IMPROVEMENTS ON THE ESTIMATE OF SEISMIC CHARGES

SIMÓN GERSHANIK

1. Assuming that buildings behave under seismic accelerations as vertical clamped bars, that the accelerations are only horizontal and of a translatory character, we can put for the seismic charges q:

$$q = \rho S \left(\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 X_B}{\partial t^2} \right) + 2 \varepsilon' \frac{\partial u}{\partial t} \qquad [1]$$

the meaning of simbols being as follows:

- ρ the mean density of the building
- S its mean transverse section
- u its deformation (fig. 1)
- X_B horizontal ground displacement at the place B where the building is champed
- t time, and
- ε' a damping coefficient of the huildings.

Therefore q, due to a ground acceleration $\frac{\partial^2 X_B}{\partial t^2}$ can be determined if we know the expression of u.

To get u three different assumptions may he considered: *a*) the deformations are principally due to shear; *b*) they are principally due to flexure; and *c*) they are due to both causes. This paper deals only with assumptions *a*) and *b*).

a) In this case, as is well known, u is given by the differential equation:

$$\frac{\partial^2 u}{\partial z^2} = k \frac{\partial^2 u}{\partial t^2} + 2 \varepsilon_1 \frac{\partial u}{\partial t} + k \frac{\partial^2 X_B}{\partial t} \qquad [2]$$

(where $k = \frac{\rho}{\mu}$; $\varepsilon_1 = \frac{\varepsilon'}{\mu S}$; μ rigidity modulus) with the following boundary conditions

$$u = 0 \quad \text{when} \quad z = 0$$
$$\frac{\partial u}{\partial z} = 0 \quad \text{when} \quad z = l \qquad [3]$$

b) Deformation u is in this case given by:

$$\frac{\partial^4 u}{\partial z^4} = c \frac{\partial^2 u}{\partial t^2} + 2\,\varepsilon_2 \frac{\partial u}{\partial t} - c \frac{\partial^2 X_R}{\partial t^2} \qquad [4]$$

where $c = \frac{\rho S}{E J}$: $\varepsilon_2 = \frac{\varepsilon'}{E J}$, $E =$ elasticity modulus and J moment of inertia of S refered to the neutral axis of the building. The boundary conditions now are:

$$u = 0 \; : \; \frac{\partial u}{\partial z} = 0 \quad \text{when } z = 0$$

$$\frac{\partial^2 u}{\partial z^2} = 0 \; ; \; \frac{\partial^3 u}{\partial z^3} = 0 \quad \text{when } z = l$$

Assuming that building are at rest until seismic accelerations begins, we have in addition, that equations [2] and [4] must satisfy the initial conditions

$$u = 0 \,, \quad \frac{\partial u}{\partial t} = 0 \quad wh\;n \; t = 0 \qquad [5]$$

2. — Equations [2] and [4] have the form

$$\frac{\partial^n u}{\partial z^n} = - r(z) \qquad [6]$$

If we admit that the second member is a continuous function of z in the interval $0 < z < l$ (l height of the building) then, according to the theory of Green's functions, we may put the solution of [6] and therefore the solutions of [2] and [4] too, in the form (1) (2):

$$u = \int_0^l K(z, \zeta)\, r(\zeta)\, d\xi \qquad [7]$$

$K(z, \zeta)$ being Green's function of the equation

$$\frac{\partial^n u}{\partial z^n} = 0$$

As n is an even number and the boundary equations are homogeneous, Green's K function should be symetrical with respect to z and ζ. On the other hand $K(z, \zeta)$ may be considered as a kernel of the integral equation of second kind

$$v(z) = \lambda \int_0^l K(z, \zeta)\, v(\zeta)\, d\zeta$$

From this, and from [7] it follows that the solution of [6] is

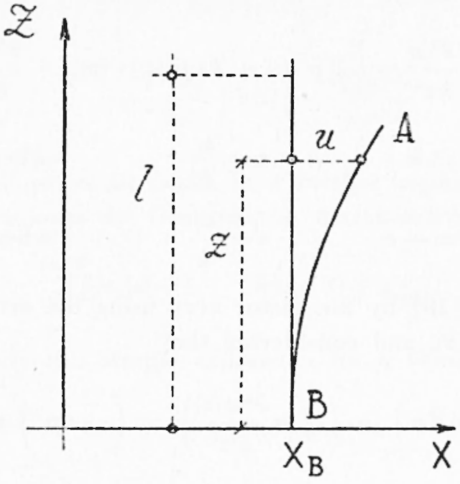

Fig. 1

a function represented according to the source $r(\zeta)$ by means of the symetrical kernel K. By Hilberts expansions theorem ([3]) it can, consequently be expanded in an uniformly convergent series of eigenfunctions $v(z)$ of this kernel.

