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APPENDIX TO

APPLICATION OF AN ENSEMBLE EARTHQUAKE RATE MODEL IN ITALY, CONSIDERING SEISMIC CATALOGS AND FAULT MOMENT RELEASE

Maura Murru*[,1], Giuseppe Falcone[1], Matteo Taroni[1] and Rodolfo Console[1,2]

[1] Istituto Nazionale di Geofisica e Vulcanologia, Roma, Italy
[2] Center of Integrated Geomorphology for the Mediterranean Area, CGIAM, Potenza, Italy

Appendix A: Estimation of the tapered *Gutenberg-Richter* distribution parameters with the Weichert (1980) method

We expand the Weichert (1980) method to estimate the annual rate, the *b*-value and the corner magnitude of the tapered *Gutenberg-Richter* (*G-R*) distribution. The original method uses a fixed maximum magnitude to estimate the annual rate and the *b*-value using an MLE approach. We are still using an MLE approach, but we simply use the likelihood as in the following equation:

$$L(X|\theta,\lambda_{TOT}) = \prod_{i=1}^{I} \frac{\left[\lambda_{TOT}\cdot Pr_i^{\theta}\cdot \mathrm{T}_i\right]^{N_i}}{N_i!}\, e^{-\lambda_{TOT}\cdot Pr_i^{\theta}\cdot \mathrm{T}_i} = \lambda_{TOT}{}^{N}\cdot e^{-\lambda_{TOT}\sum_{i=1}^{I}\left[Pr_i^{\theta}\cdot \mathrm{T}_i\right]} \prod_{i=1}^{I} \frac{\left[Pr_i^{\theta}\cdot \mathrm{T}_i\right]^{N_i}}{N_i!} \quad \textbf{(A1)}$$

where X is the dataset, i.e the number of events falling into each of the I-th magnitude bin N_i and the corresponding completeness time interval T_i, θ is the set of parameters describing the magnitude distribution used (in our case, the tapered, with the *b*-value and corner magnitude parameters), λ_{TOT} is the cumulative annual rate and $Pr_i^{\theta} = [F(M_{i+1},\theta) - F(M_i,\theta)]$ is the probability of having an event of magnitude between M_i and M_{i+1}, given the cumulative magnitude distributions F.

Essentially, compared to the original Weichert 1980 method, we simply substitute the cumulative distribution *F*, which was originally a truncated *G-R*, with a tapered *G-R* distribution. Once the distribution is changed, we have an additional parameter (corner magnitude) to estimate.

We apply this method to the declustered Italian combined catalog (CPTI15 + Instrumental), using a uniform temporal completeness for all the completeness sub-regions, excluding the "Outside zone", which has a temporal completeness too short compared to the other zones. In order to obtain a uniform

temporal completeness, we take for each threshold magnitude the most recent year (see Table S1 in the Supporting Material). Finally, the following results are obtained:

Table 1A. Parameters estimated for the declustered CPTI15 + Instrumental catalog with the modified Weichert method, considering HAC and SAC.

Parameters	HAC	SAC
λ_{TOT}	4.9	5.2
b-value	0.99	0.99
corner magnitude	7.3	7.3

For the events in the "Outside zone", we estimate the annual rate merely by computing the ratio between the number of events observed with $M_w \geq 4.45$ and the corresponding temporal completeness: 0.78 events per year are obtained for the Historical Analysis of the Completeness (HAC) and 0.82 events per year for the Statistical Analysis of the Completeness (SAC).

To estimate the uncertainties associated with the parameters of the tapered *G-R*, we use the Keller *et al*. (2014) approach, based on the Monte Carlo Markov Chain (MCMC) sampling. In Figure 1A we show the observed annual rate of events (HAC) used in the estimation and the 95% confidence intervals of the tapered *G-R* model; in Figures 2A, 3A and 4A we show the distribution of the estimated parameters (build with 10^5 samplings); finally in Figure 5A it is shown the scatter plot of cumulative annual rate λ_{TOT} and the *b*-value. Looking at the uncertainty distribution of the *b*-value, annual rate and corner magnitude parameters (Fig. 2A, 3A and 4A), it is clear that the corner magnitude is the parameter with the larger uncertainty: in fact, it strongly depends on the biggest (and rare) events in the catalog, and it is harder to constrain (Zöller and Holschneider, 2016). The scatter plot in Fig. 5A shows that the annual rate λ_{TOT} and the *b*-value parameters are correlated if we use the Weichert (1980) estimation approach: the inclination of the cloud point means a positive correlation between the two parameters.

