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Appendix A

Derivation of Decoupled Elastic Wave Equations

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The derivation of the decoupled wave equations is given here. For a better explicit demonstration, we enumerate the following identifies:

$$\begin{cases} \mathbf{k}\times(\tilde{\mathbf{k}}(\tilde{\mathbf{k}}\cdot\tilde{\mathbf{U}}))=0 \\ \mathbf{k}\cdot(\tilde{\mathbf{k}}\times(\tilde{\mathbf{k}}\times\tilde{\mathbf{U}}))=0 \end{cases} . \tag{A-1}$$

Replacing the wavefield vector in equation (2) with the linear superposition expression above, we see that

$$\frac{\partial^2 \tilde{\mathbf{U}}}{\partial t^2}+c_p^2\mathbf{k}(\mathbf{k}\cdot(\tilde{\mathbf{U}}_p+\tilde{\mathbf{U}}_s))-c_s^2\mathbf{k}\times(\mathbf{k}\times(\tilde{\mathbf{U}}_p+\tilde{\mathbf{U}}_s))=0 . \tag{A-2}$$

Incorporating equations (4) and (5) into equation (A-2), the expression is given by:

$$\frac{\partial^2 (\tilde{\mathbf{U}}_p+\tilde{\mathbf{U}}_s)}{\partial t^2}+c_p^2\mathbf{k}(\mathbf{k}\cdot(\tilde{\mathbf{U}}_p-\tilde{\mathbf{k}}\times(\tilde{\mathbf{k}}\times\tilde{\mathbf{U}})))-c_s^2\mathbf{k}\times(\mathbf{k}\times(\tilde{\mathbf{k}}(\tilde{\mathbf{k}}\cdot\tilde{\mathbf{U}})+\tilde{\mathbf{U}}_s))=0 . \tag{A-3}$$

Taking equation (A-1) into consideration, we obtain:

$$\left(\frac{\partial^2}{\partial t^2}+c_p^2\mathbf{k}(\mathbf{k}\cdot)\right)\tilde{\mathbf{U}}_p+\left(\frac{\partial^2}{\partial t^2}-c_s^2\mathbf{k}\times(\mathbf{k}\times)\right)\tilde{\mathbf{U}}_s=0 . \tag{A-4}$$

In equation (A-4) the first term $\left(\frac{\partial^2}{\partial t^2}+c_p^2\mathbf{k}(\mathbf{k}\cdot)\right)\tilde{\mathbf{U}}_p$ is proportional to $\mathbf{k}$, and the second term $\left(\frac{\partial^2}{\partial t^2}-c_s^2\mathbf{k}\times(\mathbf{k}\times)\right)\tilde{\mathbf{U}}_s$ is orthogonal to $\mathbf{k}$. The two terms are linearly independent. Consequently, each term equals zero.

$$\begin{cases} \frac{\partial^2\tilde{\mathbf{U}}_p}{\partial t^2}+c_p^2\mathbf{k}(\mathbf{k}\cdot\tilde{\mathbf{U}}_p)=0 \\ \frac{\partial^2\tilde{\mathbf{U}}_s}{\partial t^2}-c_s^2\mathbf{k}\times(\mathbf{k}\times\tilde{\mathbf{U}}_s)=0 \end{cases} . \quad \text{(A-5)}$$