POLARISATION OF THE S - PHASE OF SEISMOGRAMS (*)

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It has been observed by F. Neumann (1), P. Byerly (2) and I. Lehmann (3) that in seismograms of earthquakes recorded at moderate distances, the S phase appears first as SH, followed some 10-14 seconds later by SV. Miss Lehmann attributes this phenomenon to the small angle that the S — oscillations make with the horizontal, while Prof. Byerly suggests that some kind of double refraction may be the cause. Both agree that the interval does not seem to depend on epicentral distance.

Evidently more observational evidence is needed, but a preliminary examination can be made of the possibility that the phenomenon arises because the outer part of the Earth's mantle is not elastically isotropic. The simplest hypothesis we can make to conform to this assumption is perhaps that all directions at right angles to the vertical are equivalent; this type of solid is termed « transversely isotropic ». The assumed type of anisotropy is sufficiently general to give an indication of the order of magnitude of the anomaly to be expected. Further, it is unlikely that the outer layers are very markedly anisotropic, else this departure from isotropy would probably have been noted long ago. It should suffice then, to take as a standard of comparison a transversely isotropic material whose elastic constants are known: such a crystal is beryl, the constants of which were determined by Voigt (4).

The seismological implications of a transversely isotropic medium have been investigated, and one important fact is that if all directions perpendicular to the vertical are equivalent then (neglecting the curvature of the Earth) the velocities of SH and SV are different, and they depend on the angle of incidence of the wave. The difference in the time of propagation through a surface layer of thickness h can easily be calculated. Let suffixes H, V refer to SH, SV respectively; let c be the wave-velocity in any medium and i the angle of incidence

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in that medium. If $r$ is the radial distance from the centre of the Earth, then for any ray, in the standard notation,

$$p = \frac{r \sin i}{c} = \frac{dT}{d\Delta}$$

Suppose that a disturbance originating at $A$ generates $SH$ and $SV$ waves arriving at $B$ along paths $PQ_mB$ and $PQ_VB$. Let $P_M$ and $Q_VN$ be perpendicular to $P_n M$ and $Q_n N$ respectively, where refraction is supposed to occur at the base of a surface layer.

Then from the diagram $P_n M = NQ_n = h (\tan i_v - \tan i_n) \sin i$, so that the difference of the time of transmission of $S$ along the paths $P_v Q_v$ and $P_n Q_n$ in the lower material, supposed isotropic, is $2(h/c) \sin i (\tan i_v - \tan i_n) = 2dT_i$ say.

Alternatively, if $O$ is the centre of the Earth and $R$ the radius at the discontinuity, while the angle $Q, OQ_n$ is $d\Delta$, then $Q_v Q_n = R$. $d\Delta = R(d\Delta/dT_i) dT_i = c \, dT_i / \sin i$, giving the same result as before.

Thus, in all, the difference in the times of transmission of $SH$ and $SV$

$$T_n - T_v = \frac{2h}{c_n} \sec i_n - \frac{2h}{c_v} \sec i_v + \frac{2h}{c} \sin i (\tan i_v - \tan i_n)$$

where $c, i$ refer to the medium immediately below the junction.

To see the type of variation of $c_v$ and $c_n$ with inclination $0^\circ$ to the vertical, it will suffice to work with the density and elastic constants of beryl, and to scale down the results so as to correspond to a velocity 3.40 km/sec. in the layer for $SV$ at $\theta = 0^\circ$ and $\theta = 90^\circ$. In
the underlying material we take $c=4.38$ km/sec.; thus the model corresponds to a granitic continent resting on ultrabasic material.

The requisite formulae for $c_n$ and $c_v$ are found in the paper already cited (9). For a transversely isotropic body in which the strain-energy function $W$ is given by

$$
2W = A(e_{xx}^2 + e_{yy}^2) + C e_{xx}^2 + 2F(e_{xx} + e_{yy}) e_{xx} + 2(A-2N) e_{xx} e_{yy} + L(e_{xx}^2 + e_{yy}^2) + N e_{xz}^2,
$$

the velocity $c$ of a wave corresponding to $HS$ is given by

$$
0 \cdot c_n^2 = Nl^2 + Ln^2
$$

where $n = \cos \vartheta$; $l = \sin \vartheta$. For waves of $SV$ type

$$
2c_v^2 = Al^2 + Cn^2 - \frac{1}{4} \left( (A-L) n^2 - (C-L) n^2 \right) + 4J^2 l^2 n^2
$$

in which $J = F + L$.

Voigt's values, in dynes/cm², are

$$
A = 2.694 \times 10^{12}; \quad C = 2.363 \times 10^{12}; \quad A - 2N = 0.961 \times 10^{12};
$$

$$
F = 0.661 \times 10^{12}; \quad L = 0.653 \times 10^{12}; \quad N = 0.866 \times 10^{12}.
$$

From the foregoing formulae, the values of $c_n$ and $c_v$, scaled so that they correspond to material in which $c_n^2$ and $c_v^2$ for $\vartheta = 0^\circ$ are $11.56 \times 10^{10}$, are given in the following table:

