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A semi-empirical model for magnetic storm dynamics

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Abstract

The near-Earth electromagnetic environment represents a far-from-equilibrium system characterized by sudden irregular energy relaxation events. For a broad class of complex systems, time series can be interpreted in terms of a superposition of stochastic and deterministic components occurring at different time scales. In this work we use the generalization of the SYM-H index provided by the SuperMAG collaboration (SMR), which is meant for monitoring the global variation of the horizontal component of the Earth's magnetic field in the near-equatorial regions. The aim of this work is to model the SMR dynamics via stochastic differential equations thus providing a semi-empirical model whose parameters are retained from data. As a first step we test the Markov condition on the SMR data sample, which represents the basic condition for our stochastic modeling, and we show that such a requirement is accurately satisfied by SMR time series. This allows us to infer the model parameters for the SMR index through the Kramers–Moyal analysis. Finally, we give evidence that a purely diffusive process is not representative of the observed dynamics and then a model based on jump-diffusion processes must be considered to correctly reproduce the dynamical features of the SMR index.

Keywords: Magnetospheric dynamics; Geomagnetic indices; Markov processes; Stochastic differential equations; Complex timeseries reconstruction

1. Introduction

Earth's magnetosphere and its current systems respond to the continuously changing solar wind conditions and constitute a complex system characterized by nonlinear interactions between components evolving on different spatiotemporal scales [Baker et al., 1990; Klimas et al., 1996]. In this context, magnetospheric substorms and magnetic storms are not simply a passive response to changing interplanetary medium conditions, but the Earth's magnetospheric system displays a rich and nonlinear dynamics showing features of chaotic and critical systems [Tsurutani et al., 1990; Vassiliadis et al., 1990; Consolini et al., 1996, 1997, 2002a; Uritsky et al., 2002; Alberti et al., 2018]. The dynamical state of the magnetosphere is then defined by the interplay between externally driven processes, associated with solar wind changes, and internal processes [Stumpo et al., 2023], comprising reorganization and relaxation events, occurring on different spatial and temporal scales [e.g., Consolini et al., 2001, 2002b; Alberti et al., 2017]. For all these reasons, magnetic storm modeling represents a challenging and fundamental task in the context of space weather science [see Balasis et al., 2023 and references therein]. In fact, the most intense storm events are generally caused by the passage in the near-Earth environment of the interplanetary counterpart of coronal mass ejections (CMEs) ejected from the solar corona [see Pulkkinen, 2007; and references therein]. These events are commonly accompanied by an interplanetary shock and may also be responsible for high-energy particle acceleration thus producing radiation storms at the Earth [see Ryan et al., 2000, and references therein]. Furthermore, if the magnetic field associated with the CME has a southward component, the interplanetary plasma can enter the magnetosphere intensifying its current systems. In particular, the intensification of the ring current circulating in the magnetospheric equatorial plane is responsible for the occurrence of geomagnetic storms, usually monitored through the SYM-H geomagnetic index [Tsurutani et al., 1997]. In such a framework, good understanding and reliable modeling of dynamical features of the SYM-H index constitute a prerequisite for investigation and prediction of magnetic storms.

A plethora of different models of geomagnetic index dynamics have been introduced since the late 50s up to now and we can classify them in two classes. The first class is represented by physics-based models, which are generally deterministic models which seek coupling between magnetospheric dynamics and the external driver, i.e., solar wind. The second class is represented by statistical-based models, which aim to provide synthetic time series of several geomagnetic indices and magnetic storm predictions, based on the main statistical features of the observed dynamics.

Concerning physics-based models, several attempts were made to reproduce the disturbance storm time (Dst) index [Sugiura, 1964], i.e., the original hourly proxy of ring-current enhancement. Dessler and Parker [1959] and Sckopke [1966] introduced a linear relation between the disturbance field generated during magnetic storms and the enhancement of ring current. Afterward, starting from considerations about the evolution of the total kinetic energy of ring-current particles, $E(t)$, Burton et al. [1975] derived the following equation:

$$\frac{d}{dt}Dst^{*} = Q(t) - \frac{1}{\tau}Dst^{*}(t) \tag{1}$$

where Dst^{*} indicates the corrected Dst index after subtracting contributions from other magnetospheric current systems, $Q(t)$ is an injection term and τ is a characteristic relaxation time, which, according to O'Brien and McPherron [2000], can be related to the charge-exchange lifetimes, so that is a function of time depending on hydrogen density in the geocorona. Both the expansion and the recovery times depend on the injection term $Q(t)$, which can be one of the solar wind-magnetosphere coupling functions, so that the interplanetary conditions completely determine the long-timescale evolution of magnetic storms. For instance, O'Brien and McPherron [2000] used as coupling function the product between the solar wind velocity and the south component of the interplanetary magnetic field $V_{IMF}B_S$. Other coupling functions can be used as described in Gonzalez et al. [1994]. Later Vassiliadis et al. [1999a, b] proposed an ARMA model for the evolution of Dst^{*} [Klimas et al., 1997].

