ANNALS OF GEOPHYSICS, 68, 4, DM447, 2025; doi:10.4401/ag-9247
OPEN ACCESS

APPENDIX TO

THREE-DIMENSIONAL FAST INVERSION OF GRAVITY AND ITS GRADIENT TENSOR DATA IN WAVENUMBER DOMAIN

Sheng Liu[1,2,3], Shi Chen[3], Honglei Li[3], Yiju Tang[1], Fangchao Lu[1], Changrui Zhou[1], Quan Lou[1], Shuanggen Jin[4], Songbai Xuan[4], Renbao Yu[5]

[1] Henan University of Urban Construction, Pingdingshan, China
[2] Hebei Key Laboratory of Earthquake Dynamics, Sanhe, China
[3] Institute of Geophysics, China Earthquake Administration, Beijing, China
[4] Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai, China
[5] Xuchang Seismological Bureau, Xuchang, China

Appendix

The numerical integral procedure of Eq. (5) may be given as follows.

$$\boldsymbol{U}(u,v)=\frac{2\pi G\Delta\rho}{w}e^{wz_0}\boldsymbol{E}(u,v) \tag{A1}$$

First, $\boldsymbol{E}(u,v)$ can be integrated for ξ, η and h, respectively.

$$\boldsymbol{E}(u,v)=\int_{h_0}^{h_1}d\xi\int_{n_0-b}^{n_0+b}d\eta\int_{\xi_0-a}^{\xi_0+a}e^{-wh}e^{-i(\xi u+\eta v)}\,dh\,, \tag{A2}$$

Using the following change of variables

$$\begin{cases}\xi'=\xi-\xi_0\\ \eta'=\eta-\eta_0\end{cases} \tag{A3}$$

We can obtain

$$\boldsymbol{E}(u,v)=e^{-i(\xi_0u+\eta_0v)}\int_{h_0}^{h_1}d\xi'\int_{-\boldsymbol{b}}^{\boldsymbol{b}}d\eta'\int_{-\boldsymbol{a}}^{\boldsymbol{a}}e^{-wh}e^{-i(\xi'u+\eta'v)}\,dh\,, \tag{A4}$$

Firstly, the following expression can be obtained for integration concerning ξ':

$$\boldsymbol{E}(u,v) = e^{-i(\xi_0 u+\eta_0 v)} \int_{h_0}^{h_1} dh \int_{-\boldsymbol{b}}^{\boldsymbol{b}} \frac{ie^{-wh-i\eta' v}}{u} (e^{-iua} - e^{iua}) d\eta' \tag{A5}$$

Based on the Euler's formula, $\begin{cases} e^{-ia} = \cos a - i \sin a \\ e^{ia} = \cos a + i \sin a \end{cases}$, we have the relationship $e^{-ia} - e^{ia} = 2i \cdot \sin(ua)$; therefore,

$$\boldsymbol{E}(u,v) = e^{-i(\xi_0 u+\eta_0 v)} \frac{2\sin(ua)}{u} \int_{h_0}^{h_1} dh \int_{-b}^{b} e^{-wh-i\eta' v} d\eta' \tag{A6}$$

Secondly, the following expression can be obtained for integration with respect to η':

$$\boldsymbol{E}(u,v) = e^{-i(\xi_0 u+\eta_0 v)} \frac{4\sin(ua)\sin(vb)}{uvw} \int_{h_0}^{h_1} e^{-wh} dh\,, \tag{A7}$$

Finally, the transformed expression is given as

$$\boldsymbol{E}(u,v) = e^{-i(\xi_0 u+\eta_0 v)} \frac{4\sin(ua)\sin(vb)\,(e^{-wh_0} - e^{-wh_1})}{uvw}, \tag{A8}$$

which yields Eq. (6):

$$\boldsymbol{U}(u,v) = 8\pi G\Delta\rho \frac{e^{wz_0}(e^{-wh_0} - e^{-wh_1})}{w^2} \frac{\sin(ua)}{u} \frac{\sin(vb)}{v} e^{-i(\xi_0 u+\eta_0 v)} \tag{A9}$$