experience from the oceanic wave-guide may be of value in interpreting the corresponding channels in the solid earth, as the oceans are easily accessible for direct measurements.

The preliminary study of T phases at Kiruna (Båth, 1954 a) has got strong confirmation from a large number of later cases. At Kiruna T phases are only obtained from a limited region between Jan Mayen and Spitsbergen, but hitherto from no other part of the Atlantic or the Arctic. Recently (for an earthquake on February 22, 1956, origin time = 00.07.37 GMT, epicentre location 73°1/2 N, 8°E, i. e. in the Arctic Ocean, SW of Spitsbergen) we received very strong T phases at Kiruna and also at Skalstugan and the first observed at Uppsala, for which the land path amounts to approx. 2/3 of the total path. I believe that much more information could be obtained from other areas concerning T phases, in the European area for instance from earthquakes in the Mediterranean. But short-period instruments with high magnification are necessary for their observation.

For the earth the channel-wave problem is much more complicated, as there are several channels and both longitudinal and transverse waves can exist. In Table 4 our present knowledge is summarized concerning channel waves in the earth.

Lg and *Rg* waves, which exist only for unbroken continental paths, were discovered by Press and Ewing (1952) for American paths. Båth (1954 b) found from a study of these waves over Euroasiatic paths that *Lg* consists of two (or possibly three) distinct waves, *Lg1* and *Lg2*. This result was then confirmed by Gutenberg (1955 b). In my view, however, the channel-wave study is still only in its first beginnings. The constitution of the continental crust is probably much more complicated than we realize to-day. Our knowledge of the various possibilities for wave propagation in the crust is very meagre, and still we can at most produce hypotheses.

Table 4

Wave	Group velocity km/sec	Periods sec	
		Mean period	Period range
Li	3.8	*	*
Lg1	3.54	5.8	2–11
Lg2	3.37	6.8	2–12
(*Lg*)	3.22	*	*
Rg	3.07	9.2	3–16
Pa	7.9–8.0	—	5–12
Sa	4.4	—	10–30

* Comparable to *Lg1* and *Lg2*.

In more recent studies I have found repeatedly waves of *Lg*-type (i. e. with wave motion similar to *Lg1* and *Lg2* and propagated only over continental paths) with velocities 3.8 and 3.22 km/sec. Fig. 4 shows an example of the *Lg* wave with group velocity 3.8 km/sec. This new wave will be denoted *Li*, where *i* stands for the intermediate layer in the same way as *g* in *Lg* refers to the granitic layer. In his most recent study of wave velocities Gutenberg (1955 a) has given two low-velocity layers in the crust. It appears probable that the *Li* wave (with velocity 3.8 km/sec) is guided by the lower of the two channels, i. e. above the Mohorovicic discontinuity, at least to judge from the velocities given (see fig. 1).

Some results compiled by Bullen (1947, pp. 193-194) indicate that as a mean for Europe and central Asia the velocities of *Sg* and *S** are 3.33 and 3.77 km/sec respectively. I consider it very likely that our *Lg2* is identical with *Sg* and the newly discovered *Li* with velocity 3.8 km/sec is identical with *S**. Both *Sg* and *S** are probably guided waves or channel waves in the granitic and intermediate layers resp. For this

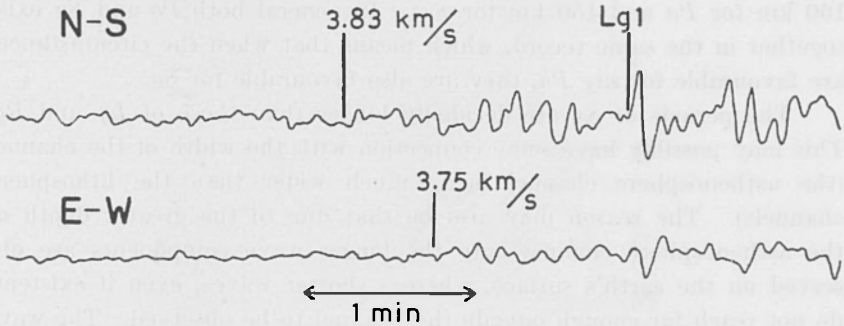

Fig. 4. – Records of *Li* in addition to *Lg 1* written by the Uppsala Wiechert seismograph for an earthquake in China. Epicentre at 39° N, 104° E. Origin time – 00.59.57 on July 31, 1954. Magnitude $\simeq$ 7.

reason they can be observed also at large distances, then called *Lg* or *Li* waves, and not only in records of near-by earthquakes. Further detailed studies of the *Li* wave are needed, especially to see if it only propagates over continental paths. The granitic layer is missing in the ocean bottom, and it is therefore natural that *Lg1* and *Lg2* do not propagate over an oceanic structure. But an intermediate layer of reduced thickness (around 5 km) exists also in the ocean bottoms. Preliminary investigations show that *Li* in spite of this does not propagate over oceanic areas. The temperature and the pressure conditions are undoubtedly different in the oceanic and the continental intermediate layers, and it appears probable that the oceanic intermediate layer is unable to act as a wave guide. Another possibility is that the energy distribution is disturbed too much at the continental edge.

The asthenosphere channel waves *Pa* and *Sa* were discovered by Caloi (1953, 1954) and independently by Press and Ewing (1955) and later studied also by Gutenberg (1955 b). We have given our observations of *Pa* and *Sa* on the records at Uppsala and Kiruna in our annual bulletins from 1954 on. The velocities and periods agree with those of the other authors. The *Pa* and *Sa* waves are often not so conspicuous as *Lg* and *Rg* and moreover they are for certain distances masked by other waves. But nevertheless, there seems to be no doubt about their existence. *Pa* and *Sa* belong to deeper channels and are observed for both oceanic and continental paths. The depth of the earthquake focus seems to be of importance for the existence of *Pa* and *Sa*. It is a matter of observation that the clearest *Pa* and *Sa* are obtained for earthquakes with foci in or near the axis of the resp. channels (around 100 km for *Pa* and 150 km for *Sa*). In general both *Pa* and *Sa* exist together in the same record, which means that when the circumstances are favourable for say *Pa*, they are also favourable for *Sa*.

The periods of *Sa* are decidedly longer than those of *Lg* and *Rg*. This may possibly have some connection with the width of the channel (the asthenosphere channel being much wider than the lithosphere channels). The reason may also be that due to the greater depth of the asthenosphere channel only the longer wave components are observed on the earth's surface, whereas shorter waves, even if existent, do not reach far enough outside the channel to be observed. The wave lengths for *Pa* and *Sa* lie in the ranges 40-96 km and 44-132 km respectively, as computed from the data given in Table 4, i. e. the depths of the asthenosphere channels for *P* and *S* amount approximately to 1-3 wave lengths. A series of seismographs with different but well-defined and very limited frequency response would probably do great service in the further investigation of the various channel waves.

Press and Ewing (1955) have found some evidence of two different *Sa* waves. One of them corresponds to the *Sa* wave already described, although they find a velocity of about 4.58 km/sec, the other appears to be associated with the long-period *G* waves with a velocity of about 4.40 km/sec. Their " whispering-gallery " hypothesis for the propagation of *Pa* and *Sa* appears less likely as there will be no total reflexions against the Mohorovicic discontinuity for incidence from below.