<table>
<thead>
<tr>
<th>$\vartheta^\circ$</th>
<th>$c_n \times 10^{10}$</th>
<th>$c_v \times 10^{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>10</td>
<td>3.41</td>
<td>3.48</td>
</tr>
<tr>
<td>20</td>
<td>3.46</td>
<td>3.68</td>
</tr>
<tr>
<td>30</td>
<td>3.54</td>
<td>3.905</td>
</tr>
<tr>
<td>40</td>
<td>3.62</td>
<td>4.04</td>
</tr>
<tr>
<td>50</td>
<td>3.71</td>
<td>4.02</td>
</tr>
<tr>
<td>60</td>
<td>3.79</td>
<td>3.855</td>
</tr>
<tr>
<td>70</td>
<td>3.86</td>
<td>3.64</td>
</tr>
<tr>
<td>80</td>
<td>3.90</td>
<td>3.47</td>
</tr>
<tr>
<td>90</td>
<td>3.92</td>
<td>3.40</td>
</tr>
</tbody>
</table>

The differing trends of $c_n$ and $c_v$ with $\vartheta$ are evident from this table.

In what follows the layer will be treated as thin, so that for propa-
gation in this layer the Earth may be considered as plane. Then \( i_{n} \) and \( i_{v} \) are given by

\[
\frac{\sin i}{c} = \frac{\sin i_{n}}{c_{n}} = \frac{\sin i_{v}}{c_{v}}
\]

where \( c = 4.38 \text{ km/sec.} \) and \( (\sin i)/c = (dT/d\Delta \sigma)/110.7 \)

Since \( dT/d\Delta \) is known from seismological tables, the angle \( i \) can be computed for various epicentral distances \( \Delta \).

To complete the calculations we first consider \( SH \), and put \( \Phi = i_{n} \) in Table I. Thus \( (\sin i_{n})/c \) can be computed as a function of \( i \), and from the foregoing equation \( i_{n} \) is found as a function of \( \Delta \). The calculations for \( SV \) are much longer, but the general principle is the same. We thus obtain Table II. It will be seen that the differences between \( i_{n} \) and \( i_{v} \) are not large. The value of \( (T_{n} - T_{v})/2h \) can now be computed directly.

<table>
<thead>
<tr>
<th>( \Delta ) degrees</th>
<th>( dT \Delta ) sec/deg</th>
<th>( i_{n} ) degrees</th>
<th>( i_{v} ) degrees</th>
<th>( i ) degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>24.4</td>
<td>56.3</td>
<td>58.7</td>
<td>74.5</td>
</tr>
<tr>
<td>20</td>
<td>19.4 Sr</td>
<td>39.5</td>
<td>45.0</td>
<td>50.0</td>
</tr>
<tr>
<td>30</td>
<td>18.7 Sr</td>
<td>38.0</td>
<td>43.9</td>
<td>47.7</td>
</tr>
<tr>
<td>40</td>
<td>15.8</td>
<td>30.0</td>
<td>31.8</td>
<td>38.8</td>
</tr>
<tr>
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<td>15.0</td>
<td>28.5</td>
<td>32.3</td>
<td>36.4</td>
</tr>
<tr>
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<td>26.4</td>
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<td>19.0</td>
<td>20.7</td>
<td>24.5</td>
</tr>
<tr>
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<td>9.0</td>
<td>16.2</td>
<td>17.2</td>
<td>20.9</td>
</tr>
</tbody>
</table>

Since the object of this investigation is to ascertain only the order of magnitude of the effect of departure from isotropy, it suffices to consider three values of \( (T_{n} - T_{v})/2h \). For \( \Delta = 20^\circ \) (Sr) it is 0.039, for \( \Delta = 50^\circ \) it is 0.035, and for \( \Delta = 80^\circ \) it is 0.025. Thus, the variation with epicentral distance is not large. However, the time difference is quite small, e.g. for \( h = 33 \text{ km.} \), about the maximum permissible thickness of the granitic layer, \( T_{n} - T_{v} \) amounts only to about 1.6 sec. Thus the explanation in terms of departure from iso-
Polarization of the S - Phase of Seismograms

In earthquakes recorded at moderate distances it has been observed that S phase appears first as SI\(_\text{II}\), followed some 10 to 14 seconds later by SV. The object of this paper is to try to decide whether double refraction is likely to be the explanation of this phenomenon.

A simple model to consider would be a "transversely isotropic" material, symmetrical about the radial direction. Formulae for the velocities of SI\(_\text{II}\) and SV waves are available; these velocities depend on the angle that the ray makes with the normal. It is unlikely that the Earth could be as markedly anisotropic as the mineral beryl, which is transversely isotropic; accordingly, this material, of which the five elastic constants are known, is taken as an extreme example, and the velocities of SI\(_\text{II}\) and SV for different angles of incidence are "scaled down" so as to match the velocity of distortional waves in granite. It is then possible to calculate the difference in the time taken by waves from one point of the surface of the Earth to another point on the surface according as the S wave in the surface layer is of SI\(_\text{II}\) or SV type.

It is found that, even in this extreme case, a layer of anisotropic rock some 30 km thick would account only for a time difference of...
about 1 1/2 seconds. Thus, if the Earth were as strongly anisotropic as beryl (which is unlikely) one would need the layer of preferred orientation to extend to a depth of about 300 km. This is difficult to believe, and thus no great credence can be attached to an explanation in terms of double refraction.

BIBLIOGRAPHY

(2) P. Byers, Bull. Seis. Soc. Amer. 28, 12 (1938).
(4) W. Voigt, Lehrbuch der Kristallphysik (1928).
(6) R. Stoneley, Loc. cit. equations (11) o (11 a).