The modeling of Dst index dynamics through these models provides a quite accurate description of the dynamics of this ring-current index. However, as shown by O'Brien and McPherron [2000] the deviations of the deterministic solutions of the previous equations from Dst are not a simple Gaussian noise, thus suggesting that magnetospheric state is characterized by much more complex dynamics. With reference to the complex dynamics in early 2000s Wanliss and colleagues [Wanliss, 2004, 2005; Wanliss and Dobias, 2007; Wanliss and Weygand, 2007; Wanliss and Uritsky, 2010; Wanliss et al., 2005] investigated the nonlinear character of 1-minute version of Dst index: the SYM-H index, which is a proxy of the symmetric part of the ring-current [Iyemori, 1990]. The studies evidenced that the SYM-H index show scale-invariant features [Wanliss, 2004; Balasis et al., 2006; Balasis et al., 2011; Alberti et al., 2021].

Alternative approaches to physical models are based on statistical properties of the geomagnetic dynamics and are of fundamental importance to improve our capabilities in modeling such complex dynamics. For instance, statistical models based on neural networks have shown great effectiveness in reproducing and predicting the main features of equatorial indices such as Dst and SYM-H, by using different sets of solar wind observations as input variables [Pallocchia et al., 2006; Bhaskar et al., 2019; Siciliano et al., 2021; Cristoforetti et al., 2022]. All these models are based on the training of neural networks on interplanetary magnetic field and plasma measurements and sometimes involving the autoregressive part of the geomagnetic index itself. As shown in numerous works, such models can reproduce the time profile of Dst or SYM-H [Pallocchia et al., 2006; Siciliano et al., 2021] indices and predict the occurrence of geomagnetic storms with remarkable performances.

Another class of statistical models which is powerful in reproducing reliable features of magnetospheric activity is represented by stochastic differential equations (SDEs). This approach has been adopted especially in high-latitude index modeling, e.g., the auroral electrojet indices AE, AL, and AU. Hnat et al. [2005], for instance, introduced a model based on the Langevin equation of auroral index fluctuations suggesting that they are driven by solar wind up to time scales of about 4 hours. A similar approach was also adopted by Rypdal and Rypdal [2010] in modeling the fluctuations of AE, pointing out that a multifractal model firstly introduced by Consolini et al. [1996] is more accurate in reproducing the scaling properties of the AE index fluctuations instead of the α-stable motion (also called fractional Lévy flight) proposed by Watkins et al. [2005]. A great advantage of SDE-based models is that, in principle, they can be implemented considering only the geomagnetic index itself without any knowledge of the external driving state. This aspect is of great relevance for forecasting purposes. In this context a complete example of SDE-based modeling of the AE index has been introduced by Pulkkinen et al. [2006] using a semi-empirical approach based on the knowledge of the AE index itself and relying on a generalized Langevin equation. Drift and diffusion terms of the generalized Langevin equation are then estimated from data and the synthetic time series obtained by integrating the SDE are satisfactorily in reproducing the main features of the AE index dynamics. However, it has been recently pointed out that the Fokker-Planck equation associated with the Langevin process represents only a rough approximation of the master equation governing this stochastic process. A systematic analysis of high-order statistics, in fact, highlights how terms higher than diffusion are not negligible in the Kramers-Moyal (KM) expansion [see Benella et al., 2022 for details].

As it is evident from this brief review, based on our knowledge, most of the efforts made in SDE modeling of the magnetospheric activity have been aimed at simulating high-latitude indices. Since an equivalent and extensive study of the near-equatorial magnetospheric dynamics has not been developed yet, the main purpose of this work is to bridge the gap by performing a statistical analysis of the dynamical properties of the generalization of the SYM-H index provided by the SuperMAG collaboration (SMR) by using a historical data sample and then by developing a semi-empirical model of the SMR index based on SDEs. The work is organized as follows: Section 2 introduces the methods for complex time series reconstruction adopted in this work, Section 3 describes data and results obtained in the case of SMR modeling. In Section 4, we present the main conclusions and discuss future perspectives.