Figure 1A also shows the difference between the uncertainties on annual rate and corner magnitude.

In the left part of the plot the 95% confidence interval bounds are close to the observed seismicity, i.e. the cumulative annual rate is well constrained. On the contrary in the right part of the plot the

95% confidence interval bounds are far from the observed seismicity, i.e. the corner magnitude estimation has a large uncertainty.

As final comment, we can also say that the Italian seismic catalog, that lasts about 1000 years, is temporally long enough to have a robust statistical estimation of the annual rate and the *b*-value, but it is inadequate to have a clear constrain of the corner magnitude of the tapered *G-R* distribution.

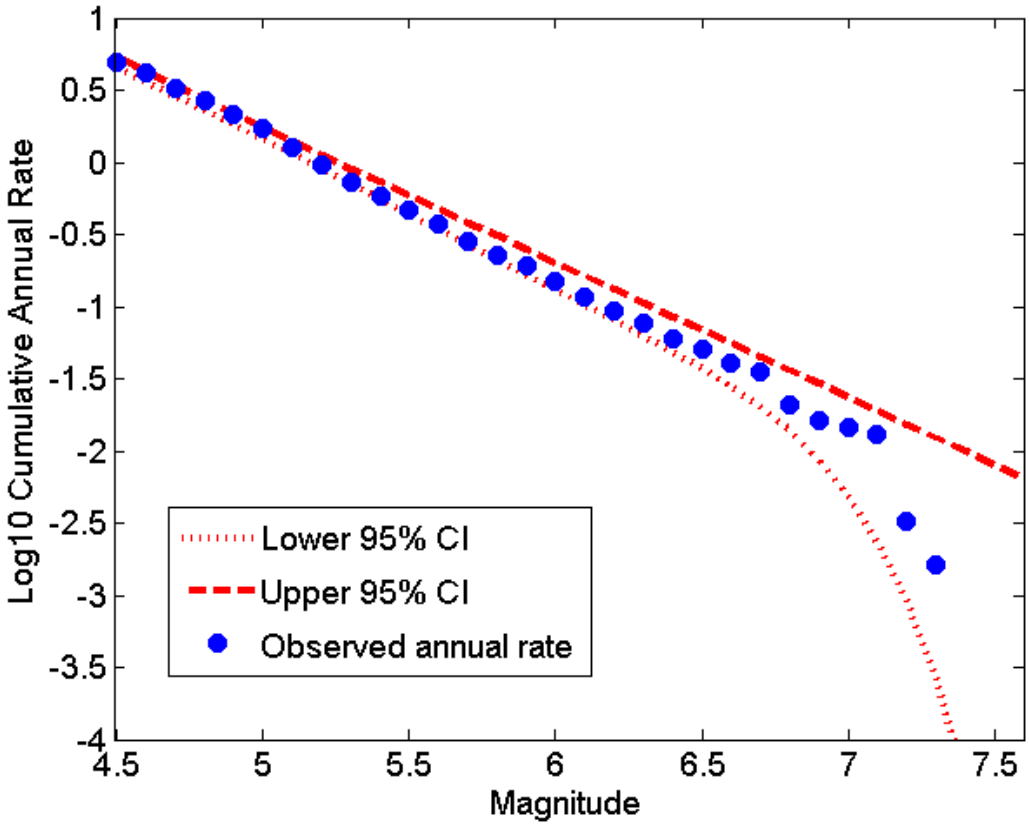

Figure 1A. Cumulative observed annual rate (blue dots) and 95% confidence intervals (upper and lower) of the estimated model (dashed red curves).

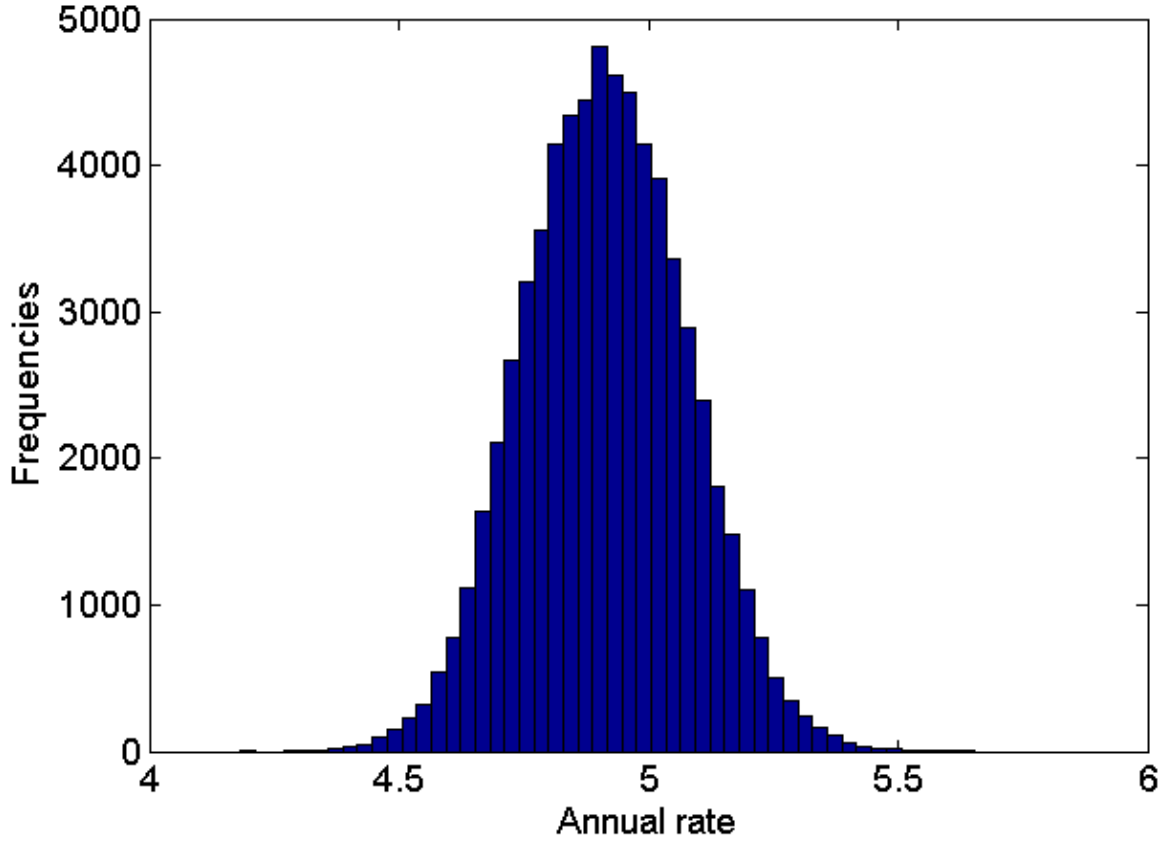

Figure 2A. Distribution of the annual rate ($\boldsymbol{\lambda_{TOT}}$), computed through the MCMC approach (10^5 samplings), for the Weichert (1980) estimation of the Tapered *G-R* parameters.

