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APPENDIX TO

TESTING OBSERVABLES FOR TELESEISMIC SHEAR-WAVE SPLITTING INVERSIONS: AMBIGUITIES OF INTENSITIES, PARAMETERS, AND WAVEFORMS

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A1. Apparent splitting parameters

To derive expressions for the apparent splitting parameters, we rewrite the splitting equation (10) in terms of radial and transverse displacement components. In view of Fig. 1a, with the angular difference $\delta\varphi = \beta - \varphi$ between back-azimuth and fast axis, we obtain

$$\begin{pmatrix} u^{(r)} \\ u^{(t)} \end{pmatrix} = \begin{pmatrix} -\cos\delta\varphi & \sin\delta\varphi \\ -\sin\delta\varphi & -\cos\delta\varphi \end{pmatrix} \begin{pmatrix} \mathrm{e}^{+i\omega\delta t/2} & 0 \\ 0 & \mathrm{e}^{-i\omega\delta t/2} \end{pmatrix} \begin{pmatrix} -\cos\delta\varphi & -\sin\delta\varphi \\ \sin\delta\varphi & -\cos\delta\varphi \end{pmatrix} \begin{pmatrix} u_0^{(r)} \\ u_0^{(t)} \end{pmatrix}. \tag{A1}$$

For XKS phases in a radially symmetric Earth, it is usually assumed that $u_0^{(t)} = 0$ upon entering the anisotropic domain in the upper mantle beneath the receiver. Multiplication of the matrices yields

$$\begin{pmatrix} u^{(r)} \\ u^{(t)} \end{pmatrix} = \begin{pmatrix} \cos\theta + i\sin\theta\cos 2\,\delta\varphi & i\sin\theta\sin 2\,\delta\varphi \\ i\sin\theta\sin 2\,\delta\varphi & \cos\theta - i\sin\theta\cos 2\,\delta\varphi \end{pmatrix} \begin{pmatrix} u_0^{(r)} \\ u_0^{(t)} \end{pmatrix}, \tag{A2}$$

where $\theta = \omega\delta t/2$. Assuming an unsplit incoming wave with $u_0^{(r)} \sim \mathrm{e}^{i\omega t}$ and $u_0^{(t)} = 0$, at relatively long periods $\delta t \ll T$, such that $\cos\theta \approx 1$ and $\sin\theta \approx \omega\delta t/2$, the radial component is unchanged, and the transverse component can be expressed by

$$u^{(t)} \sim i\omega\frac{\delta t}{2}\sin 2\,\delta\varphi \tag{A2a}$$

which, in the time domain, corresponds to the derivative of the radial component multiplied by $(\delta t/2)\sin(2\,\delta\varphi)$. This result is also used in the approximate representation of the splitting intensity (see eq. 21). Note that the opposite sign results from the definition of the transverse component used here.

In short eq. (A2) can be written as

$$\boldsymbol{u}^{(r,t)} = \boldsymbol{S}^{(r,t)}(\varphi, \delta t)\ \boldsymbol{u}_0^{(r,t)}. \tag{A3}$$

For two anisotropic layers, we have

$$\boldsymbol{u}^{(r,t)} = \boldsymbol{S}_2^{(r,t)}(\varphi_2, \delta t_2)\ \ \boldsymbol{S}_1^{(r,t)}(\varphi_1, \delta t_1)\ \boldsymbol{u}_0^{(r,t)} = \boldsymbol{S}_{2,1}^{(r,t)}\ \boldsymbol{u}_0^{(r,t)}. \tag{A4}$$

To derive apparent splitting parameters, we define an apparent splitting matrix $\boldsymbol{S}_a^{(r,t)}$ similar to eq. (A2) and equate

$$\boldsymbol{S}_{2,1}^{(r,t)} = k\boldsymbol{S}_a^{(r,t)}(\varphi_a, \delta t_a), \tag{A5}$$

$$\Leftrightarrow \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} = k \begin{pmatrix} \cos\theta_a + i\sin\theta_a \cos 2\,\delta\varphi_a & i\sin\theta_a \sin 2\,\delta\varphi_a \\ i\sin\theta_a \sin 2\,\delta\varphi_a & \cos\theta_a - i\sin\theta_a \cos 2\,\delta\varphi_a \end{pmatrix}, \tag{A6}$$

where k is a complex number that allows for an arbitrary time shift and φ_a and δt_a (as obtained from $\delta\varphi_a$ and θ_a) are apparent splitting parameters as first defined by Silver & Savage (1994). Note that $S_{22} = S_{11}^*$ and $S_{21} = -S_{12}^*$, where the * denotes the complex conjugate (Rümpker & Silver 1998). In view of (A3) and assuming $u_0^{(t)} = 0$, elimination of k yields two equations

$$\tan\theta_a = \frac{S_{21}^I}{S_{11}^R \sin(2\delta\varphi_a) - S_{21}^R \cos(2\delta\varphi_a)} = \frac{S_{21}^R}{S_{21}^I \cos(2\delta\varphi_a) - S_{11}^I \sin(2\delta\varphi_a)}, \tag{A7}$$

from which we obtain

$$\tan(2\delta\varphi_a) = \frac{(S_{21}^R)^2 + (S_{21}^I)^2}{S_{11}^R S_{21}^R + S_{11}^I S_{21}^I}. \tag{A8}$$

A2. Elastic constants for the laterally and vertically varying medium

For the modeling presented in this paper we start with the generic elastic constants for a general transversely isotropy medium with a vertical (fast) axis of symmetry, as defined in Rümpker & Kendall (2002), which allows for a convenient scaling of the elastic constants

$$c_{IJ} = \begin{pmatrix} c_{11} & c_{11} - 2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11} - 2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix}. \tag{A9}$$

The density normalized elastic constants are given by

$$\begin{gathered} c_{11} = \left(v_P - \frac{1}{2}a_1 v_P\right)^2, \quad c_{33} = \left(v_P + \frac{1}{2}a_1 v_P\right)^2, \\ c_{44} = \left(v_S + \frac{1}{2}a_1 v_S\right)^2, \quad c_{66} = \left(v_S - \frac{1}{2}a_1 v_S\right)^2, \\ c_{13} = (1 + a_1 a_0)(v_P^2 - 2v_S^2). \end{gathered} \tag{A10}$$

We then apply a rotation such that the (fast) symmetry axis is oriented horizontally, at first along the 2-direction, which corresponds to geographic North in our modeling (back-azimuth $\beta = 0$). A second rotation (with respect to the vertical axis) is applied to align the fast axis with the specific value of φ in the anisotropic domain. The 1-direction is oriented horizontally along the profile (corresponding to geographic East) and the 3-direction is oriented vertically.

The curvature parameter a_0 (eq. A10) is set to –1. The strength of the anisotropy is controlled by the parameter a_1, according to $a_1 = v_S \delta t / \Delta z$, such that vertically propagating fast and slow shear waves accumulate the required delay time, δt, over the total thickness ($\Delta z = 100$ km) of the anisotropic domain. Note that our simplified formulation (A10) implies that the strength of anisotropy is the same for both P and S waves. The bulk isotropic P and S-wave velocities within each domain are the same as in the isotropic ($a_1 = 0$) section of the mantle ($v_P = 8.3$ km/s, $v_S = 4.5$ km/s). For the isotropic crust, we assume $v_P = 6.2$ km/s, $v_S = 3.5$ km/s.