2. Methods

Methods based on SDEs are powerful in reproducing both qualitative and quantitative features of “real world” time series for several reasons. First, the parameters of a stochastic model can be inferred from data with general procedures which allow us to disentangle the deterministic part of the dynamics from the stochastic contribution. SDE models have been used in many fields such as turbulence, surface science, finance, economy, neuroscience, and so forth (see Friedrich et al. 2011, for a comprehensive review on this topic). A paradigmatic example of SDE modeling is the generalized Langevin equation:

$$dx_t = -U'(x)dt + b(x)dW_t \tag{2}$$

which models the motion of a random walker within a potential well $U(x)$, described through a Wiener process W_t and a diffusion term $b(x)$, which depend on x. Such a stochastic process constitutes a Markov process since information about past dynamics, necessary for reaching the state x at the time t_n, is fully contained in the previous time step t_{n-1}. From the mathematical point of view, the Markov property is expressed in terms of the n-points transition probability of the process $x(t)$ as

$$p[x_n, t + n\tau | x_{n-1}, t + (n-1)\tau; \dots; x_0, t] = p[x_n, t + n\tau | x_{n-1}, t + (n-1)\tau]. \tag{3}$$

Eq. (3) implies that knowing the initial distribution and the transition probabilities we can define the complete n-point transition probability distribution function (PDF) of the process. For a process satisfying Eq. (3), the Chapman-Kolmogorov equation (CKE) have the following expression:

$$p(x_2, t + 2\tau | x_0, t) = \int_{-\infty}^{+\infty} p(x_2, t + 2\tau | x_1, t + \tau) p(x_1, t + \tau | x_0, t) dx_1. \tag{4}$$

This equation expresses the probability of observing the state x_2 at time $t + 2\tau$ given the previous observation of x_0 at time t for a Markov process. The differential form of Eq. (4) is called *master equation*, and it expresses the time evolution of the PDF. The master equation can be expressed through the KM expansion as [Risken, 1996]

$$\frac{\partial}{\partial t} p(x, t + \tau | x_0, t) = L_{KM}(x) p(x, t + \tau | x_0, t), \tag{5}$$

where L_{KM} is the KM operator which includes all the k-order KM coefficients $D^{(k)}(x)$

$$L_{KM}(x) = \sum_{k=1}^{\infty} \left(-\frac{\partial}{\partial x}\right)^k D^{(k)}(x), \tag{6}$$

and the KM coefficients are defined in terms of the conditional moments as follows:

$$D^{(k)}(x) = \lim_{\tau \to 0} \frac{1}{\tau} \langle [x(t + \tau) - x(t)]^k \rangle |_{x(\mathrm{t}) = x}. \tag{7}$$

It can be shown that for the continuous diffusion process introduced in Eq. (2) all the KM coefficients with $k \geq 3$ vanish and the KM expansion reduces to the Fokker-Planck equation [Risken, 1996]. Processes that exhibit non-vanishing higher-order KM coefficients are generally characterized by a greater degree of complexity in the stochastic fluctuations, which, instead of being continuous, may exhibit discontinuities and jumps. The simplest approach to incorporate these effects in the model is by superimposing a jump process onto the Langevin dynamics, i.e.,

$$dx_t = a(x)dt + b(x)dW_t + \xi dJ_t \tag{8}$$

where $a(x)$ is the drift term, $b(x)$ is the diffusion term of the Wiener process W_t, and ξ is the jump size of the Poisson jump process J_t [Tabar, 2019]. In this case, the jump process is fully described by two parameters, i.e., the jump size variance $\sigma^2{}_\xi(x)$ and the jump rate $\lambda(x)$, which can be estimated non-parametrically from the data by calculating high-order KM coefficients:

$$\sigma^2{}_\xi(x) = \frac{6D^{(6)}(x)}{D^{(4)}(x)}, \qquad \lambda(x) = \frac{8D^{(4)}(x)}{\sigma^4{}_\xi(x)}. \tag{9}$$

Such a framework can be generalized to multivariate processes [Tabar, 2019]. For instance, let us introduce two different stochastic processes x_1 and x_2, the evolution equation for the coupled SDEs can be written in vectorial form as follows:

$$dx = Adt + DdW + \Xi dJ. \tag{10}$$

In Eq. (10) $x = (x_1, x_2)$ is the state vector, $A = \big(a_1(x), a_2(x)\big)$ indicates the drift vector, D and Ξ are the diffusion and jump matrices and $W = \big(W_1(t), W_2(t)\big)$ and $J = \big(J_1(t), J_2(t)\big)$ indicate the bivariate Wiener and Poisson processes, respectively. In the spirit of the SDE approach, all the parameters of the diffusion and jump matrices may depend on x and the Wiener and Poisson processes are assumed to be independent and uncorrelated in time. For a bivariate process, diffusion and jump matrices contain eight unknown variables, to which are added two stochastic unknown parameters, i.e., the jump rates $\lambda_1(x)$ and $\lambda_2(x)$, and two drift terms $a_1(x)$ and $a_2(x)$. Thus, we end with twelve unknown functions which can be evaluated by using a bivariate extension of the KM coefficients, i.e.,

$$D^{(k_1,k_2)}(x_1, x_2) = \lim_{\tau\to 0}\frac{1}{\tau}\langle[x_1(t+\tau) - x_1(t)]^{k_1}[x_2(t+\tau) - x_2(t)]^{k_2}\rangle|_{x_1(t)=x_1,\, x_2(t)=x_2}. \tag{11}$$

In this fashion, the KM coefficients become matrices that contain all the information about the correlation between the different processes.

