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Energetic Constraints and Kinetic Closure in Dynamic Rupture: An Analytical Framework for Earthquake Source Physics

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Abstract

Dynamic fracture and earthquake rupture propagation are governed by the balance between elastic energy release, fracture resistance, and elastodynamic wave radiation. In this study, we present a compact energetic framework that synthesizes established results from dynamic fracture mechanics and earthquake source theory into a physically transparent formulation. The framework couples bulk elastodynamics with an energetic admissibility condition and employs a phenomenological kinetic closure to relate energy excess to rupture speed, providing an explicit description of rupture evolution without prescribing detailed frictional constitutive laws. Within this setting, elastodynamic radiation constrains rupture velocity through mode-dependent dynamic energy corrections, leading to asymptotic saturation within sub-Rayleigh rupture regimes. Analytical relations are derived linking rupture speed to stress drop, rupture size, and fracture energy, yielding interpretable scaling trends for dynamically propagating ruptures. The proposed framework is intended as an analytical reference for interpreting rupture dynamics and energy partitioning, and for benchmarking numerical and constitutive models of earthquake rupture.

Keywords: Dynamic Fracture; Earthquake Rupture; Fracture Energy; Energy Release Rate; Rupture Velocity

1. Introduction

The dynamics of fracture propagation in elastic media underlies a wide range of geophysical phenomena, most prominently earthquake rupture on tectonic faults. Earthquakes represent macroscopic expressions of dynamic brittle failure, during which elastic strain energy accumulated over long geological timescales is rapidly released through the propagation of a rupture front. Understanding the physical principles governing rupture initiation, propagation, and arrest remains a central challenge in earthquake physics and geodynamics (Scholz, 2019).

Classical fracture mechanics established the energetic foundation for brittle failure through the seminal work of Griffith (1920), who formulated crack growth as an energy balance between elastic energy release and the surface energy required to create new fracture surfaces. This framework was later generalized through the concepts of stress intensity factors and energy release rate (Irwin, 1957), providing a quantitative description of crack-tip stress fields and fracture criteria. These ideas were extended to dynamic conditions by Freund (1990), who demonstrated that crack propagation is fundamentally constrained by elastodynamic wave speeds and by the redistribution of stress and energy near the moving crack tip. Subsequent theoretical and experimental studies further clarified the role of near-tip nonlinear elasticity, dynamic instabilities, and energy flux localization during rapid fracture propagation (e.g., Fineberg and Marder, 1999; Bouchbinder et al., 2008; Bouchbinder et al., 2014).

In parallel, seismological models of earthquake rupture adopted related concepts to describe fault slip and seismic radiation. Early analytical solutions for shear dislocations in elastic media (Kostrov, 1964) and dynamic rupture models incorporating slip-weakening friction (Andrews, 1976) established the physical link between rupture velocity, stress drop, and radiated seismic energy. Subsequent work emphasized the role of fracture energy and energy partitioning in controlling rupture dynamics and stability (Rice, 1980; Abercrombie and Rice, 2005). These studies highlighted that earthquake rupture cannot be fully understood without explicitly considering energetic constraints.

Recent reviews and perspective studies have emphasized both the successes and the limitations of classical energetic fracture mechanics when applied to earthquake rupture, particularly regarding rupture complexity, heterogeneity, nonlinear fault-zone processes, and the role of additional dissipative mechanisms (e.g., Cocco et al., 2023). These works clarify that many open questions lie not in the basic energetic balance itself, but in how energetic constraints interact with constitutive assumptions, rupture kinetics, and observational interpretation across scales.

Despite this conceptual overlap, fracture mechanics and earthquake source physics have largely evolved along parallel but only partially connected paths. Many earthquake rupture models prescribe rupture kinematics or frictional laws without explicitly enforcing crack-tip conditions or global energy balance (e.g., Day et al., 2005; Lapusta et al., 2000). Conversely, classical fracture mechanics formulations often assume idealized cracks in homogeneous or weakly heterogeneous media, conditions that only approximate the complexity of natural fault systems (Bonnet et al., 2001).