The low-velocity layers shown in fig. 1 have a sharp discontinuity on one side when in the lithosphere but not in the asthenosphere. This agrees with the observations of channel waves. In the lithosphere channels longitudinal waves could not be guided for any length due to rapid loss of energy on repeated reflexions at the discontinuities or at the surface of the earth. It is also a fact that no longitudinal waves in the lithosphere channels exist at greater distances (Båth, 1954 b, p. 316). On the other hand, in the asthenosphere channel, as shown in fig. 1, both longitudinal and transverse waves could be expected. As a matter of fact, they both exist (*Pa* and *Sa*).

We know that P waves are never totally reflected (as P waves) at the surface of the earth or at an internal discontinuity. SH waves are totally reflected (as SH waves) at the surface, and SV waves are

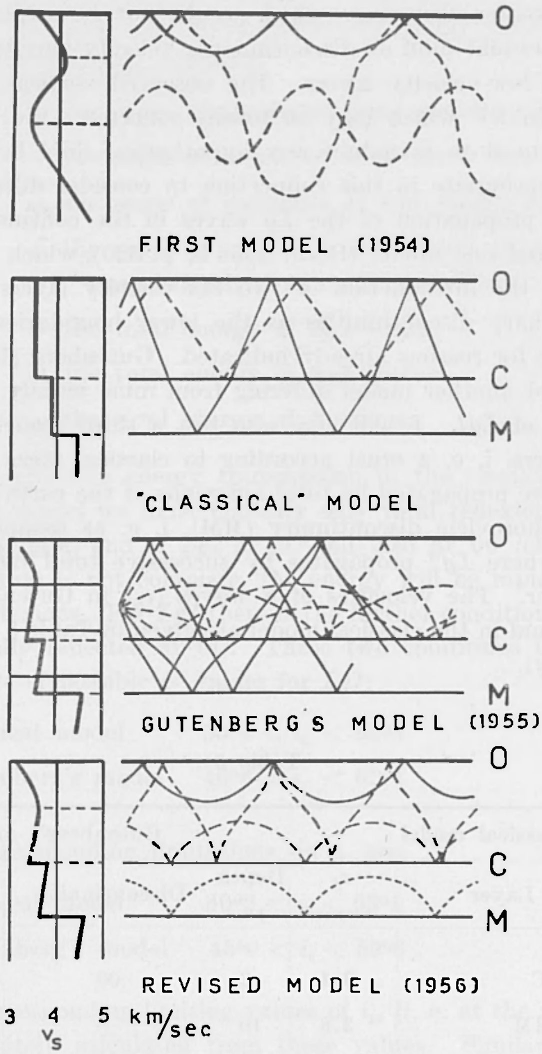

Fig. 5. – Solid curves = *Lg 1*, sdashed curves = *Lg 2*, dotted curves = *Li*.

totally reflected at the surface, if the angle of incidence surpasses approx. 35°. Both SH and SV are totally reflected at internal discontinuities between solids for angles of incidence surpassing the limiting angle for total reflexion (as in optics). Lithosphere channels with a sharp discontinuity on one side (the lower side) therefore seem to explain most observations hitherto concerning *Lg*. They explain the absence of guided *P* waves at greater distances, which could not be explained if there were only gradual (and no discontinuous) velocity variations in passing through the low-velocity layers. The observed vertical component of *Lg* is due to *SV* which may be totally reflected. We also note that there is no need to introduce any hypothetical fluid layers.

It is appropriate in this connection to consider different possibilities for the propagation of the *Lg* waves in the continental crust. I have presented one model (Båth, 1954 b, p. 320), which should now be modified by the introduction of two low-velocity layers in the crust, both with sharp discontinuities as the lower boundaries (Conrad and Mohorovicic) for reasons already indicated. Gutenberg (1955 b, p. 290) has presented another model differing from mine mainly in the way of propagation of *Lg1*. I will here also test a third model without low-velocity layers, i. e. a crust according to classical ideas and in which *Lg1* waves are propagated by total reflexions at the earth's surface (00) and the Mohorovicic discontinuity (MM), i. e. as assumed by Gutenberg, and where *Lg2* propagates by successive total reflexions in the granitic layer. The velocities of *S* waves (v_S) in Gutenberg's (1955 b) new model and in the classical model are given in Table 5 (CC = Conrad discontinuity).

Table 5

Classical model			Gutenberg's model		
Depth km	Layer	v_S km/sec	Depth km	Discontinuity	v_S km/sec
0–22	00-CC	3.4	0	00	3.28
22–34	CC-MM	3.8	10	—	3.56
> 34	immediately below MM	4.4	22	above CC	3.36
			22	below CC	3.79
			34	above MM	3.56
			34	below MM	4.61

All three models are capable to give the velocities of *Lg* and *Li* waves in close agreement with observations. This is naturally a necessary but by no means a sufficient requirement for any model.

We will now try to proceed further by energy considerations. We introduce the following notation:

i_C = the angle of incidence from above against CC,

i_M = the angle of incidence from above against MM,

i_0 = the angle of incidence at the earth's surface,

i_{10} = the angle of incidence at a depth of 10 km, the assumed depth of an earthquake focus,

E_{Lg1} = the total energy of *Lg1* waves,

E_{Lg2} = the total energy of *Lg2* waves,

E_S = the total energy of *S* waves.

For an effective energy transmission in the classical model or in Gutenberg's model we must consider only total reflexions of *Lg1* both at 00 and at MM, and of *Lg2* at CC and also at 00 (classical model). If the reflexion is not complete, the energy will be rapidly lost on the repeated reflexions. For *Lg1* there is the further condition that it should not be totally reflected at CC. These two conditions impose the following limits of possible i_C values for *Lg1*:

Classical model $50^{\circ}4 < i_C < 63^{\circ}5$

Gutenberg's model $46^{\circ}6 < i_C < 62^{\circ}4$.

The corresponding limitations on i_0 are:

Classical model $50^{\circ}2 < i_0 < 63^{\circ}1$

Gutenberg's model $45^{\circ}0 < i_0 < 59^{\circ}6$.

The corresponding limiting values of i_{10} (i. e. at the assumed focus) are immediately calculated from these values. Similar computations can easily be made for *Lg2*. The results are given in Table 6.

We assume spherically symmetrical energy radiation from the focus and take into account that energy passes into *Lg1* and *Lg2* by radiation both upward and downward from the focus. Diffraction and scattering phenomena are neglected. The energy ratios of *Lg* to *S* are then given by the ratios of the area on a sphere around the focus cut out by *Lg*

Table 6

Model	*Lg1*	*Lg2*
Classical . .	$50^o2 \leqslant i_{10} \leqslant 63^o1$ upward $50^o4 \leqslant i_{10} \leqslant 63^o5$ downward	$63^o1 \leqslant i_{10} \leqslant 90^o$ upward $63^o5 \leqslant i_{10} \leqslant 90^o$ downward
Gutenberg .	$50^o2 \leqslant i_{10} \leqslant 69^o6$ upward $50^o2 \leqslant i_{10} \leqslant 69^o6$ downward	*a)* $69^o6 \leqslant i_{10} \leqslant 90^o$ downward *b)* $69^o6 \leqslant i_{10} \leqslant 90^o$ upward $69^o6 \leqslant i_{10} \leqslant 90^o$ downward

a) *Lg2* is assumed to propagate altogether within the granitic low-velocity layer.

b) *Lg2* includes both the case *a)* and the assumed *Lg2* reflected both at 00 and CC (see fig. 4 by Gutenberg, 1955 b, p. 290).

to the total area of the sphere. Our results are given in Table 7, where all energy ratios refer to the immediate vicinity of the focus (indicated by subscript 0).