Although the idea of a bivariate model is intriguing, from a statistical standpoint it is also challenging for the reasons mentioned above. Therefore, the primary focus of this work is to provide a stochastic model for the SMR time series only, regardless of any potentially coupled variable. Nonetheless, acquiring a comprehensive understanding of multivariate models will be valuable for the final discussions in the current study.

3. Results

Data used in this work are provided by the SuperMAG worldwide collaboration, a scientific community currently operating in the unification of ground-based magnetic field observations gathered by about 350 stations [Gjerloev, 2009; Gjerloev, 2012; Newell and Gjerloev, 2012]. Data from ground-based observatories with different sampling time, with or without baseline subtraction and different coordinate systems are standardized by the SuperMAG collaboration, which provide data with a time resolution of 1-minute rotated into the geomagnetic coordinate system where the H component is directed towards the local north.

With this procedure is then possible to calculate a geomagnetic index which constitute a generalization of the usual SYM-H index used to investigate magnetic storms. The SuperMAG generalization of the SYM-H index is called SMR. The statistical analysis carried out in this work to develop a semi-empirical model for SMR is performed on a 25-year time interval, from 1990 to 2015. A SMR data sample of about 70 days, during which an intense magnetic storm reaching values < –200 nT occurs, is reported in Figure 1.

The first step in the analysis of the SMR index is to test the Markov property on the data sample used in the analysis. With reference to Eq. (4), we can use the CKE to assess the validity of the Markov condition on the SMR index, Figure 2. Indeed, by evaluating and comparing both members of Eq. (4), we found a remarkable agreement between the level curves obtained for the empirical transition probability (blue) and the one predicted from the CKE (red). This is also emphasized when examining the vertical cuts at –30 nT at $\tau = 10, 60, 1440$ min reported in the bottom panels. Hence, we conclude that the SMR index satisfies the Markov property for any time scale $\tau \geq 1$ min. Furthermore, the 4-th order KM coefficient estimated from SMR index dynamics does not vanish (not shown), thus implying that all the KM expansion must be included in the model. This suggests that the dynamics of the SMR index is characterized by jumps/discontinuities possibly associated with the occurrence of magnetic storms. Therefore, we assume that the dynamics of the SMR index can be described by the jump-diffusion (JD) model of Eq. (8). By assuming that jumps follow a normal distribution in amplitude and a Poisson distribution in time, Eq. (9) provides a non-parametric estimation of the drift, diffusion and jump terms inferred from the data. All the parameters obtained from the SMR time series are reported in Figure 3 (circles). Although the estimation of high-order KM coefficients turns out to be quite noisy, the overall trend of drift, diffusion and jump terms have been parametrized through the polynomial functions reported below (red lines in Figure 3):

$a(x) = -1.61 \times 10^{-5} - 7.48 \times 10^{-6}x - 1.92 \times 10^{-7}x^2 - 3.73 \times 10^{-9}x^3$
$b^2(x) = 0.002 + 2.19 \times 10^{-5}x + 3.92 \times 10^{-6}x^2$
$\sigma_\xi^2(x) = 491.4 + 2.87x$
$\lambda(x) = 2.76 \times 10^{-8}x^2.$

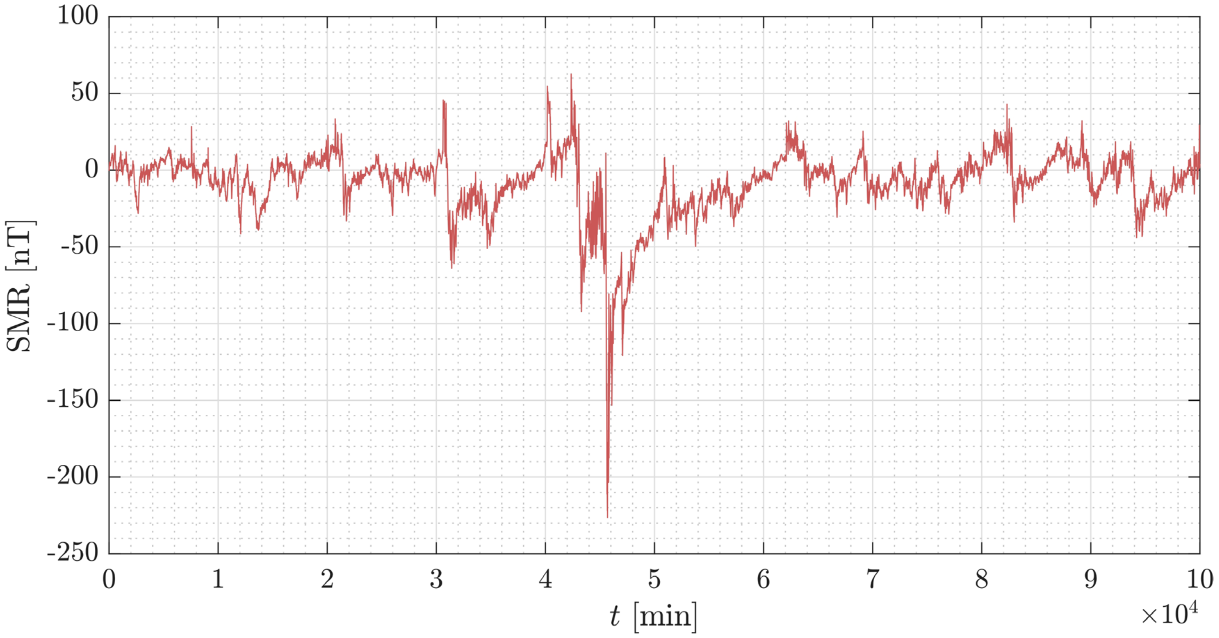

Figure 1. SMR data sample of about 70 days. A typical magnetic-storm event is observed in this data sample, where values of the SMR index reach values well below −200 nT.

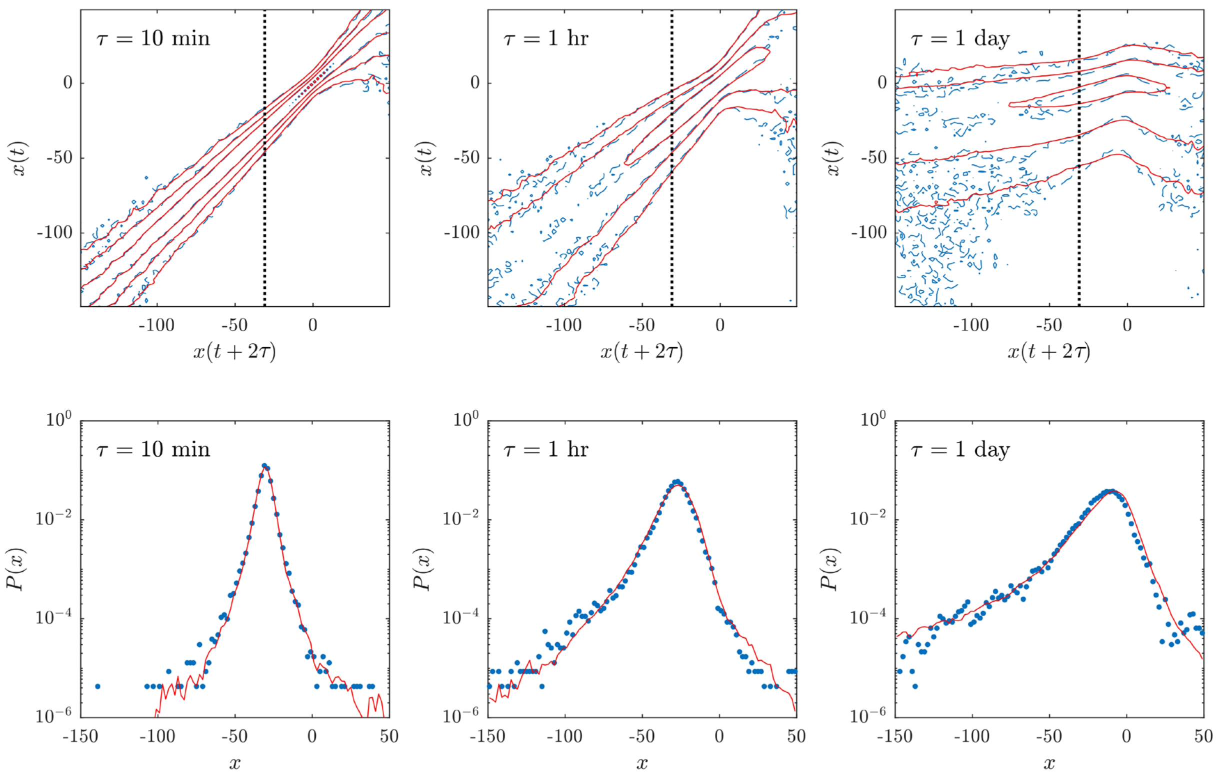

Figure 2. CKE test results for different time-scale separations τ. In the top panels are reported the empirical transition probabilities (blue) along with the prediction of the CKE (red) for 10 min, 1 hour and 1 day time separations. The bottom panels illustrate the comparison between the empirical transition probabilities (blue) and prediction of the CKE (red) along the vertical cuts at −30 nT (dotted lines in the top panels).