There fore we may write

$$u = \sum_{j=1}^{j=\infty} p_j\, v_j(z) \qquad [8]$$

$v_j(z)$ being such eigenfunctions, and p_j coefficients independent of z.

As u is a function of t, we can differentiate the expression [7] with respect to this variahle. We then get expressions similar to [7]; from this it follows that the two derivates my also be expanded in uniformly convergent series of $v(z)$.

By integration of the series representing $\frac{\partial^2 u}{\partial t^2}$ with respect to t, we obtain another representing $\frac{\partial u}{\partial t}$, and by integration of this we obtain another representing u.

This, as well as the fact that u is given by [8], allows us to write:

$$\frac{\partial u}{\partial t} = \sum_{j=1}^{j=\infty} p'_j(t)\, v_j(z) \; : \quad \frac{\partial^2 u}{\partial t^2} = \sum_{j=1}^{j=\infty} p_j''\, v_j(t) \qquad [9]$$

Substituting this in [2] and [4] we obtain:

$$\frac{1}{M}\,\frac{\partial^n u}{\partial z^n} = \sum_{j=1}^{\infty} \{[p''(t) + 2\varepsilon p'(t)]\, v(z)\}_j + \frac{\partial^2 X_B}{\partial t^2} \qquad [10]$$

where $M = k \qquad \varepsilon = \frac{\varepsilon_1}{k}$ when $n = 2$

$M = -c \qquad \varepsilon = \frac{\varepsilon_o}{k}$ when $n = 4$

Multiplying [10] by the factor $v(z)$, using the orthogonality properties of the $v(z)$, and considering that

$$\left(\int_0^l \frac{\partial^n u}{\partial z^n} v(z)\, dz\right)_1 = \left(\int_0^l u\, \frac{\partial^n v(z)}{\partial z^n}\right)_j = \left(-\lambda p \int_0^l v^2(z)\, dz\right)_j$$

we easily obtain

$$p'' + 2\varepsilon p' + \omega^2 p = -\beta \frac{\partial^2 X_B}{\partial t^2} \qquad [11]$$

where:

$$\beta = \frac{\int_0^1 v(z)\, dz}{\int_0^1 v^2(z)\, dz} \; ; \quad \omega^2 = \frac{\lambda}{M}$$

When applied to [8], the initial conditions require that

$$p(t) = O \qquad p'(t) = O \qquad \text{when} \quad t = O$$

From this and from [11] we draw the conclusion that each coefficient p represents the movement of a dynamic system initially at rest, and forced by $\beta \frac{\partial^2 X_B}{\partial t^2}$. Using Rayleigh's formulae ([4]) ([5]) we can put:

$$p = \beta \frac{T'}{2\pi} \int_0^t \frac{\partial^2 X_B}{\partial t^2}(\tau)\, e^{-\varepsilon(t-\tau)}\, sen \frac{2\pi}{T'}(t-\tau)\, d\tau \qquad [12]$$

where

$$T' = \frac{2\pi}{\sqrt{\omega^2 - \varepsilon^2}}$$

3. — According to [1], [9] and [11], it is evident that

$$q = \rho S \left[\sum_j^{\infty} \left\{ (p'' + z\,\varepsilon\, p')\, v(z) \right\}_y + \frac{\partial^2 X_B}{\partial t^2} \right] = -\rho S \left[\sum_{jy}^{\infty} \left\{ \left(\omega^2 p + \right.\right.\right.$$

$$\left.\left.\left. + \beta \frac{\partial^2 X_B}{\partial t^2} \right) v(z) \right\}_j - \frac{\partial^2 X_B}{\partial t^2} \right]$$

As we shall prove in detail in a further paper to be published in the « Publicaciones del Observatorio Astronómico de Eva Perón »,

$$\sum_{j=1}^{j=\infty} \beta_j v_j(z) = 1 \qquad \text{when } O < z \leq 1 \qquad [13]$$

Consequently this simple expression for q remains:

$$q = -\rho\, S \sum_j^{\infty} \left(\omega^2\, p\, v(z) \right)_j \qquad [14]$$

which shows that in order to obtain q, we have to know the function $p(t)$.

This function can be determined with the help of formulae [12]. As $\frac{\partial^2 X_B}{\partial t^2}$ is an empirical function, the integral can be calculated only numerically or mechanically.

The first way was employed with a different aim, by A. Blake ([6]), some years ago. Because of the work involved it is scarcely attractive. Use of the second way was recently made by Alford Housner and Martel ([7]). These investigators, studying the seismic action on buildings from another viewpoint determined integrals similar to those in [12], by means of the electrical analog computer of the California Institute of Technology, for values of $T = 2\ \pi / \omega$ ranging from 0.1 to 3.0 seconds and for values of ε / ω ranging from 0.0 to 0.4, using accelerograms of several strong earthquakes. Special attention was paid by them to the maximum valne X_m of the integral for each T. With their results, they constructed curves of X_m as a function of T, which they called « spectrum » of the earthquakes.