Table 7

Model	$(E_{Lg1})_0 : (E_S)_0$	$(E_{Lg2})_0 : (E_S)_0$	$(E_{Lg2})_0 : (E_{Lg1})_0$
Classical	0.19	0.45	2.37
Gutenberg	0.29	*a)* 0.17	*a)* 0.60
		b) 0.35	*b)* 1.19

For explanation of *a)* and *b)* see note of Table 6.

The computed values of $(E_{Lg2})_0 : (E_{Lg1})_0$ can be compared with my observations (Båth, 1954 b, p. 312), noting, however, that my observations do not refer to the conditions near the focus but are obtained at greater distance and also include values for slightly different focal depths. In both the classical and in Gutenberg's model the total energy of *Lg2* is independent of distance, whereas E_{Lg1} should decrease due to losses in crossing CC. If the existence of CC is assumed and it has been assumed to be of great importance for the propagation of *Lg2*, we cannot naturally ignore its influence on *Lg1*. The consequence is that in both the classical model and in Gutenberg's model $E_{Lg2} : E_{Lg1}$ should increase as the wave motion propagates. This makes the classical model still more improbable as $(E_{Lg2})_0 \cdot (E_{Lg1})_0$ deviates by far too much from the observations. This fact together with the incapability to transmit *Li* waves (no total reflexion possible at CC for incidence from below in the classical model) is sufficient to exclude the classical model. Low-velocity layers appear as necessary. $(E_{Lg2})_0 : (E_{Lg1})_0$ as computed for Gutenberg's model (especially case b) agrees well with my observations. But it is expected that the energy losses of *Lg1* at CC will also here make the ratios much larger than observed. On the other hand, in my model for *Lg*-propagation (Båth, 1954 b, p. 320) both E_{Lg2} and E_{Lg1} as well as their ratio are independent of distance. I have used my own material (see the list of observations in Båth, 1954 b, pp. 324-342) to test this. Plotting

$$\varepsilon'_{Lg2} - \varepsilon'_{Lg1} = \log \frac{E_{Lg2}}{E_{Lg1}}$$

for 24 earthquakes at normal depth against distance (ranging from 14° to 61°), there was clearly no variation with distance, i. e. $E_{Lg2} : E_{Lg1}$ is independent of distance. This fact is more in favour of my model than of Gutenberg's.

In Gutenberg's model for *Lg1* there are total reflexions at 00 and MM but both reflexion and refraction at CC. The original energy of *Lg1* (considering first only the *SH* motion) is therefore kept within the crust but arrives at a given point on the surface along different paths within the crust. We use the following notation, considering only *SH* waves:

f_1 = the fraction of energy reflected at CC for a wave incident from above (*SH*),

f_2 = the fraction of energy reflected at CC for a wave incident from below (*SH*),

$(E_{Lg1})_{00}$ = the energy of *Lg1* near the focus per unit angle i,

$(E_{Lg1})_0$ = the total energy of *Lg1* near the focus,

E_{Lg1} = the total energy of *Lg1* at an epicentral distance Δ,

$(E_{Lg2})_0$ = the total energy of *Lg2* near the focus,

E_{Lg2} = the total energy of *Lg2* at an epicentral distance Δ.

After n reflexions from below, the energy of *Lg1* received at the surface in the form of *SH* waves is

$$E_{Lg1} = \int_{i_1}^{i_2} \left[\sum_0^n (f_1 + f_2 - 1)^n - f_2 \sum_0^{n-1} (f_1 + f_2 - 1)^n - f_1{}^n \right] (E_{Lg1})_{00}\, di$$

where i_1 and i_2 are the limits of the angle of incidence at the focus between which *Lg1* is obtained (see above).

The derivation is given below, using the following notation:

E_n = the energy at the surface after n returns to the surface, corresponding to the original energy E_0 per unit angle of incidence i at the focus,

E'_{n-1} = the energy of the wave in the intermediate layer after $n - 1$ reflexions against MM,

a = $f_1 + f_2$.

For a given original angle of incidence f_1 and f_2 are constant along the path. We then have

$$E_n = f_1 E_{n-1} + (1 - f_2) E'_{n-1}$$

$$E_{n-1} = f_1 E_{n-2} + (1 - f_2) E'_{n-2}$$

$$E'_{n-1} = (1 - f_1) E_{n-2} + f_2 E'_{n-2}$$

E'_{n-1} and E'_{n-2} are eliminated:

$$E_n = a E_{n-1} + (1 - a) E_{n-2}$$

Summing from $n = 2$ to n

$$\sum_2^n E_n = a \sum_2^n E_{n-1} + (1 - a) \sum_2^n E_{n-2}$$

we get

$$E_n = (a - 1) E_{n-1} + (1 - f_2) E_0$$

Multiplying by $(a-1)^m$ and summing from $m=0$ to $m=n-1$

$$\sum_{m=0}^{m=n-1} (a-1)^m E_{n-m} = \sum_{m=0}^{m=n-1} (a-1)^{m+1} E_{n-m-1} + \\ + \sum_{m=0}^{m=n-1} (1-f_2)\ (a-1)^m E_0$$

we obtain

$$E_n = (a-1)^n E_0 + (1-f_2) E_0 \sum_0^{n-1} (a-1)^n =$$

$$E_0 \sum_0^n (a-1)^n - f_2 E_0 \sum_0^{n-1} (a-1)^n$$

E_n as computed here includes also part of *Lg2*, namely the part which is not totally reflected at CC. Obviously then

$$E_n = (E_{Lg1})_i + (E_{Lg2})_{\substack{\text{not totally}\\ \text{refl. at CC}}} = (E_{Lg1})_i + f_1{}^n E_0$$

$$E_{Lg1} = \int_{i_1}^{i_2} (E_{Lg1})_i\, di = \int_{i_1}^{i_2} \left[\sum_0^n (f_1+f_2-1)^n - \right. \\ \left. - f_2 \sum_0^{n-1} (f_1+f_2-1)^n - f_1{}^n \right] (E_{Lg1})_{00}\, di$$

as $E_0 = (E_{Lg1})_{00}$. This is the formula which should be proved.

For greater epicentral distance, i. e. $n \to \infty$, the expression for E_{Lg1} simplifies to the following

$$E_{Lg1} = \int_{i_1}^{i_2} \frac{1-f_2}{2-f_1-f_2} (E_{Lg1})_{00}\, di$$

Combined with the obvious relation

$$(E_{Lg1})_0 = \int_{i_1}^{i_2} (E_{Lg1})_{00}\, di = (E_{Lg1})_{00} \int_{i_1}^{i_2} di$$

valid for spherically symmetrical energy radiation from the focus, this equation gives

$$E_{Lg1} = (E_{Lg1})_0 \frac{\displaystyle\int_{i_1}^{i_2} \frac{1 - f_2}{2 - f_1 - f_2}\, d\,i}{i_2 - i_1}$$

Combining this equation with the following

$$E_{Lg2} = (E_{Lg2})_0$$

we obtain that

$$\frac{E_{Lg2}}{E_{Lg1}} = \frac{(E_{Lg2})_0}{(E_{Lg1})_0} \frac{i_2 - i_1}{\displaystyle\int_{i_1}^{i_2} \frac{1 - f_2}{2 - f_1 - f_2}\, d\,i} .$$

The ratio E_{Lg2}/E_{Lg1} corresponds to my observations (Båth, 1954 b, p. 312), $(E_{Lg2})_0/(E_{Lg1})_0$ corresponds to the theoretical results calculated above (Table 7). The factor on the right-hand side is > 1. If the focus is at the surface (00) then $i_2 = 59^o6$ and $i_1 = 45^o0$ in Gutenberg's model (see above).