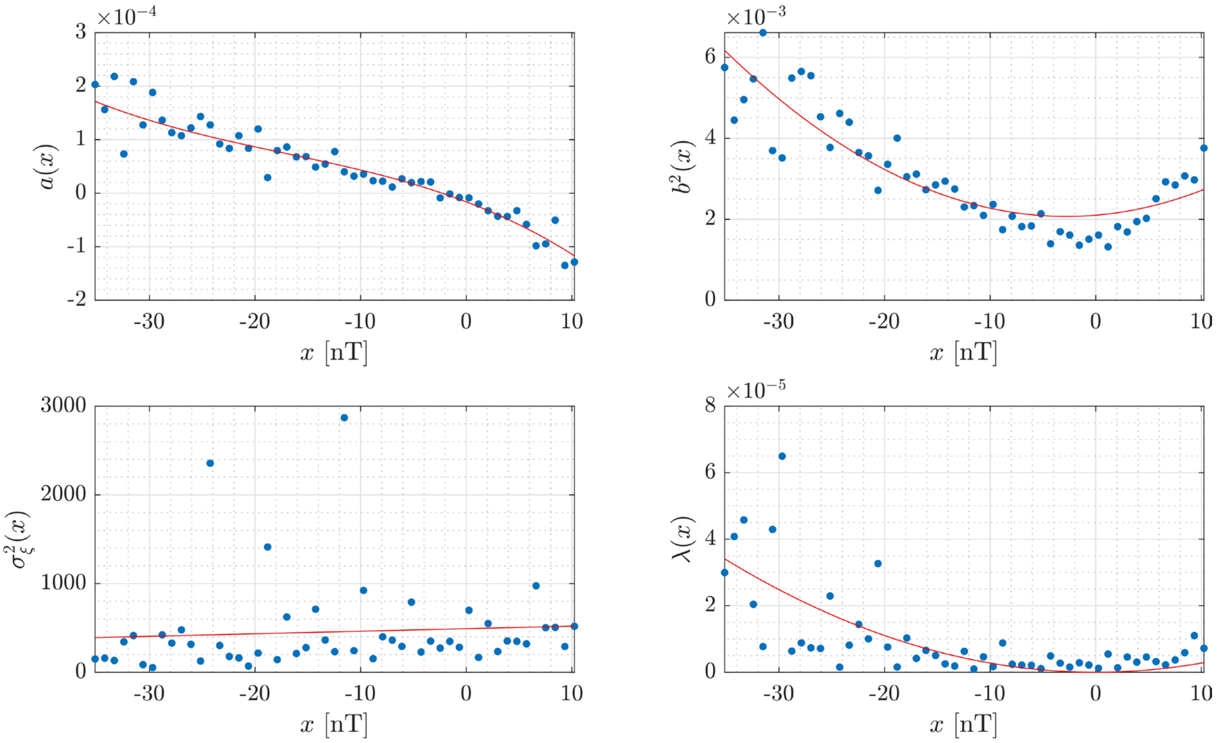

Figure 3. Main parameters of the JD process associated with SMR index dynamics (circles). Red lines indicate the polynomial functions used in the parametrization.

The range of SMR values used in this estimation corresponds to the core of the distribution of the signal, i.e., we used the 95% confidence bounds estimated from the cumulative distribution function of the SMR index.

In Figure 4 we report a sample of 10^4 data points generated through the stochastic JD model. The SDE has been integrated with $\Delta t = 5$s and then resampled to 1-minute, to be properly compared with the SMR index. The Wiener process W_t (red) and the Poisson process J_t (green) obtained through the stochastic integration on this sample are reported in the bottom panel. From a qualitative point of view, we can notice how many features of the real SMR index are present also in the reconstruction performed through the JD process. In the data sample here shown, the stochastic process can mimic a magnetic storm event with values of the synthetic index reaching about −140 nT in an asymmetric configuration, i.e., with a sudden decrease followed by a more gradual recovery phase. The event duration is also compatible with typical magnetic storm timing which is of the order of several days (~5 days in the case of Figure 4). However, fundamental signatures, such as the presence of the sudden impulse preceding the main phase of the storm, are absent in the sample generated through the JD model. This aspect will be discussed below.

From the statistical side we perform a comparison between the PDFs of SMR and JD model in the interval $[-150, 50]$ nT, Figure 5. This interval is larger than the one used to estimate diffusion and jump parameters. The core of the PDFs is fairly in agreement, exhibiting a maximum around $x = 0$. On the other hand, the model clearly tends to overestimate positive fluctuations of the index in the interval $[10, 40]$ nT and, conversely, to underestimate negative ones in the interval $[-90, -10]$ nT. These differences may be explained in terms of the accuracy in drift, diffusion and jump term parametrization. As highlighted in the discussion of Figure 3, indeed, in the case of SMR high-order moments make their estimate quite noisy and the simple polynomial fits we introduced may affect the goodness of the JD model, representing a strong simplification. Moreover, the PDF associated with JD model outcome exhibits a cutoff < -100 nT, meaning that the model is not able to produce extreme storm events. The presence of such a cutoff is expected since the JD model parameters are set in the variability interval of the SMR index where the statistics is sufficient to estimate high-order moments. Then, with this approach is not possible to capture the "real world" phenomena which typically constitute the tails of the PDFs [Benella et al., 2022; Cristoforetti et al., 2022].

