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Appendix B

The Error between True Velocity and Numerical Velocity

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Given that a plane wave is supported in a uniform and infinite medium, the particle velocity components can be defined as

$$\mathbf{v}(x,z)=(v_x,v_z)e^{i(\mathbf{k}\cdot\mathbf{x}-\omega t)} \tag{B-1}$$

Substituting equation (B-1) into equation (28) yields that

$$k_x \sin c(v_p(x)|\mathbf{k}|\Delta t/2)=\sum_{l=1}^{L}\mathbf{G}_{px}(\mathbf{x},l)\sin((2l-1)k\Delta x/2)\,, \tag{B-2}$$

$$k_z \sin c(v_p(x)|\mathbf{k}|\Delta t/2)=\sum_{l=1}^{L}\mathbf{G}_{pz}(\mathbf{x},l)\sin((2l-1)k\Delta z/2)\,. \tag{B-3}$$

For simplicity, let $\Delta x=\Delta z=\Delta h$. If we define $\eta_{px}=\sum_{l=1}^{L}G_x^p(\mathbf{x},l)\sin((2l-1)k_x\Delta h/2)$, $\eta_{pz}=\sum_{l=1}^{L}G_z^p(\mathbf{x},l)\sin((2l-1)k_z\Delta h/2)$, equation (B-2) and equation (B-3) can be formulated as

$$\eta_{px}=k_x \sin c(v_p(x)|\mathbf{k}|\Delta t/2) \tag{B-4}$$

$$\eta_z=k_z \sin c(v_p(x)|\mathbf{k}|\Delta t/2)\,. \tag{B-5}$$

Then, by the respective summations of the items on the both sides, which are shown in equation (B-2) and equation (B-3), we can obtain

$$\frac{\sin^2(v_p\,|\mathbf{k}|\,\Delta t/2)}{v_p^2\Delta t^2/4}=\eta_{px}^2+\eta_{pz}^2\ . \tag{B-6}$$

If we employ the dispersion relation ω=kv, equation (B-6) can be transformed as,

$$\omega=\frac{2\arcsin(\sqrt{(\eta_{px}^2+\eta_{pz}^2)v_p^2\Delta t^2/4})}{\Delta t}\ . \tag{B-7}$$

The P-wave numerical velocity is given by

$$v_{pnum}=\frac{2\arcsin(\sqrt{(\eta_{px}^2+\eta_{pz}^2)v_p^2\Delta t^2/4})}{\Delta tk}\ . \tag{B-8}$$

If ξ_p means the ratio of the numerical velocity to the real velocity for P-waves, then it follows

$$\xi_p=\frac{2\arcsin(\sqrt{(\eta_{px}^2+\eta_{pz}^2)v_p^2\Delta t^2/4})}{\Delta tkv_p}\ . \tag{B-9}$$

By inspection, it is clear that the velocity term is the only primary distinction between equation (28) and equation (30). Thus, the procedure of the deduction for the ratio of the numerical velocity to the real velocity for S-waves is almost the same to one of the P-wave. Therefore, we give the ratio of these two terms directly.

$$\xi_s=\frac{2\arcsin(\sqrt{(\eta_{sx}^2+\eta_{sz}^2)v_s^2\Delta t^2/4})}{\Delta tkv_s}\ , \tag{B-10}$$

where $\eta_{sx}=\sum_{l=1}^{L}G_x^s(\mathbf{x},l)\sin((2l-1)k_x\Delta h/2)$, $\eta_{sz}=\sum_{l=1}^{L}G_z^s(\mathbf{x},l)\sin((2l-1)k_z\Delta h/2)$.