It results from these calculations that E_{Lg1} (considering only the dominant *SH* motion) approaches a definite value during the propagation in the model assumed by Gutenberg (1955 b), i. e. the limit given above for $n \to \infty$. The factor by which $(E_{Lg2})_0/(E_{Lg1})_0$ should be corrected may also bring this ratio in good agreement with the observed E_{Lg2}/E_{Lg1}.

Nevertheless, this model seems to be less likely for the following two reasons. In *Lg1* there are also *SV* waves. When an *SV* wave strikes CC we get both reflected and refracted *SV* and *P*. The energy of the *P* waves thus obtained is to be considered as an energy loss for *Lg1*. This means that in this model the total energy of *Lg1* (considering both *SH* and *SV*) is steadily decreasing with increasing distance, and that E_{Lg2}/E_{Lg1} should increase, which is contrary to the observations. It is also a fact that *Lg1* often has a clear vertical component even for large distances, which would not be the case if *SV* lost part of its energy into *P* waves along the path.

Another reason which makes this model unlikely is the following. In the model considered above, the *Lg1* wave reaches the surface along different paths (including both refractions and reflexions at CC in addition to reflexions at 00 and MM). It is clear that the travel time for these different paths will be slightly different. This means that the energy E_{Lg1} (as computed here) will be spread over an interval of time. The motion will then show a small, gradual beginning already in advance of *Lg1* proper. This result is in contrast to the observed fact that *Lg1* mostly has a very sharp onset with the largest amplitudes already at the start.

The energy computations above have been made only for models where reflexions and refractions occur. In models where also diffraction and scattering play an important rôle, no similar computations can easily be made. It may finally also be remarked that no model so far explains an (*Lg*) wave with a group velocity of 3.22 km/sec. It is not improbable that this wave at least in part consists of *Rg* motion related to *Lg1* (compare Båth, 1954 b, p. 302 and p. 322). The various models are shown in fig. 5, including also my own revised model.

We could possibly believe that the most pronounced low-velocity layer (on the inner side of the core) should also be a good guide for channel waves, not observable on the surface of the earth. However, it is easily seen that this low-velocity layer is only a very poor wave guide. *S* waves do not exist due to the liquid nature of the core, and *P* waves loose their energy rapidly due to repeated reflexions against the boundary of the core. The velocity of transverse waves immediately outside the core boundary is smaller than the velocity of longitudinal waves immediately inside (see Bullen, 1947, p. 210), and this circumstance precludes total reflexion. It is also a matter of fact that core waves with one or two parts of the path within the core *PKP*, *SKS*, *PKKP*, *SKKS* etc.) are well observed, whereas there are no observations of *PKKKP* and only a few not very well estabhshed observations of *SKKKS* (see Gutenberg and Richter, 1934, pp. 118-119).

In this connection I should like to suggest more studies of channel waves at the seismological observatories around the world and not least in Europe with the largest of all continents in front of us. The waves *Li*, *Lg*, and *Rg* are slower than the usual surface waves (*LQ* and *LR*) and therefore arrive later. In practice I always read the most prominent phases of a seismogram with no knowledge whatsoever where the phases should be found. I think this is essential in all seismogram analyses and not least in channel-wave studies. The identification of channel waves is then made by checking their group velocity and period (see Table 4). For certain distances care must be taken due to simultaneous arrival of various body waves. Reports on clear cases of channel waves in the regular bulletins would be very helpful for the whole problem.

Shadow zones

A low-velocity layer may produce a shadow zone for seismic waves on the surface of the earth. This is well known for the low-velocity layer inside the outer core. For the lithosphere and asthenosphere low-velocity layers the corresponding shadow zones are not so pronounced, and there are still contradictory statements, although such shadow zones were found already about thirty years ago by Gutenberg. For a given velocity-depth relation both the position and the width of the shadow zone depend on the focal depth (see fig. 6, first constructed

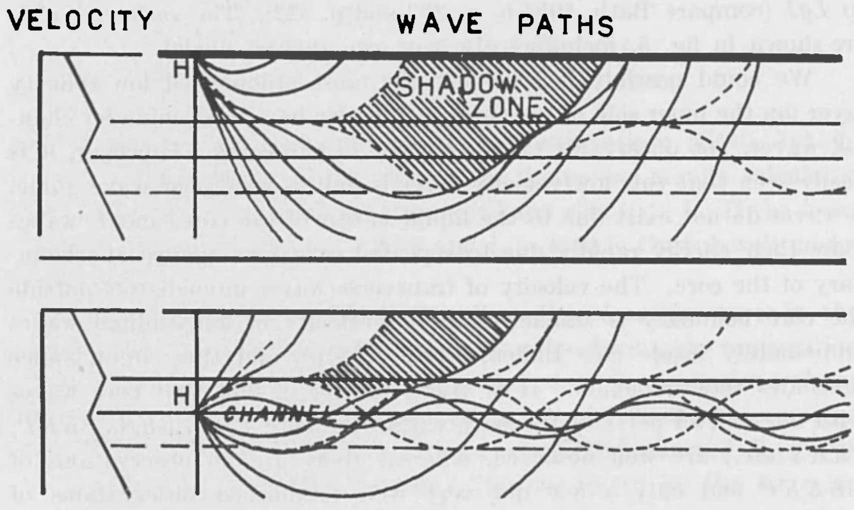

Fig. 6

by Gutenberg, 1954 b). Numerical computations have been made for the asthenosphere channel for P waves by the present author, which I hope to present in the future.

Several studies on amplitude variation with distance for P and S waves have been made by Gutenberg. See e. g. his paper of 1948, which contains several references to earlier papers. Recent studies of the amplitude variation of P waves with distance in the interval $2^{\circ}-24^{\circ}$ have been made by De Bremaecker (1955 a). Constructing a mean curve based on a number of curves for different earthquakes, he finds in spite of large scatter, indications of a minimum in the amplitude-distance curve around 7°1/2 distance. As this result is based only on earthquakes in a very limited area, it would be valuable to investigate if the result found has general validity or if it is of a local character.

Since we have been working with short-period seismographs of high sensitivity, the shadow-zone effects have shown to be very pronounced at our stations (Uppsala, Kiruna, and lately Skalstugan). For a zone over Central Europe (including e. g. Switzerland, Austria, southwestern Germany, France etc.) our instruments generally do not record the earthquakes, and it does not matter how high we make the magnification. This corresponds to the asthenosphere low-velocity layer. On the other hand, shocks of similar magnitude (5 – 5 1/2) in the New Zealand area are usually recorded with clear *PKP* phases at our stations. Within the shadow zones there is no possibility to get any direct body wave energy by the classical theory of refraction and reflexion; diffraction and scattering are the mechanisms for energy transmission by means of body waves into the shadow zones, and the energies thus conveyed are greatly reduced compared with the surroundings outside the shadow zones.

