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APPENDIX TO

ON THE RELEVANCE OF THE FORESHOCKS IN FORECASTING SEISMIC MAINSHOCKS

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1. A: Geometric-growth model of energy accumulation in focus

In Apostol [2006] a typical earthquake is considered, with a small focal region localized in the solid crust of the Earth. The dimension of the focal region is so small in comparison to our distance scale, that we may approximate the focal region by a point in an elastic body. The movement of the tectonic plates may lead to energy accumulation in this point-like focus. The energy accumulation in the focus is governed by the continuity equation (energy conservation)

$$\frac{\partial E}{\partial t} = -\boldsymbol{v} gradE \tag{A1}$$

where E is the energy, t denotes the time and $\boldsymbol{v}$ is an accumulation velocity. For such a localized focus we may replace the derivatives in equation (A1) by ratios of small, finite differences. For instance, we replace $\partial E/\partial x$ by $\Delta E/\Delta x$ for the coordinate x, etc. Moreover, we assume that the energy tends to zero at the borders of the focus, such that $\Delta E = -E$, where E is the energy in the centre of the focus. Also, we assume a uniform variation of the coordinates of the borders of this small focal region, given by equations of the type $\Delta x = u_x t$, where $\boldsymbol{u}$ is a small displacement velocity of the medium in the focal region. The energy accumulated in the focus is gathered from the outer region of the focus, as expected. With these assumptions equation (A1) becomes

$$\frac{\partial E}{\partial t} = \left(\frac{v_x}{u_x} + \frac{v_y}{u_y} + \frac{v_z}{u_z}\right)\frac{E}{t} \tag{A2}$$

Let us assume an isotropic motion without energy loss; then, the two velocities are equal, $\boldsymbol{v} = \boldsymbol{u}$, and the bracket in equation (A2) acquires the value 3. In the opposite limit, we assume a one-dimensional motion. In this case the bracket in equation (A2) is equal to unity. A similar analysis holds for a two dimensional accumulation process. In general, we may write equation (A2) as

$$\frac{\partial E}{\partial t} = \frac{1E}{rt} \tag{A3}$$

where r is an empirical (statistical) parameter; we expect it to vary approximately in the range (1/3,1). We note that equation (A3) is a non-linear relationship between t and E. The parameter r may give an insight into the geometry of the focal region. Also, it reflects the structural condition of the focal region, by the relation between the two velocities $\boldsymbol{v}$ and $\boldsymbol{u}$. We call this model a geometric-growth model of energy accumulation in the focal region.

It is shown in Appendix B that the parameter r is related to the Gutenberg-Richter parameter β and the Hanks-Kanamori constant $b = 3.45$ ($3/2$ in decimal logarithms) through $\beta = br$.

A special attention is given to shearing faults, which are typical earthquake sources. The energy accumulation takes place along one direction, say $u_x = v_x$, but the mass conservation requires, on the average, a motion in opposite directions along, say, the perpendicular y-axis [Apostol, 2019]. This makes $u_y = 2v_y$ (2 from the two opposite directions), which, together with $u_z = 0$, leads to $r = 2/3$. Indeed, this is the mean value of the ratio $r = \beta/b$, accepted as reference value ($\beta = 2.3$, $b = 3.45$, $r = 2/3$, see the main text).

The integration of equation (A3) needs a cutoff (threshold) energy and a cutoff (threshold) time. During a short time t_0 a small energy E_0 is accumulated. In the next short interval of time this energy may be lost, by a relaxation of the focal region. Consequently, such processes are always present in a focal region, although they may not lead to an energy accumulation in the focus. We call them fundamental processes (or fundamental earthquakes, or E_0-seismic events). It follows that we must include them in the accumulation process, such that we measure the energy from E_0 and the time from t_0. The integration of equation (A3) leads to the law of energy accumulation in the focus

$$t/t_0 = (E/E_0)^r \tag{A4}$$

The time t in this equation is the time needed for the accumulation of the energy E, which may be released in an earthquake (the accumulation time). This is the time-energy accumulation equation referred to in the main text.

2. B: Gutenberg-Richter law. Time probability

The well-known Hanks-Kanamori law reads

$$\ln \overline{M} = const + bM \tag{B1}$$

where $\overline{M}$ is the seismic moment, M is the moment magnitude and $b = 3.45$ ($3/2$ for base 10). In Apostol [2019] the relation $\overline{M} = 2\sqrt{2}E$ has been established, where $\overline{M} = \left(\Sigma_{ij} M_{ij}^2\right)^{1/2}$ (mean seismic moment), M_{ij} is the tensor of the seismic moment and E is the energy of the earthquake. If we identify the mean seismic moment with $\overline{M}$ we can write

$$\ln E = const + bM \tag{B2}$$

(another *const*), or

$$E/_{E_0} = e^{bM} \tag{B3}$$

where E_0 is a threshold energy (related to *const*). Making use of equation (A4), we get