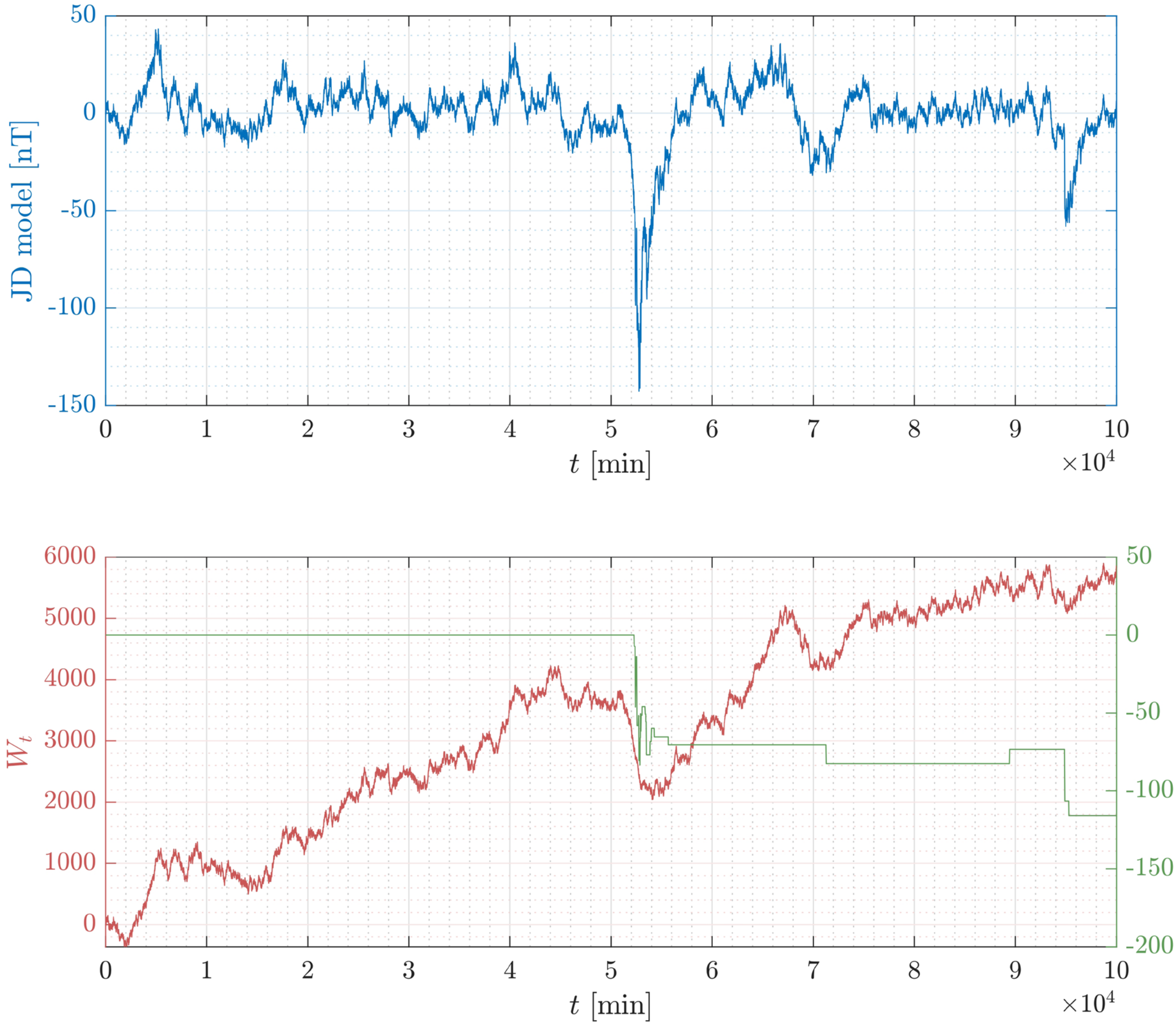

Figure 4. Data sample obtained from stochastic integration of Eq. (9) with the parameters reported in Figure 3. In the top panel is reported the JD model of the SMR index integrated over a time interval corresponding to about 70 days. The stochastic contribution to the dynamics coming from diffusion and jump processes is depicted in the bottom panel. The Wiener process corresponding to the JD model realization is reported in red and the Poisson process in green.

4. Conclusions and Future Perspectives

In this work we present a comprehensive statistical analysis of the equatorial SMR geomagnetic index. The time series of such a generalized geomagnetic index can be envisioned as a stochastic process, exhibiting sudden jumps, especially in conjunction with geomagnetic storms. The investigation of the properties of the transition probability associated with this process reveals the markovian nature of equatorial magnetospheric activity for $\tau > 1$ min. In other words, all the information on the history of the process as observed at time t_n is contained in the previous time step t_{n-1}, with $\tau = t_n - t_{n-1}$. Because of the Markov condition, the transition probability of the process is expressed as a KM expansion of the master equation. This allows us to infer information about the different terms governing the dynamics in the SDE-based model straight from the data through non-parametric methods. The approximation we make consists in assuming that the master equation describes the statistics of a jump-diffusion process, whose jumps follow a Poisson distribution in time and a normal distribution in amplitude. In this framework the KM coefficients can be recursively defined and the SDE has a jump term whose amplitude and rate can be estimated empirically [Anvari et al., 2016; Tabar, 2019]. We show that the SMR model is capable of reproducing some of the key features of the real SMR index dynamics, especially when a magnetic storm occurs. In fact, when the diffusion term pulls the dynamical state towards negative values of the index, the jump rate increases and the probability of observing a discontinuity in the

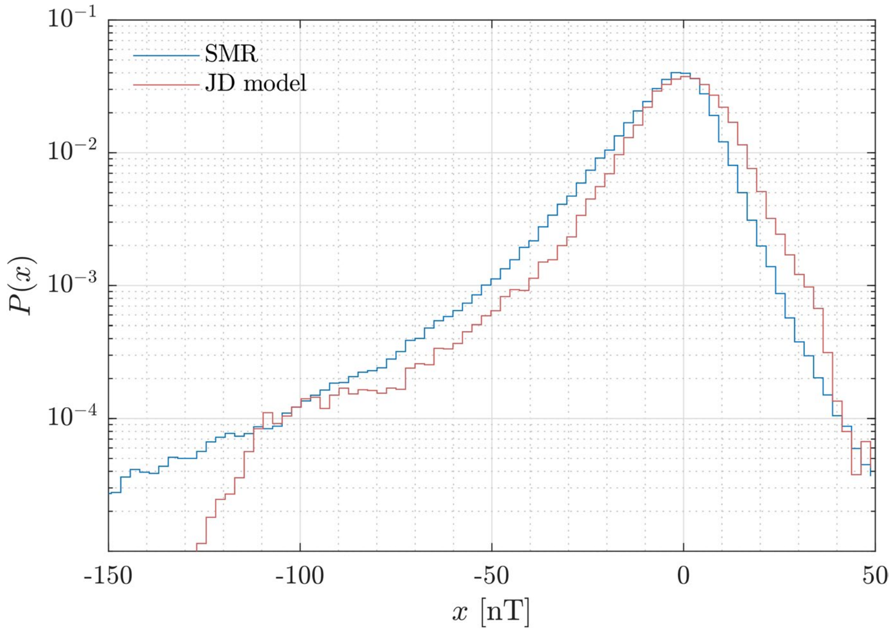

Figure 5. Comparison between the SMR index PDF (blue) and JD model PDF (red).

dynamics, which mimic the occurrence of a magnetic storm, is higher (Figure 3). On the other hand, a simple Markov process does not allow to reproduce the coincidence and/or correlation between the sudden impulse and the onset of the magnetic storm. As a matter of fact, the time concatenation of these events is tightly related to the interaction with the external driver. Since information about solar wind conditions are not included in the JD model, the correlation between sudden impulse and magnetic storm occurrence cannot be reproduced.

To this end, we would discuss the possibility of including the external forcing in future stochastic modeling, in terms of z component of the interplanetary magnetic field, dynamical pressure of the solar wind, high-latitude geomagnetic indices, one of the solar wind-magnetosphere coupling functions $Q(t)$ introduced in Eq. (1). In this context further investigation is necessary to assess the feasibility and the possible success of a multivariate SDE modeling, and for this reason such a discussion is posed as a future perspective. As stated in Eq. (11), the KM coefficients are matrices containing information about the correlation between different processes. In this case, we need to sample high-order moments of the PDFs of both processes x_1 and x_2 to fix twelve unknown functions, and this makes the bivariate model very sensitive to the goodness of data and statistical sampling of the processes in their variability domains. The relevance of a bivariate stochastic modeling in a space weather perspective can be assessed by relating the stochastic process $x_1 = SMR$ to a possible driver, thus representing the coupled stochastic process, e.g., $x_2 = B_z$, solar wind dynamic pressure, solar wind-magnetospheric coupling functions, etc. Indeed, if the off-diagonal terms of the diffusion and jump matrices of the bivariate process are different from zero, the state of one variable can influence the state of the other one, then reproducing a complex and intertwined dynamics, potentially capable to mimic the interplay between different processes. In this way, it is possible to assess the response of the stochastic model of the SMR index to an external forcing, thus exploring the predictive capabilities of SDEs in the context of space weather. Thus, bivariate stochastic modeling constitutes a promising perspective as it provides a comprehensive statistical framework for modeling, and possibly predicting, magnetic storm dynamics by considering the interactions between different processes and their responses to external factors, shedding light on the complex interactions governing space weather phenomena.

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