I have made some comparisons of records obtained from the Grenet-Coulomb seismograph at Uppsala and from the Hiller seismograph at Stuttgart, which are both short-period vertical and operated with similar characteristics. Comparisons have been made of amplitudes of *P* waves from earthquakes at Jan Mayen and in Greece (see Table 8).

In Table 8 the upper part contains earthquakes in the region of Jan Mayen, the lower part earthquakes in Greece. The mean values of $\left(\frac{w}{T}\right)_U : \left(\frac{u}{T}\right)_S$ obtained from Table 8 are for Jan Mayen earthquakes 1.19 ± 0.14 (standard error), and for Greek earthquakes 4.30 ± 0.78. The number of observations are 12 and 7 resp. The mean epicentral distances are the following

	Uppsala	Stuttgart
Jan Mayen	17°	25°
Greece	22°	13°

The influence on the ratio $\left(\frac{w}{T}\right)_U : \left(\frac{w}{T}\right)_S$ of the different angles of incidence (due to the different epicentral distances) has been eliminated

Table 8

w = the ground amplitude (expressed in microns) of the vertical component of P,

T = the period of P, expressed in sec,

U = Uppsala

S = Stuttgart

Date	Origin time GMT	Epicentre location		$\left(\frac{w}{T}\right)_U$	$\left(\frac{w}{T}\right)_S$	$\left(\frac{w}{T}\right)_U : \left(\frac{w}{T}\right)_S$
1954, Aug. 20 . .	19.21.33	70° 1/2 N, 15°	W	0.13	0.09	1.44
	20.24.15	71 1/2 N, 14	W	0.13	0.07	1.86
	20.42,3	71 1/2 N, 14	W	0.13	0.10	1.30
	22.39,9	71 1/2 N, 14	W	0.07	0.07	1.00
	22.59.16	71 N, 14	W	0.08	0.08	1.00
Aug. 21 . .	00.25.35	71 N, 13 1/2	W	0.10	0.11	0.91
	04.13.14	71 N, 14 1/2	W	(0.03)	0.09	0.33
Aug. 22 . .	10.08.02	71 N, 14 1/2	W	0.08	0.15	0.53
	18.21.12	70 1/2 N, 14	W	0.15	0.10	1.50
	23.52,1	71 1/2 N, 14	W	0.11	0.06	1.83
Aug. 23 . .	09.32.37	70 1/2 N, 14	W	(0.03)	0.04	0.75
Aug. 24 . .	06.18,2	71 1/2 N, 14	W	0.60	0.33	1.82
1954, Apr. 30 . .	12.55.39	39 1/2 N, 22	E	0.05	0.01	5.00
	13.02.36	39 1/2 N, 22	E	5.87	0.97	6.05
	19.33.30	39 1/2 N, 22	E	0.13	0.04	3.25
May 25 . .	22.03.33	39 1/2 N, 22	E	0.10	0.13	0.77
Aug. 3 . .	18.18.10	40 N, 25	E	0.45	0.12	3.75
1955, Apr. 13 . .	20.45.45	37 1/4 N, 22 1/4	E	1.00	0.28	3.57
Apr. 19 . .	16.47.19	39 1/4 N, 23	E	1.08	0.14	7.71

by computing the total amplitudes a from the vertical and horizontal components, w and u resp., of P in the following way:

$$a = (w^2 + u^2)^{1/2} = w\left(1 + \frac{u^2}{w^2}\right)^{1/2}$$

where

$$\frac{u^2}{w^2} = \frac{4\,(2 + 3\,\alpha^2)}{(1 + 3\,\alpha^2)^2}$$

and

$$\alpha = \cot\, i$$

where i = the angle of incidence (see Jeffreys, 1926, pp. 326-327). The angles of incidence have been computed from the well-known relation

$$\frac{r \sin i}{v} = \frac{d\,t}{d\,\Delta}$$

using Jeffreys-Bullen's (1940) travel times of P for a surface focus and putting $v = 7$ km/sec. The resulting mean ratios for the total amplitudes of P are

for Jan Mayen $\quad \left(\frac{a}{T}\right)_U : \left(\frac{a}{T}\right)_S = 1.56 \pm 0.18$

and for Greece $\quad \left(\frac{a}{T}\right)_U : \left(\frac{a}{T}\right)_S = 3.11 \pm 0.56$

These mean values are significantly different from each other; in fact, an application of Fisher's t-test (1950, pp. 122 and 174) shows that the probability that this difference should be obtained by pure chance is only 0.01-0.02. With the notation J = Jan Mayen and G = Greece, our result is therefore that

$$\left[\frac{\left(\frac{a}{T}\right)_U}{\left(\frac{a}{T}\right)_S}\right]_G - \left[\frac{\left(\frac{a}{T}\right)_U}{\left(\frac{a}{T}\right)_S}\right]_J = 1.55$$

and significantly different from zero.

In expressing the amplitudes in the form of ratios as above, we are independent of the effects of different ground conditions (Uppsala:

granite, Stuttgart: sandstone) and possible instrumental inconsistencies. It is obvious that any such effects would mean that both the ratios

$$\left[\left(\frac{a}{T}\right)_U : \left(\frac{a}{T}\right)_S\right]_G \quad \text{and} \quad \left[\left(\frac{a}{T}\right)_U : \left(\frac{a}{T}\right)_S\right]_J$$

are multiplied by the same factor. The difference of the ratios as well as their standard error should also be multiplied by the same factor, i. e. the significance of the result is unchanged. This is easily proved mathematically. If each observation (last column of Table 8) is multiplied by a factor k, it is clear that t (Fisher's t-test, 1950, p. 122) is independent of k and equal to its value before introducing the factor k.

If the radiation of P waves were undisturbed by low-velocity layers we should expect that

$$\left[\frac{\left(\frac{a}{T}\right)_U}{\left(\frac{a}{T}\right)_S}\right]_G < 1 \quad \text{and} \quad \left[\frac{\left(\frac{a}{T}\right)_U}{\left(\frac{a}{T}\right)_S}\right]_J > 1$$

and a significantly negative difference should have been obtained above. In undisturbed radiation the amplitudes decrease with increasing distance both for geometrical reasons and due to extinction. The observations given here demonstrate very clearly that the radiation does not correspond to such undisturbed conditions.

It is difficult to see how these striking properties could be explained without assuming a shadow zone (in this case due to the asthenosphere low-velocity layer for P). In fig. 7 the limits of these shadow zones are given approx. for Uppsala (full circles) and for Stuttgart (dashed circles). We see that Jan Mayen is inside this shadow zone or close to its outer limit for Uppsala but outside for Stuttgart, whereas the reverse is the case for the Greek earthquakes studied. Immediately outside the shadow zone there is formed a caustic with a high concentration of energy.

It needs to be emphasized that no correlation is to be expected between the amplitudes of body waves and the existence of channel waves at a given station. The amplitudes of body waves depend on the location of the station in relation to a possible shadow zone, whereas the clearness of channel waves in a record depends on the focal depth and the nature of the path.