$$t = t_0 e^{brM} = t_0 e^{\beta M} \tag{B4}$$

where $\beta = br$. From this equation we derive the useful relation $dt = \beta t_0 e^{\beta M} dM$, or $dt = \beta t dM$. If we assume that the earthquakes are distributed according to the well-known Gutenberg-Richter distribution,

$$dP = \beta e^{-\beta M} dM \tag{B5}$$

we get the distribution

$$dP = \beta \frac{t_0}{t} \frac{1}{\beta t} dt = \frac{t_0}{t_2} dt \tag{B6}$$

[Apostol, 2006]. This law shows that the probability for an earthquake to occur between t and $t + dt$ is $\frac{t_0}{t^2} dt$; since the accumulation time is t, the earthquake has an energy E and a magnitude M given by the above formulae (equations (B2) and (B3)). The law given by the equation (B5) is also derived [Apostol, 2021] from the definition of the probability of the fundamental E_0-seismic events ($dP = -\frac{\partial}{\partial t} \frac{t_0}{t} dt$). We note that this probability assumes independent earthquakes.

3. C: Time-time correlations

In general, if two earthquakes are mutually affected by various conditions, and such an influence is reflected in the above equations, we say that they are correlated to each other. Of course, multiple correlations may exist, *i.e.* correlations between three, four, etc. earthquakes. We limit ourselves to two-earthquake (pair) correlations. Very likely, correlated earthquakes occur in the same seismic region and in relatively short intervals of time. The physical causes of mutual influence of two earthquakes are various. In Apostol [2021] three types of earthquake correlations are identified. In one type the neighbouring focal regions may share energy. Since the energy accumulation law is non-linear, this energy sharing affects the occurrence time. We call these correlations time-magnitude correlations (or energy-energy correlations), as described in the main text. They are a particular type of dynamical correlations. In a second type of correlations, to be described below, two earthquakes may share their accumulation time, which affects their total energy. We call such correlations time-time, or purely dynamical correlations. Both these correlations affect the earthquake statistical distributions; in this respect, they are also statistical correlations. Finally, additional constraints on the statistical variables (*e.g.*, the magnitude of the accompanying seismic event be smaller than the magnitude of the main shock) give rise to purely statistical correlations.

Let us assume that an earthquake occurs in time t_1 and another earthquake follows in time t_2. The total time is $t = t_1 + t_2$, such that these earthquakes share their accumulation time, which affects their total energy. These are time-time (or purely dynamical) correlations. According to equation (5) (and the definition of the probability), the probability density of such an event can be obtained

$$-\frac{\partial}{\partial t_2} \frac{t_0}{(t_1 + t_2)^2} = \frac{2t_0}{(t_1 + t_2)^3} \tag{C1}$$

(where $t_0 < t_1 < +\infty, 0 < t_2 < +\infty$). By passing to magnitude distributions ($t_{1,2} = t_0 e^{\beta M_{1,2}}$), we get

$$d^2P = 4\beta^2 \frac{e^{\beta(M_1+M_2)}}{(e^{\beta M_1} + e^{\beta M_2})^3} dM_1 dM_2 \tag{C2}$$

(where $0 < M_{1,2} < +\infty$, corresponding to $t_0 < t_{1,2} < +\infty$, which introduces a factor 2 in equation (C1)). This formula (which is a pair, bivariate statistical distribution) is established in Apostol [2021]. (We note that there is no restriction upon M_2 in comparison with M_1, in contrast to the time-magnitude correlations). If we integrate equation (C2) with respect to M_2, we get the distribution of a correlated earthquake (marginal distribution)

$$dP = \beta e^{-\beta M_1} \frac{2}{(1 + e^{-\beta M_1})^2} dM_1 \tag{C3}$$

If we integrate further this distribution from $M_1 = M$ to $+\infty$, we get the correlated cumulative distribution

$$P(M) = \int_M^\infty dP = e^{-\beta M} \frac{2}{1 + e^{-\beta M}} \tag{C4}$$

From $M \gg 1$ the correlated distribution becomes $P(M) \simeq 2e^{-\beta M}$ and $\ln P(M) \simeq \ln 2 - \beta M$, which shows that the slope β of the logarithm of the independent cumulative distribution (Gutenberg-Richter, standard distribution $e^{-\beta M}$) is not changed (for large magnitudes); the correlated distribution is only shifted upwards by ln 2. On the contrary, for small magnitudes ($M \ll 1$) the slope of the correlated distribution becomes $\beta/2$ ($P(M) \simeq 1 - \frac{1}{2}\beta M + \cdots$ by a series expansion of equation (C4)), instead of the slope of the Gutenberg-Richter distribution ($e^{-\beta M} \simeq 1 - \beta M + \cdots$). The time-time correlations modify the slope of the Gutenberg-Richter standard distribution for small magnitudes. This is the roll-off effect referred to in the main text.