Natural faults are embedded in heterogeneous, anisotropic, and evolving crustal environments, where stress loading is spatially variable and rupture dynamics are influenced by multiscale material heterogeneity (Ben-Zion and Sammis, 2003). Observational evidence from seismic inversions and laboratory experiments suggests that rupture propagation is often intermittent, spatially irregular, and subject to arrest or branching (Mai and Beroza, 2002; Kame and Yamashita, 2003). These features highlight that rupture complexity and heterogeneity are intrinsic components of fracture physics rather than secondary perturbations and therefore call for energetic frameworks capable of accommodating evolving fault-zone complexity while preserving physical consistency (e.g., Sornette et al., 1990; Steinhardt and Rubinstein, 2022; Sagy et al., 2025).

Recent numerical advances have enabled increasingly realistic simulations of dynamic rupture in complex media. Boundary integral methods (Cochard and Madariaga, 1994; Geubelle and Rice, 1995), spectral element approaches (Komatitsch and Tromp, 1999), and phase-field formulations of fracture (Bourdin et al., 2008; Borden et al., 2016) have all contributed to improved modeling capabilities. However, numerical realism alone does not guarantee physical insight. Without a clear analytical framework, it remains difficult to identify the fundamental constraints that govern rupture velocity, stability, and energy partitioning across different tectonic settings.

From a geophysical standpoint, several fundamental questions remain unresolved. What physical mechanisms determine whether a rupture accelerates toward limiting wave speeds or arrests prematurely? How does material heterogeneity modulate the balance between energy release and dissipation during rupture propagation? To what extent can earthquake rupture dynamics be interpreted as a general manifestation of dynamic fracture processes in elastic continua, once energetic constraints are separated from closure-dependent assumptions?

Addressing these questions benefits from a formulation that is grounded in continuum mechanics, energetically consistent, and readily connected to seismological observables. Here we present a compact synthesis of the energetic structure underlying dynamic fracture propagation in elastic media, emphasizing how classical energy balance and elastodynamic radiation constrain admissible rupture evolution. Fracture propagation is treated as an evolving boundary-value problem governed by elastodynamics and constrained by energy balance and dissipation; rupture speed is obtained once an explicit kinetic closure relating energy excess to advance rate is specified. This perspective is intended to clarify what follows directly from energetic admissibility and what depends on additional constitutive or kinetic assumptions.

In particular, the formulation distinguishes between (i) relations that follow directly from elastodynamics and energy balance, and (ii) closure assumptions that must be introduced to relate energy excess to rupture propagation. This distinction is essential for interpreting which aspects of rupture dynamics are physically constrained and which depend on additional modeling choices.

Because natural earthquakes are predominantly shear ruptures, the energetic correction associated with rupture velocity cannot be treated as mode independent. In classical dynamic fracture mechanics, the velocity-dependent reduction of the energy release rate depends on the fracture mode and on the admissible propagation regime. In particular, tensile Mode I cracks, in-plane shear Mode II cracks, and anti-plane shear Mode III cracks are characterized by different dynamic correction factors and, in some cases, different limiting velocities. Accordingly, the present framework is formulated in terms of a mode-dependent dynamic factor, rather than a universal function of rupture velocity. The sub-Rayleigh formulation developed below is therefore intended as a general energetic structure whose quantitative realization depends on the selected fracture mode. To make this distinction explicit, the manuscript discusses both Mode-I-type sub-Rayleigh behavior and the analytically tractable Mode III anti-plane shear case, while treating Mode II and supershear rupture as important regimes requiring separate dynamic correction factors.

The proposed framework allows rupture velocity and evolution to be expressed in terms of energetic constraints, once a kinetic closure is specified. The present work does not aim to introduce new fracture physics or new constitutive rupture laws. Instead, its contribution lies in the explicit analytical integration of bulk elastodynamics, energetic admissibility, dynamic correction factors, and phenomenological rupture kinetics into a unified and physically transparent framework. In much of the existing literature, these elements are often treated separately, embedded implicitly within numerical implementations, or discussed primarily within specific constitutive formulations. Here they are reorganized into a compact analytical structure that makes explicit which aspects of rupture dynamics follow directly from energetic constraints and which depend on closure assumptions. In this sense, the proposed formulation is intended primarily as an interpretative and benchmark framework that clarifies the energetic architecture underlying dynamic rupture propagation across different rupture models and physical regimes.

The paper is organized as follows. Section 2 introduces the physical assumptions and governing equations for elastic media and fracture energetics. Section 3 presents the mathematical formulation of dynamic fracture propagation, including boundary integral and variational representations. Analytical results and scaling relations are derived in Section 4. Section 5 discusses the implications of the proposed framework for earthquake rupture dynamics and seismological observations. Section 6 summarizes the main conclusions and outlines directions for future research.

2. Physical Background and Governing Assumptions

We consider a deformable elastic medium occupying a domain $\Omega \subset \mathbb{R}^d$, $d = 2{,}3$, representing the effective volume surrounding an evolving fracture or fault segment. The medium is characterized by a mass density $\rho(x)$ and elastic properties that may vary spatially, reflecting the heterogeneous and anisotropic nature of the Earth's crust (Ben-Zion and Sammis, 2003; Scholz, 2019).

The motion of the medium is governed by the balance of linear momentum,

$$\rho(x)\,\ddot{u}_i(x,t) = \sigma_{ij,j}(x,t), \tag{1}$$

where $u_i(x,t)$ is the displacement field and σ_{ij} is the Cauchy stress tensor. Body forces are neglected, as dynamic fracture and earthquake rupture processes are dominated by stress gradients generated internally during rupture propagation (Aki and Richards, 2002).

Under the assumption of infinitesimal strains, the strain tensor is defined as

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j}+u_{j,i}\right), \tag{2}$$

and the constitutive relation for a linear elastic medium reads

$$\sigma_{ij} = C_{ijkl}(x)\,\varepsilon_{kl}, \tag{3}$$

where C_{ijkl} is the fourth-order stiffness tensor. This formulation naturally accommodates isotropic and anisotropic elasticity as well as spatial heterogeneity, which are known to influence rupture complexity and seismic radiation (Mai and Beroza, 2002; Ampuero and Rubin, 2008).

The present formulation adopts a continuum small-strain elastic description in which dissipative and inelastic processes are represented implicitly through the effective fracture energy G_c and the phenomenological kinetic closure. This separation between elastic energy release and dissipation constitutes a modeling assumption rather than a rigorously derived decomposition. In particular, the constitutive relation introduced in Eq. (3) is local and does not include gradient terms, internal length scales, or nonlocal damage evolution. Consequently, while spatial heterogeneity in C_{ijkl} and G_c can be prescribed externally, the framework does not explicitly capture scale-dependent constitutive behavior emerging from damage-zone physics or evolving material structure (e.g., Zaccagnino et al., 2025). Similarly, the treatment of inelastic deformation through an effective energetic contribution to G_c implicitly assumes an approximate additive separation between elastic and dissipative processes within a small-strain setting. In natural earthquake ruptures, however, microcracking, frictional sliding, off-fault damage, and stiffness degradation may alter the effective elastic response itself, making the separation between elastic strain energy release and dissipation only approximate. The present framework should therefore be interpreted as an intentionally simplified energetic representation designed to isolate the large-scale energetic structure of dynamic rupture propagation rather than a complete constitutive description of fault-zone physics.

The elastic strain energy density is given by

$$\Psi(\varepsilon_{ij}) = \frac{1}{2}\,\sigma_{ij}\varepsilon_{ij}, \tag{4}$$

and the total elastic energy stored in the domain Ω is

$$E_{el} = \int_{\Omega} \Psi\, d\Omega. \tag{5}$$

The total potential energy of the system is defined as

$$\Pi = E_{el} - W_{ext}, \tag{6}$$

where W_{ext} denotes the work done by external tractions. Figure 1 provides a schematic representation of the energetic partitioning at the rupture front and does not describe the geometric evolution of the fracture or changes in domain topology. In the absence of fracture growth, variations in Π are associated with boundary loading and seismic wave radiation. When a fracture propagates, however, the topology of the domain changes, leading to a reduction of the total potential energy and to the release of elastic energy available to drive rupture (Griffith, 1920; Rice, 1968).

Fracture propagation is governed by an energetic balance between elastic energy release and fracture resistance. The energy release rate G is defined as

$$G = -\frac{\partial \Pi}{\partial a}, \tag{7}$$

where a denotes a characteristic measure of fracture extension, such as crack length in two dimensions or fracture area in three dimensions.

The resistance to fracture growth is quantified by the fracture energy G_c, representing the total energy dissipated per unit fracture advance. In geophysical systems, G_c includes not only surface energy but also dissipation associated with frictional sliding, inelastic deformation, microcracking, and thermal processes within the fracture process zone (Fig. 1; Abercrombie and Rice, 2005; Viesca and Garagash, 2015). In this sense, the effective fracture energy may incorporate energy losses associated with frictional resistance and off-fault inelastic deformation (Andrews, 2005), as well as distributed microcracking and damage generation around the rupture front (Yamashita, 2000).

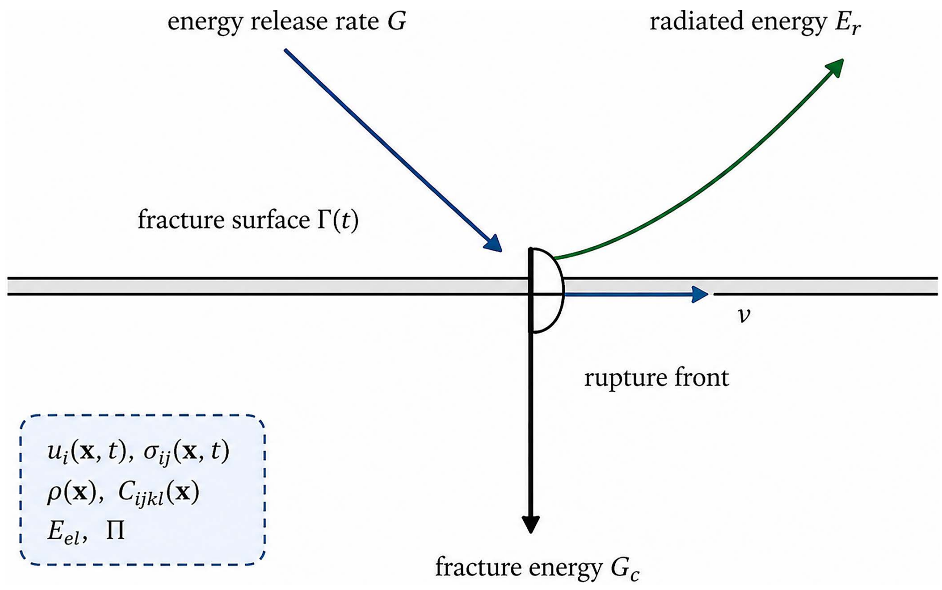

Figure 1. Energetic partitioning at a dynamic rupture front. Elastic strain energy released from the surrounding medium is focused at the rupture front, where it is partitioned into fracture energy G_c, radiated seismic energy E_r, which together govern rupture propagation. The rupture velocity v is constrained by the local balance between the energy release rate G and the effective fracture resistance, once a kinetic closure is specified.

Fracture propagation is energetically admissible when

$$G \geq G_c, \tag{8}$$

with equality corresponding to marginally stable propagation and strict inequality characterizing dynamically accelerating rupture. The energetic admissibility condition expressed by Eq. (8) should be interpreted here as a global criterion for dynamically sustained supercritical rupture propagation, as commonly adopted in earthquake rupture mechanics. Localized subcritical crack growth and progressive damage evolution may nevertheless occur under conditions for which $G < G_c$, particularly in the presence of stress corrosion, time-dependent weakening, or other slow fracture processes not explicitly considered within the present dynamic framework.

Under dynamic conditions, fracture propagation is fundamentally constrained by elastodynamic wave radiation. A moving fracture tip radiates seismic waves, which transport energy away from the rupture front and reduce the energy flux available to sustain further growth (Freund, 1990).

As a consequence, the rupture velocity v is theoretically bounded by the Rayleigh wave speed c_R in classical elastodynamic fracture mechanics. This upper bound is strictly valid for sub-Rayleigh rupture regimes and for crack modes in which the Rayleigh wave speed represents the limiting propagation velocity. However, in shear-dominated Mode II rupture, supershear propagation exceeding the shear wave speed can occur under appropriate stress conditions and fault properties, as documented in both theoretical and observational studies (Andrews, 1976; Bao et al., 2022). More recently, transonic and supershear propagation has also been demonstrated for tensile cracks when geometric nonlinearities are taken into account (Pundir et al., 2024). The present formulation is therefore restricted to sub-Rayleigh rupture regimes, where the dynamic energy release rate decreases monotonically with rupture velocity. In natural earthquake ruptures, however, observed rupture velocities are often significantly lower than this upper bound, reflecting the influence of dissipation mechanisms and heterogeneous fault properties

$$v < c_R. \tag{9}$$

Eq. (9) should be interpreted as a regime-specific constraint applicable to sub-Rayleigh rupture, rather than as a universal bound for all rupture modes. This distinction is particularly important for earthquake ruptures, which are predominantly shear-dominated processes. In anti-plane Mode III rupture, the limiting propagation velocity is the shear wave speed rather than the Rayleigh wave speed, and the dynamic correction factor exhibits a different asymptotic structure. In in-plane Mode II rupture, the energetic behavior is more complex and may include supershear propagation regimes in which rupture velocities exceed the shear wave speed. Consequently, the present framework should not be interpreted as implying a universal Rayleigh-wave limitation for all earthquake ruptures. Instead, the sub-Rayleigh relations introduced below represent one admissible energetic branch within a broader class of mode-dependent rupture dynamics. The dynamic energy release rate can be expressed as

$$G(v) = G_0 F_m(v) \qquad m \in \{I, II, III\}, \tag{10}$$

where G_0 is the quasi-static energy release rate and $F_m(v)$ is a mode-dependent dynamic correction factor describing the reduction of the energy flux available at the rupture front as rupture velocity increases. The specific form of $F_m(v)$ depends on the fracture mode and on the admissible elastodynamic regime.

For tensile Mode I cracks in the sub-Rayleigh regime, the dynamic correction factor decreases as rupture velocity approaches the Rayleigh wave speed c_R, leading to a progressive reduction of the available energy flux near the limiting velocity (Freund, 1990; Broberg, 1999). In contrast, anti-plane shear Mode III ruptures are characterized by a different energetic structure, for which the limiting velocity is the shear wave speed c_s and the dynamic correction factor takes the form

$$F_{III}(v) = \sqrt{\frac{1 - v/c_s}{1 + v/c_s}}, \tag{11}$$

which decreases asymptotically as v approaches c_s.

For in-plane shear Mode II ruptures, the dependence of $G(v)$ on rupture velocity is more complex and does not generally reduce to the same asymptotic structure observed in Mode I. In particular, supershear rupture propagation may occur under appropriate stress conditions and fault properties, implying that the Rayleigh wave speed cannot be interpreted as a universal limiting velocity for earthquake ruptures.

Accordingly, the present framework should be interpreted as a generalized energetic structure in which the functional form of $F_m(v)$ depends on the selected fracture mode and rupture regime. The sub-Rayleigh energetic relations developed below are therefore intended as representative mode-dependent realizations rather than universal dynamic correction laws.

The function $F(v)$ depends on the assumed crack model and material properties, and its asymptotic behavior reflects idealized elastodynamic solutions.

Natural fault systems exhibit a variety of dissipative processes associated with the evolving rupture interface and surrounding damage zone, including frictional sliding, off-fault damage, and thermally activated weakening processes (Andrews, 1976; Dieterich, 1979; Ruina, 1983; Ben-Zion and Sammis, 2003; Rice, 2006). These mechanisms primarily govern post-front sliding behavior, stress drop evolution, and energy dissipation behind the rupture front, whereas the admissibility of rupture propagation itself is governed at leading order by the near-tip energetic balance and associated elastodynamic fields.

Rather than prescribing a specific friction law, we adopt a phenomenological description in which the fracture energy is allowed to depend on rupture velocity, stress conditions, and spatial location,

$$G_c = G_c(x, v, \sigma_{ij}). \tag{12}$$

This formulation preserves generality while remaining consistent with the energetics of fracture and earthquake rupture.

From a mathematical perspective, dynamic fracture propagation is treated as a moving boundary-value problem. The fracture surface $\Gamma(t)$ constitutes a time-dependent internal boundary of the domain Ω, and its evolution determines the partitioning between intact and fractured regions.

The fracture tip velocity is not prescribed a priori but is described through an explicit phenomenological kinetic relation linking the local energy excess to rupture speed,

$$v(t) = V\,[G(t) - G_c], \tag{13}$$

where $V[\cdot]$ denotes a monotonic functional mapping energy excess into rupture velocity, satisfying $V(0) = 0$ and $dV/dG > 0$, ensuring consistency with energetic admissibility and dynamic constraints (Rice, 1980). Here $V[\cdot]$ denotes a phenomenological kinetic functional mapping local energy excess into rupture velocity, rather than a constant proportionality factor.

Within this formulation, the kinetic relation Eq. (13) provides a general statement linking energy excess to rupture velocity, while Eq. (19) represents a specific admissible functional realization of this relation. This distinction is important: the energetic balance constrains the admissible range of rupture dynamics but does not uniquely determine the functional form of the kinetic closure, which reflects unresolved physical processes within the fracture process zone.

3. Mathematical Formulation of Dynamic Fracture Propagation

Within the bulk medium away from the evolving fracture surface $\Gamma(t)$, the displacement field satisfies the elastodynamic Eq. (1) together with the constitutive relation Eq. (3). Figure 2 summarizes the logical structure of the formulation by organizing governing equations, boundary conditions, energetic constraints, and kinetic closure relations. It is intended as a conceptual representation of the framework rather than a physical illustration of rupture processes. On the evolving fracture surface $\Gamma(t)$, tractions satisfy

$$\sigma_{ij}\hat{n}_j = \tau_i \tag{14}$$

where $\hat{\mathbf{n}}$ is the unit normal versor to the fracture surface, $\hat{n}_j$ denotes its j-th Cartesian component, and τ_i represents the traction components acting on the evolving fracture interface. Depending on the physical rupture mode, these tractions may describe cohesive resistance opposing crack opening or frictional resistance associated with relative sliding along a fault surface. The boundary condition expressed in Eq. (14) represents a gener al traction balance at the fracture surface. Figure 2 schematically summarizes this general formulation and should not be interpreted as a restriction to a specific fracture mode.

For linear elastic media, the displacement field admits a boundary integral representation,

$$u_i(x,t) = \int_{\Gamma(t)} \int_0^t K_{ij}\,(x,\xi,t-\tau)\,\Delta u_j(\xi,\tau)\,d\tau\,d\Gamma, \tag{15}$$

where Δu_j is the displacement discontinuity across the fracture surface and K_{ij} is the elastodynamic Green's function kernel. This formulation naturally accounts for wave-mediated stress transfer and nonlocal interactions along the fracture surface (Cochard and Madariaga, 1994; Geubelle and Rice, 1995).

Sufficiently close to the fracture tip, the stress field exhibits the asymptotic form

$$\sigma_{ij}(r,\theta,t) \sim \frac{K(v,t)}{\sqrt{2\pi r}}\,f_{ij}(\theta,v), \tag{16}$$

where $K(v,t)$ is the dynamic stress intensity factor. The corresponding energy flux into the fracture tip is

$$G(t) = \frac{K^2(v,t)}{E'}\,A(v), \tag{17}$$

where E' is the effective elastic modulus and $A(v)$ captures dynamic corrections associated with rupture velocity (Freund, 1990).

An equivalent and complementary description of fracture dynamics is obtained through a variational formulation. The total energy functional is written as

$$E[u,\Gamma] = \int_{\Omega(t)} \Psi \, d\Omega + \int_{\Gamma(t)} G_c \, d\Gamma + \int_{\Omega(t)} \frac{1}{2}\rho \mid \dot{u} \mid^2 \, d\Omega. \tag{18}$$

Dynamic fracture evolution corresponds to energetically admissible paths $u(t)$, $\Gamma(t)$ satisfying stationarity with respect to admissible variations in u and inequality constraints with respect to fracture growth (Rice, 1968; Bourdin et al., 2008).

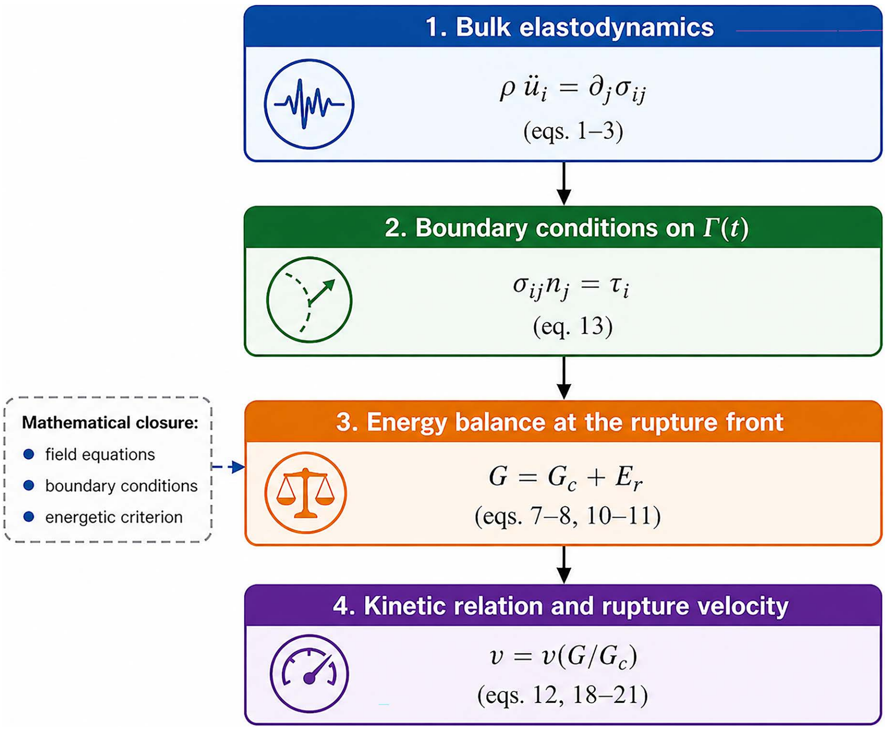

Figure 2. Mathematical structure of the dynamic fracture framework. Bulk elastodynamics and constitutive relations govern the displacement and stress fields within the elastic domain, while boundary conditions are imposed on the evolving fracture surface $\Gamma(t)$. The energetic balance at the rupture front defines admissibility conditions but does not uniquely determine rupture velocity. A kinetic relation must be specified to close the system by linking energy excess to rupture speed, highlighting the distinction between physically constrained quantities and phenomenological closure assumptions

To close the system, a dynamic relation linking energy excess to rupture velocity is required. This relation is not derived from first principles within the present framework, but must be specified as a phenomenological closure, consistent with energetic admissibility and dynamic fracture theory (Rice, 1980; Freund, 1990). In general, such relations can take different forms depending on the assumed dissipation mechanisms and near-tip physics. A simple admissible form is

$$v = c_R \left[1 - \left(\frac{G_c}{G}\right)^{\alpha}\right], \alpha > 0, \tag{19}$$

which captures both rupture onset near the fracture threshold and dynamic acceleration toward limiting wave speeds (Fig. 2). Eq. (19) should therefore be interpreted as an ansatz representing a class of admissible kinetic closures, rather than as a uniquely derived relation. Its specific functional form is not uniquely determined by first principles but is constructed to satisfy fundamental energetic constraints and asymptotic limits derived from dynamic fracture theory. It should be noted that Eq. (19) is constructed to describe rupture evolution within the sub-Rayleigh regime. Extensions to supershear propagation would require alternative kinetic relations and dynamic correction factors, reflecting the different structure of elastodynamic fields beyond the Rayleigh speed. Its functional form is chosen to satisfy key physical constraints: (i) vanishing rupture velocity as $G \to G_c$, (ii) monotonic increase of rupture speed with increasing energy excess, and (iii) asymptotic saturation as v approaches the Rayleigh wave speed. Similar phenomenological relations have been employed in dynamic fracture and earthquake rupture modeling to connect energy balance with rupture kinetics (Rice, 1980).

Different admissible kinetic closures may produce quantitatively different rupture-velocity evolutions and scaling trends, even when constrained by the same energetic admissibility condition. