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APPENDIX TO

ASSESSMENT OF NOISE IN INSAR TIMESERIES USING LEAST SQUARES VARIANCE COMPONENT ESTIMATION

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Implementation of multivariate LS-VCE model

Here, we consider the stochastic model $D\left(\text{vec}(\underline{Y})\right) = \Sigma \otimes \left(\sum_{k=1}^{p} \sigma_k Q_k\right)$ and the p-vector of unknown variance components σ_k are to be calculated using multivariate LS-VCE as follows:

$$\hat{\sigma} = N^{-1}\underline{l} \tag{A1}$$

Where entries normal matrix $N_{p\times p}$ elements and vector $\underline{l}_{p\times 1}$ are presented as

$$n_{kl} = \frac{r}{2} tr(Q^{-1}P_A^{\perp}Q_k Q^{-1}P_A^{\perp}Q_l) \tag{A2}$$

with the univariate orthogonal projector $P_A^{\perp} = I - A(A^{\mathrm{T}}Q^{-1}A)^{-1}A^{\mathrm{T}}Q^{-1}$, and

$$\underline{l}_k = \frac{m-n}{2} tr(\underline{\hat{E}}^T Q^{-1} Q_k Q^{-1} \underline{\hat{E}} (\underline{\hat{E}}^T Q^{-1} \underline{\hat{E}})^{-1}) \tag{A3}$$

Where $k, l = (1, 2, \cdots, p)$, $\underline{\hat{E}}_{m\times r} = (\underline{\hat{e}}_1, \underline{\hat{e}}_2, \cdots, \underline{\hat{e}}_r)$ are the matrix of estimated residuals (e.g., $\underline{\hat{e}}_i = P_A^{\perp}\underline{y}_i$ that $\underline{y}_i$ is the m-vector of observables for timeseries i), and the inverse of the normal matrix N^{-1} naturally gives the variance-covariance matrix of the estimated variance components (i.e., $Q_{\hat{\sigma}} = N^{-1}$).

The unknown variance components σ_k should be estimated through an iterative procedure. Therefore, the estimated unknown variance components are updated after every iteration until their changes are less than a defined threshold (i.e., $\|\hat{\sigma}^i - \hat{\sigma}^{i-1}\| \prec \varepsilon$) [Amiri-Simkooei 2009].