There is naturally close connection between the existence of shadow zones and the problem of earthquake magnitude determination. The determination of magnitudes is a complicated problem even for distant earthquakes (see e. g. Båth, 1956). For epicentral distances

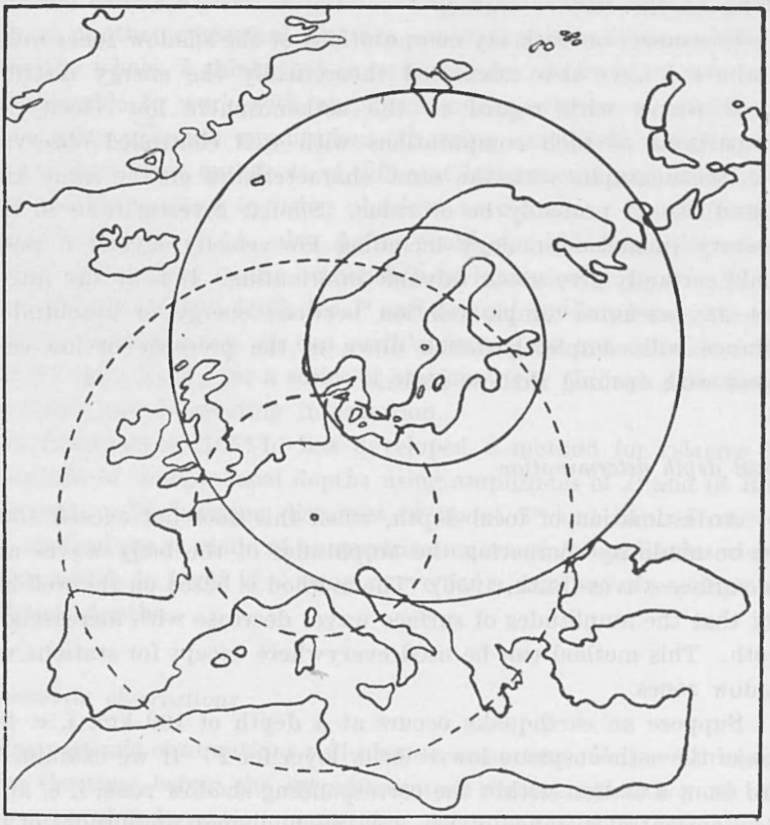

Fig. 7

$\leqslant 18^{\circ}$ the additional complication due to shadow zones for body waves comes in. Surface waves, which do not have shadow zones, are still not usable for magnitude determinations for shorter distances; the methods hitherto developed for surface waves suppose a period close to 20 sec, and for the usually much shorter periods for the shorter distances we must have better knowledge of the extinction before they can be used for magnitude determinations. Some authors have developed formulas for determining the magnitude from the maximum amplitudes in a seismogram and valid for the distance interval $2^{\circ}-30^{\circ}$. From a purely practical point of view such formulas may be of use, but theoretically they are less satisfactory. At least in an interval from $2^{\circ}-30^{\circ}$ it is clear that for the shorter distances the maximum amplitudes occur in the *S* group, whereas for the greater distances the maximum amplitudes belong to the surface waves.

In connection with my computations of the shadow zones mentioned above I have also calculated theoretically the energy distribution for *P* waves with regard to the asthenosphere low-velocity layer. Comparisons of such computations with well controlled observations (i. e. seismographs with the same characteristics on the same kind of ground) would probably be of value. Similar investigations in the laboratory (model seismology including low-velocity layers, if possible) would certainly give much valuable information. It is at any rate clear that any assumed simple relation between energy or amplitude and distance will completely break down in the presence of low-velocity layers with ensuing shadow zones.

Focal depth determination

An estimation of focal depth, when this does not exceed 100 km, can be made by comparing the amplitudes of the body waves and of the surface waves (Båth, 1956). The method is based on the well-known fact that the amplitudes of surface waves decrease with increasing focal depth. This method can be used everywhere except for stations within shadow zones.

Suppose an earthquake occurs at a depth of 100 km, i. e. in the axis of the asthenosphere low-velocity layer for *P*. If we examine a record from a station within the corresponding shadow zone, i. e. approx. at an epicentral distance of $6^{\circ}-15^{\circ}$, we find small amplitudes of the *P* waves due to the shadow zone, but also small amplitudes of the surface waves due to the focal depth. Under these circumstances the method mentioned above for focal depth determination is not applicable. A straight-forward application of the method without regard to the shadow-zone effect will lead to completely erroneous results. In the example mentioned, the record may give the misleading result that the earthquake occurred at normal depth.

More detailed computations of the amplitude ratios between *P* waves and surface waves for assumed velocity-depth relations would be desirable. Such computations should be made for various focal depths down to 100-150 km and for various epicentral distances through the shadow zone. The functional relation between focal depth and the amplitude ratio A_P/A_L will certainly be much more complicated than in the simpler case I have studied earlier. At least on a very preliminary investigation it seems possible that in some cases more than one depth-value could satisfy a given amplitude ratio A_P/A_L. The use of a second station at another epicentral distance could then solve the ambiguity.

On the whole, I think that in comparisons of the kind mentioned it is adviceable to work with amplitude ratios, whenever possible, instead of using absolute amplitudes. In using amplitude ratios we are much less dependent on effects of different instruments and of different ground conditions than in using absolute amplitudes. In addition to the amplitude ratio A_P/A_L also A_P/A_S could give valuable information, especially with regard to the asthenosphere low-velocity layer. As this layer is around 100 km depth for P and around 150 km for S, the corresponding shadow zones on the earth's surface do not coincide. The amplitude ratio A_P/A_S for a series of stations right through the shadow zones could give interesting information.

De Bremaecker (1955 b) has developed a method for relative determinations of shallow focal depths using amplitudes of P and of Rayleigh waves, valid between distances of about 2° and 24°. However, in this method use is made of an empirical curve of P amplitudes versus distance, which in itself is a mean curve for a number of earthquakes at different depths.

Macroseismic observations

Macroseismic observations still play an important rôle in seismology, and for the time before the introduction of seismographs they are the only source of information concerning earthquakes and their effects. The interpretation of macroseismic observations is therefore an important seismological task. Hitherto these interpretations have been made without regard to possible low-velocity layers and shadow zones. Because of the importance of such layers and zones it is desirable to improve the methods both for the observation and for the interpretation.

It is well known from the observations of sound waves through the atmosphere from large explosions that there may be several alternating zones of audibility and of silence, centered approximately around the point of explosion. In the macroseismic observations we could expect a similar manifestation of shadow zones, corresponding to low-velocity layers in the lithosphere. However, I do not know of any such case. It is probable that the macroseismic observations are too inaccurate and too much influenced by differences in ground and other circumstances in order to show such effects. But if anybody could find any such case which is well established, it would be of the greatest interest. Another effect of a shadow zone may be to limit off the area of perceptibility; it is not impossible that in some cases the limit of this area coincides with the inner limit of the shadow zone, and that the areas both inside and beyond the shadow zone receive too little energy to be macroseismically observed. In such a case we could expect a fairly rapid drop in observed intensity at the limit of perceptibility. In order to improve the results I would like to suggest a much more wide-spread use of standardized accelerometers. In seismic regions they could fairly easily be put up in large numbers, preferably on ground of similar nature. If only a limited number of accelerometers is available, I would like to suggest to place them along two straight lines, crossing each other at about a right angle and both passing through the region of seismic activity. The intensity distribution along the two lines for a given shock would make an epicentre determination immediately possible.