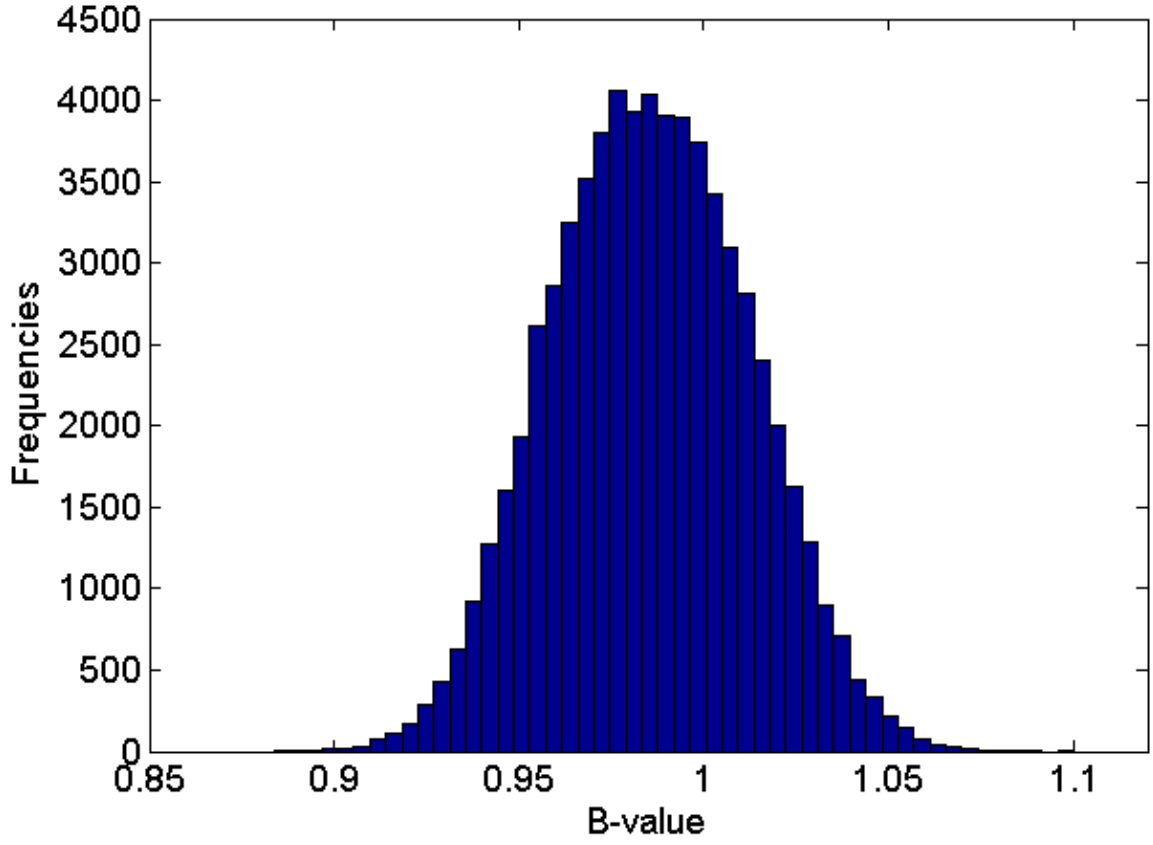

Figure 3A. Distribution of the *b*-value, computed through the MCMC approach (10^5 samplings), for the Weichert (1980) estimation of the Tapered *G-R* parameters.

