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APPENDIX TO

Seismoelectric effect in Lamb's problem

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Let $M(\zeta)$ denotes the integrand in equation (19). The singular points of $M(\zeta)$ and paths of integration in a complex plane ζ are shown in Figure 4. These singularities are associated with the branch points $\zeta = \pm i\gamma$ and $\zeta = -qD/C_t$ of the functions s_2 and λ, respectively, as well as with zeros of the denominator at the points $\zeta = \pm i\xi$. Let us cut the complex plane through the branch points and then choose a Riemann surface sheet on which the real parts of the functions s_2 and λ are both positive. Since Jordan's lemma is valid for integral (19) one can transform the integration path shown in Figure 4 with line I into that shown with line II. In such a case, the longitudinal wave makes contribution to integral (19) due to the contour integral along the cut lines starting at the branch points of the function s_2. One more contribution to integral (19) is due to the contour integral along the cut line starting at the branch point of the function λ. The implication here is that this contribution is associated with the diffusion of the pore fluid pressure. These contours are indicated by numbers 3, 4 and 5 in Figure 3. It should be noted that the integrand has not a singularity at the point $\zeta = C_t\gamma^2/(qD)$ since both the denominator and numerator of the integrand are equal to zero at this point.

The pole residues contribute to the variation of pore fluid pressure caused by Rayleigh wave propagation. Taking into account that the poles of $M(\zeta)$ are determined by zeros of the function $R(\zeta)$ in the denominator and using a residue formula we obtain

$$\operatorname*{res}_{\zeta=i\xi} M(\zeta) + \operatorname*{res}_{\zeta=-i\xi} M(\zeta) = \frac{w(i\xi)}{R'(i\xi)} + \frac{w(-i\xi)}{R'(-i\xi)} = 2\,\mathrm{Re}\frac{w(i\xi)}{R'(i\xi)}, \tag{A1}$$

where

$$w(i\xi)=\frac{i\xi^2\left(2-\xi^2\right)\exp\left(iqtC_R-qr_0\right)}{\left(qD\xi+iC_t\gamma^2\right)}\times$$
$$\left[\exp\left\{-qz\left(1+\frac{iC_R}{qD}\right)^{1/2}\right\}-\exp\left\{-zq\left(1-\frac{\xi^2}{\gamma^2}\right)^{1/2}\right\}\right] \tag{A2}$$

and the derivative $R'(i\xi)$ is given by

$$R'(i\xi)=4i\xi\left[2-\xi^2-\frac{\left(1-\xi^2/\gamma^2\right)^{1/2}}{\left(1-\xi^2\right)^{1/2}}-\frac{\left(1-\xi^2\right)^{1/2}}{\gamma^2\left(1-\xi^2/\gamma^2\right)^{1/2}}\right]. \tag{A3}$$

Substituting equations (A1)–(A3) for the pole residues into equation (19) we come to equation (20).

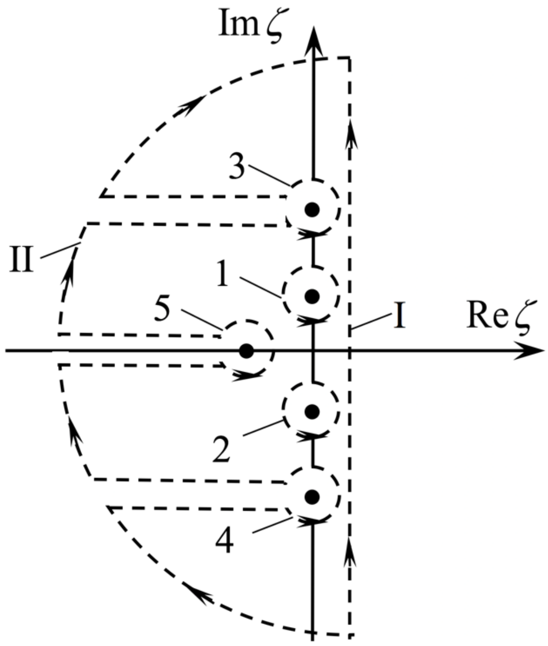

Figure 4. A schematic plot of the complex plane ζ, which contains the integration paths and the singular points of the integrand in equation (19). The initial and biased paths of integration are shown with lines I and II, respectively. Lines 1 and 2 correspond to the paths tracing around poles $\zeta=\pm i\xi$. The integration paths along bank of cuts and around the branch points $\zeta=\pm i\gamma$ and $\zeta=-qD/C_t$ are shown with lines 3, 4 and 5, respectively.

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