The isoseismal maps constructed by means of the macroseismic observations have often been used for calculation of the focal depth. Several authors have developed very ingenious methods for such calculations, but they assume either constant velocity or a velocity increasing steadily downward in the ground. The assumption of spherical energy distribution corresponds to the assumption of a constant velocity in all directions. These depth computations are generally regarded as relatively inaccurate, and they are certainly inferior to instrumental determinations when records from near-by stations are available. So far no formula for depth computations has been developed which takes account of a low-velocity layer.

Several authors, including myself, have used the intensities observed on the earth's surface for computation of energies and magnitudes of earthquakes. Most of the normal earthquakes occur in or close to the lithosphere channels. This is a circumstance of great consequence for energy estimations. For a focus in a channel it is no overestimate to say that at least 50% of the total seismic energy remains in the channel and does not reach the surface. On the other hand, large macroseismic effects from a comparatively small earthquake may be due to a position of the focus *above* the channel. It is therefore obvious that the observed intensities and the damage done may give quite misleading information on the energy actually released in an earthquake. It is true that in the usual magnitude determinations an error of 100% in measured amplitudes can be tolerated, corresponding to an error of 0.3 in the magnitude. The error of the energy then amounts roughly to 60%. If we are satisfied with such errors, the low-velocity layers are not so serious, but if we want greater accuracy they must in some way be taken into account.

Microseisms

The microseismic problem includes both the question of the mechanism for their generation and the question of their distribution from the source. The existence of channels in the crust may be of importance in dealing with the last-mentioned problem. Several investigators now believe that the lithosphere channels (in the continental crust) may be responsible for the transmission of microseisms. It has long been known that the usual microseisms propagate very far over the continents, e. g. Gutenberg (1932) showed that the microseisms in general vary in unison from Norway over the whole northern part of the Euroasiatic continent far into Siberia. In a comparison of the microseisms across the northernmost Atlantic (Båth, 1953) I found that the continental distribution of microseisms contrasted very sharply to the oceanic transmission. Even at such relatively near places as Reykjavik and Bergen the microseisms behave quite differently (see fig. 8). The poor transmission of microseisms over oceanic structures compared to continental structures was confirmed by Carder (1955) for other parts of the world. Already in the same paper (Båth, 1953) I said that " it is not excluded that the continental channel (for microseisms) is the same as for the short-period waves *Lg* and *Rg*, observed in earthquake records when the path is purely continental ". In the same paper I proposed also an hypothesis for microseismic barriers on this basis, saying (p. 133) that " I would like to mention also the possibility that irregular distribution of microseisms (barrier effects). may be due to interchanging oceanic and continental structures ". These results of mine were obtained completely independent of similar results of other authors, especially Ewing and Press (1952) and Ewing and Donn (1952), as both these papers were published first in 1954.

Concerning the comparison of the usual microseisms with *Lg* and *Rg* waves a few remarks can be made. *Lg* and *Rg* waves are completely extinguished even by a short oceanic path, whereas the microseisms seem to be able to propagate at least for some distance along an oceanic structure, even if it is very small compared to the distances along continental structures. Some authors have found strong extinction of the

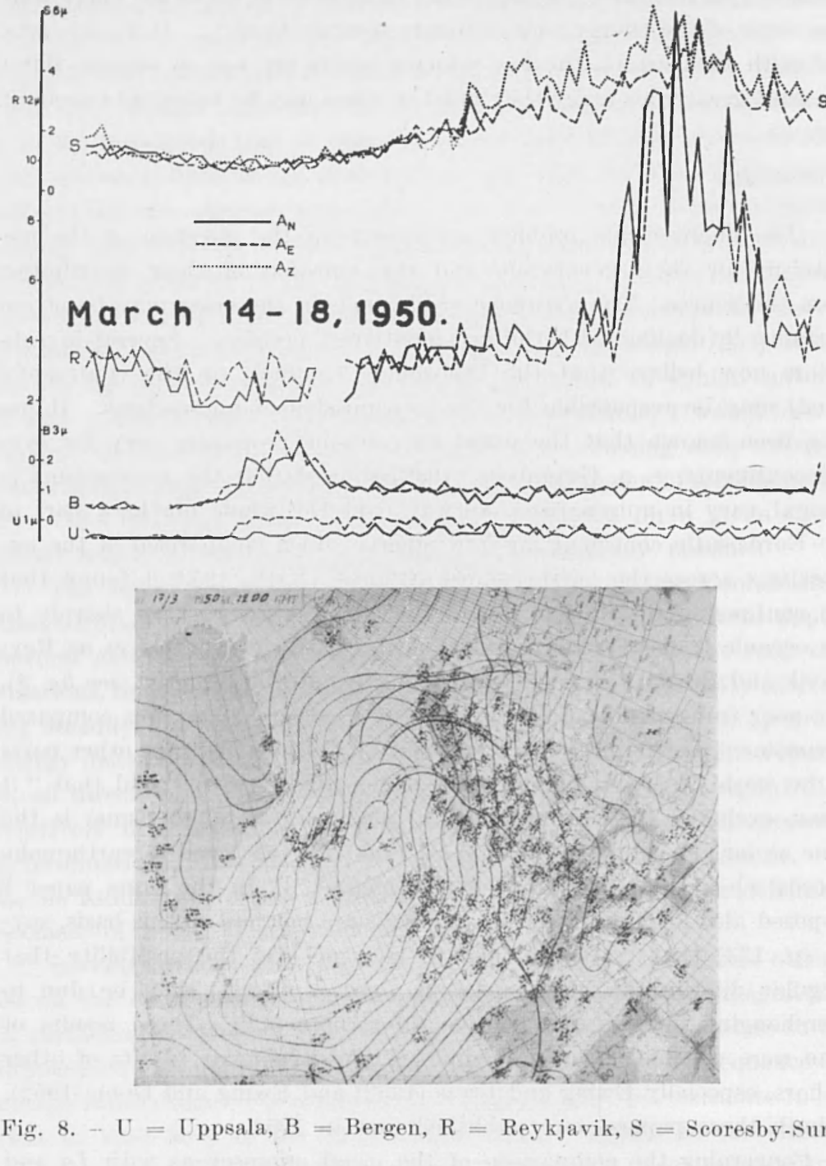

Fig. 8. – U = Uppsala, B = Bergen, R = Reykjavik, S = Scoresby-Sund.

microseisms in crossing the continental border. I have found no such effect outside the Norwegian coast (Båth, 1952). In order to decide better between transmission properties and extinction effects at continental borders it would be desirable to have a dense net of suitable seismographs in a region where oceanic and continental structures could be compared. The geophysical year would be a suitable occasion for such an investigation.

Comparing the wave types we find several points of agreement between microseisms and channel waves. The periods of the usual microseisms fall in the same range as for *Lg*. The microseisms are certainly a composite wave motion, probably containing both *Lg* and *Rg*. The *Rg* component is generally dominating, which is probably due to the generation mechanism of the microseisms. The determinations of velocity for microseisms by means of tripartite station observations have given somewhat divergent results, but on the whole the velocities fall in the same range as for *Lg* and *Rg*.