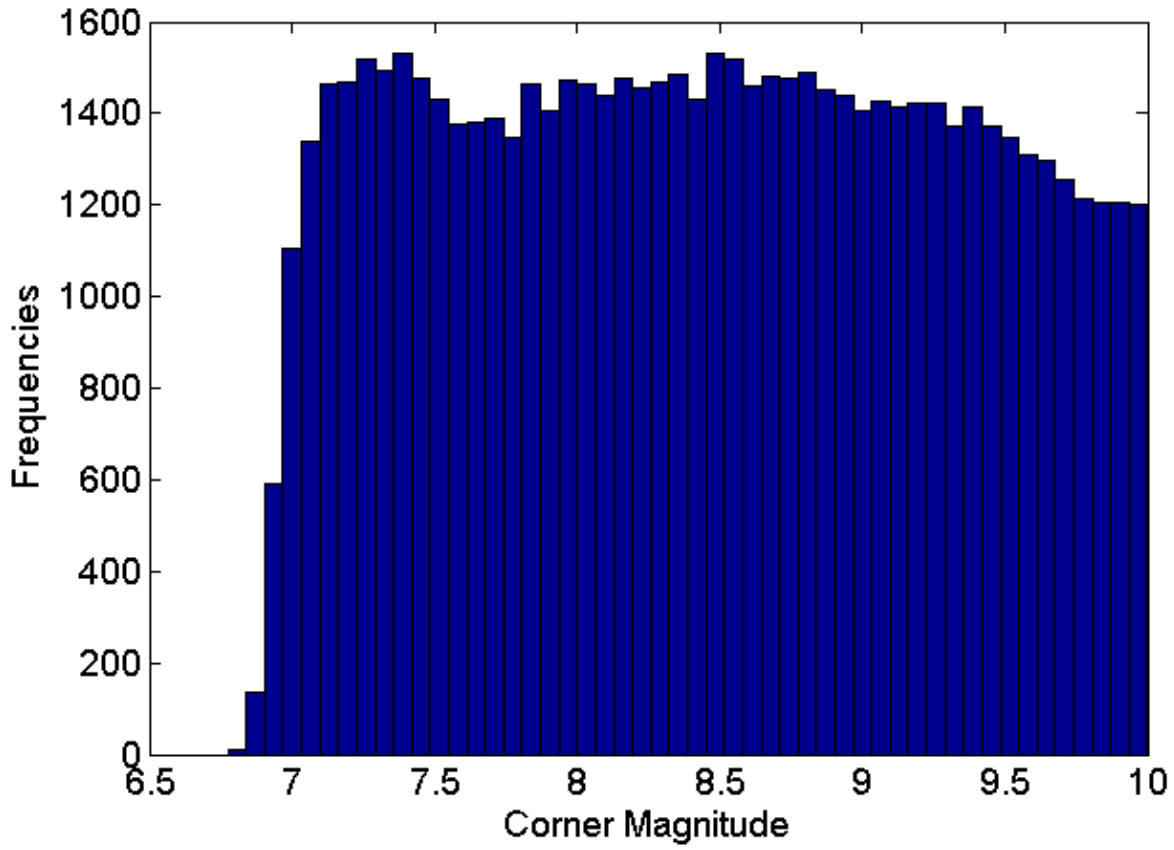

Figure 4A. Distribution of the corner magnitude, computed through the MCMC approach (10^5 samplings), for the Weichert (1980) estimation of the Tapered *G-R* parameters.

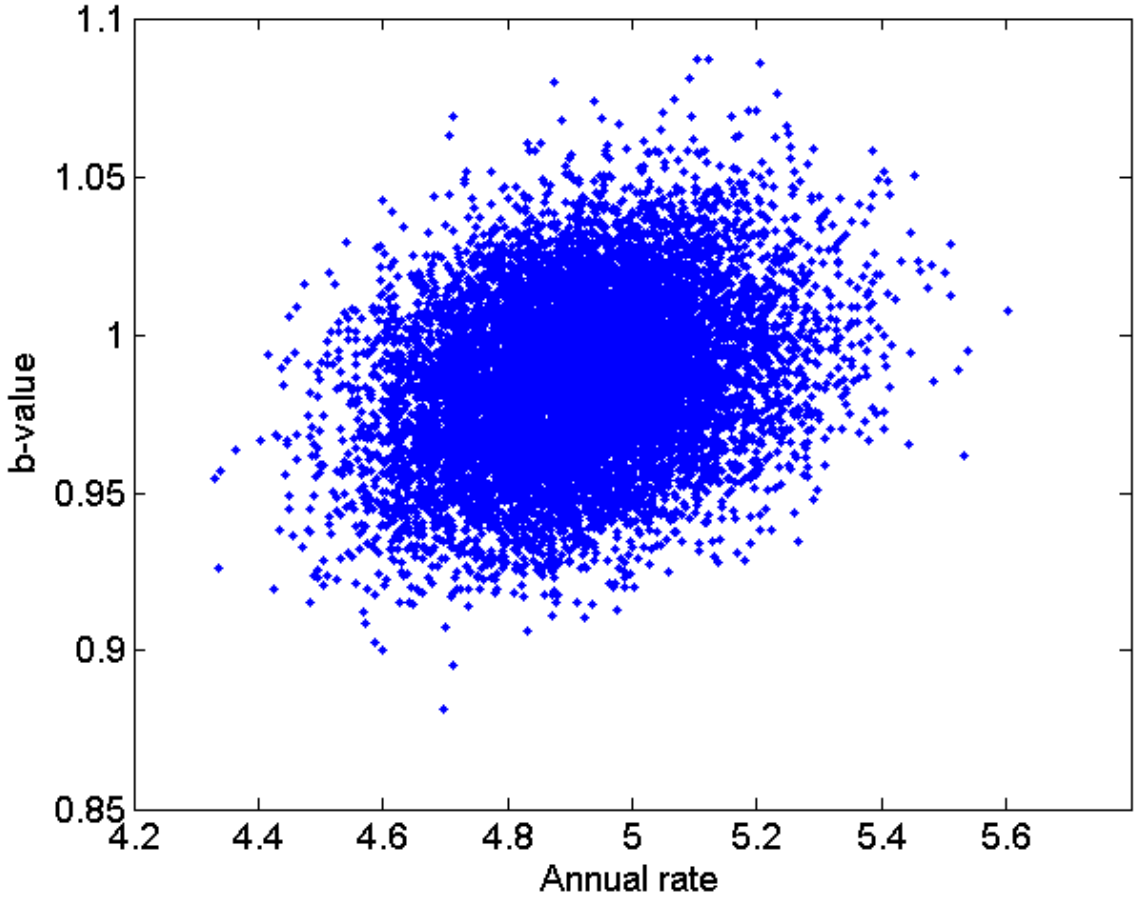

Figure 5A. Scatter plot of *b*-value and annual rate (λ_{TOT})**,** computed through the MCMC approach (10^5 samplings), for the Weichert (1980) estimation of the tapered *G-R* parameters.