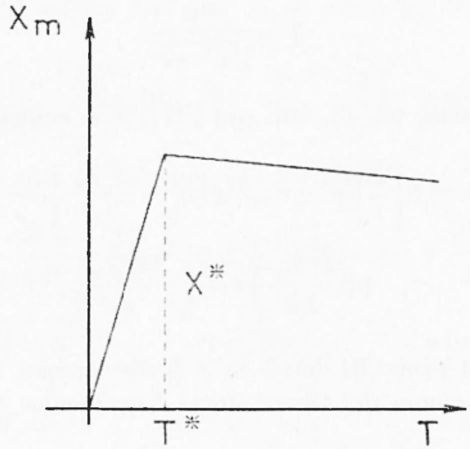

Fig. 2

Having now this spectrum at our disposal, we tried to improve the empirical estimate of seismic charges, that is customary in engineering, using values of X_m instead of the accurate values of the integrals written in [12]. With this, and the assumptions that $T' = 2\ \pi/\omega$ we obtain instead of [14]

$$q = -\rho S \sum_{j=1}^{\infty} \left(\frac{2\pi}{T} X_m \beta v(z) \right)_j \qquad [15]$$

Curves representing X_m are also complicated. But it is possible to aproximate them with simpler ones and try with these to obtain the sum pointed out in [14].

It is very interesting to note that, if we adopt for X_m the representation given in fig. 2 (which can be obtained by smoothing the true curves) then for buildings in which the maximum T is less than T^*, we get from [15] that:

$$q = -\rho\, S \frac{2\pi X^*}{T^*} \Sigma \beta\, v(z)$$

or remembering [13]:

$$q = -\rho\, S \frac{2\pi X^*}{T^*}$$

We arrive thus to the remarkable conclusion that the seismic charge q should be constant all along the building; a fact that is customarily accepted among engineers, whithout demonstration.

From measurements made in the U.S.A. (8) we have deduced that

$$T = 0{,}02 \quad l$$

$l =$ height of the buildings in metres

$T =$ in seconds

According to the refered spectrum, T^* is of the order of 0,6 seconds. Consequently the assumption that q is constant could be accurate for buildings less than 30 m height. For higher buildings it can be proved that q decreases with l.

Observatorio Astronómico
Eva Perón (formerly La Plata), May 1954

SUMMARY

Deformations u of buildings due to the acceleration of earthquakes are investigated. Two suppositions are made: a) *the deformations are principally due to shear; and* b) *they are principally due to bending.*

In both cases it is found that u may be expressed by an uniformly convergent series of eigenfunctions.

Except for a variable factor, the Fourier coefficients of these series are the same as the values X obtained recently by Alford Housner and Martel, by means of the electrical analog computer of the California Institute of Technology.

Using mean values of what they call the spectrum of earthquakes, it is found that the seismic charge is constant on buildings less than 30 m height. On buildings higher than 30 m it becomes decreasing with height.

RIASSUNTO

Vengono studiate le deformazioni di edifici, dovute all'accelerazione determinata da terremoti. Vengono fatte due ipotesi: a) *le deformazioni sono principalmente dovute a forze tangenziali;* b) *esse sono principalmente legate alla flessione.*

In entrambi i casi, si prova che esse possono venir espresse mediante una serie uniformemente convergente di autofunzioni.

Fatta eccezione di un fattore variabile, i coefficienti di Fourier di questa serie sono gli stessi che figurano nei valori ottenuti da Alford Housner e Martel in ricerche su argomenti analoghi condotte presso il « California Institute of Technology ».

Facendo uso dei valori medi del così detto spettro dei terremoti, l'Autore trova che la carica sismica su edifici di altezza minore di 30 metri è costante, mentre tende a diminuire per altezze maggiori.

REFERENCES

(1) Courant Hilbert, *Methoden der Mathematischen Physik*, I Kap V 10, 14.

(2) L. Collatz. *Eigenwertaufgaben mit Technischen anwendungen*, Kap II, 5.

(3) Courant Hilbert, *loc. cit.*, Kap. III 5, 1.

(4) Idem, Kap V 2, 3.

(5) Lord Rayleigh, *Theory of Sound*, Chap II 66, Dover Publ. 1945.

(6) A. Blake, *Proceedings of the* 1935, Meeting Eastern Sec Seism. Soc. America.

(7) Alford Housner and Martel. Bull. Seismological Soc. of America, vol. 43, Number 2.

(8) U. S. Coast and Geodetic Survey, *Earthquake investigations in California* 1934-1935, Special publication, n. 201.