Directions obtained at one station (or at a tripartite station) must be handled with care. As the microseisms prefer the continental structures, it is expected that in areas with both continental and oceanic structures notable deviations from the transmissions along great circle arcs may occur.

It is also to be expected that the study of microseisms will profit by more intense study of the *Lg* and *Rg* waves from earthquakes, where the conditions are better known than in the case of microseisms. When the constitution of the crust and its different possibilities for wave propagation have first been established by investigations of *Lg* and *Rg*, the problem of the distribution of microseisms will be easier to solve.

Oliver, Ewing, and Press (1955) have recently found a kind of short-period surface waves from certain earthquakes. No complete theoretical explanation has as yet been given for these waves. They show extreme attenuation upon passing a continental boundary but are propagated efficiently within either oceanic or continental areas. They resemble long-period microseisms (6-9 sec), and it is probable that the investigation of these waves will also throw further light on the microseismic problem.

Amplitude measurements in refraction shooting

The various indications of the existence of low-velocity layers given above are generally only indirect, in other words we have only studied various effects of such layers. It would naturally be very welcome if some more direct evidence for these layers could be produced. Further laboratory investigations of the variation of the elastic parameters for various rocks with pressure and temperature would be very helpful. However, also a more direct method applicable in nature is wanted. The layering in the crust is now accessible to direct investigation by seismic methods (refraction and reflexion shooting). But this method, based on time measurements, fails in discovering low-velocity layers. No wave can have its deepest point in such a layer, and therefore such layers are not discovered, even if present. In order to increase the efficiency of the seismic methods I therefore propose that also amplitude measurements be made. As comparable amplitudes are wanted, we are up against several difficulties mentioned above. Similar ground, preferably bed-rock, must be selected, and all the seismometers must have the same response characteristics and they must naturally all be calibrated. Refraction shootings with profiles more than 300 km in length are made in large scale in North America and at several other places. In Europe similar investigations in more limited extent are made, but extensive seismic investigations of the Alps are planned. In all these and similar investigations in all parts of the world, I propose that also amplitude measurements are tried.

On the basis of the seismic methods the constitution of the crust has been deduced at a number of places on the globe. The existence of the continental waves (*Lg1*, *Lg2*, *Rg*) is proved beyond any doubt. These waves are able to propagate over large distances (across the largest continents) with very little loss of energy. The fact that they are limited to continental paths as well as their velocities and other properties show that they are propagated within the crust itself. On the basis of these experiences no model for the continental crust can be accepted which does not provide an explanation for this energy transmission over large distances. At present I can see no other solution to this obvious problem than to assume the existence of one or preferably several low-velocity layers in the crust. The same remark applies to the asthenosphere low-velocity layers for P and S waves.

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June, 1956.

SUMMARY

The main purpose of this paper is to emphasize a series of problems for the solution of which low-velocity layers are of decisive importance. The intention is also to stimulate research on these problems. Many more problems could be mentioned, where low-velocity layers play a rôle, e. g. the interpretation of records of near-by earthquakes, the determinations of travel times, the influence on the usual surface waves (see Stoneley, 1950), etc. Nevertheless, I hope that the items treated will give the impression that low-velocity layers enter into most problems of modern seismology. There are certainly several problems which can be reasonably solved without assuming such layers. But the well-established fact that channel waves exist calls for an explanation, which in my view requires low-velocity layers. I know that several investigators do not believe in the existence of such layers. In saying that the intention of this paper is to stimulate research concerning low-velocity layers, naturally also the last-mentioned category is welcome. Well substantiated proofs against their existence are naturally also valuable as contributions to the solution of this whole complex of problems.

Among the problems studied in this paper the following will be mentioned here.

1. *The physical-chemical properties, especially the dependence of incompressibility and rigidity on temperature and pressure, of the asthenosphere low-velocity layers are deduced.*

2. *A new channel wave in the intermediate layer, Li with a velocity of about 3.8 km/sec, is presented.*

3. *Various calculations, especially of energies, lead to a revised model for channel-wave propagation in the continental crust.*

4. *New evidence for the shadow zone corresponding to the asthenosphere low-velocity layer for P waves is given, based on a comparison of records at Uppsala and Stuttgart.*

5. *The importance of shadow zones for determinations of magnitude and of focal depth is emphasized.*

6. *Shadow zones may be taken into account in the interpretation of macroseismic observations.*

7. The channels in the continental crust are believed to be of importance for the distribution of microseisms.

8. It is suggested that in refraction shootings also amplitude measurements are made in an effort to discover the shadow zones corresponding to the lithosphere low-velocity layers.

RIASSUNTO

Lo scopo principale di questo lavoro è di far risaltare una serie di problemi per la soluzione dei quali gli strati a flessione di velocità sono di importanza decisiva. — L'intenzione è anche di stimolare le ricerche su questi problemi —. Ben altri problemi potrebbero essere menzionati, nei quali gli strati a flessione di velocità giuocano un ruolo importante: per esempio, l'interpretazione di registrazioni di terremoti vicini, le determinazioni dei tempi di tragitto, l'influenza sulle usuali onde di superficie (Stoneley 1950), etc. Ciononostante, spero che gli elementi trattati daranno l'impressione che gli strati a flessione di velocità entrano in molti problemi della moderna sismologia. Vi sono certamente parecchi problemi che possono essere razionalmente risolti senza considerare tali strati. Ma il fatto, ben dimostrato, che le onde canalizzate esistono, richiede una spiegazione, che secondo me, risiede appunto sull'esistenza di strati a flessione di velocità. Io so che parecchi ricercatori non credono all'esistenza di questi strati. Dicendo che l'intenzione di questa pubblicazione è di stimolare le ricerche, riguardanti gli strati a flessione di velocità, naturalmente anche l'ultima categoria menzionata è benvenuta. Prove molto sicure contro la loro esistenza sarebbero naturalmente pure preziose, come contributi alla soluzione di questo complesso di problemi.

Fra i problemi studiati in questa pubblicazione i seguenti vengono qui menzionati:

1. Si deducono le proprietà fisico-chimiche, specialmente la dipendenza dall'incompressibilità e rigidità, dalla temperatura e dalla pressione, degli strati a flessione di velocità dell'astenosfera.

2. Si presenta una nuova onda canalizzata nello strato intermediario, Li, con una velocità di circa 3,8 km/sec.

3. Vari calcoli, specialmente delle energie, conducono ad un modello corretto per la propagazione delle onde canalizzate nella crosta continentale.

4. *Si mette in nuova evidenza la zona d'ombra corrispondente allo strato a flessione di velocità dell'astenosfera per le onde P, basata su un confronto delle registrazioni ottenute a Uppsala e a Stoccarda.*

5. *È messa in risalto l'importanza delle zone d'ombra per le determinazioni della magnitudo e della profondità ipocentrale.*

6. *Le zone d'ombra possono essere prese in considerazione nell'interpretazione delle osservazioni macrosismiche.*

7. *I canali nella crosta continentale sono ritenuti di grande importanza per la propagazione dei microsismi.*

8. *Viene suggerito che anche nella sismica a rifrazione le misure delle ampiezze siano fatte nel tentativo di scoprire le zone d'ombra, corrispondenti agli strati a flessione di velocità della litosfera.*

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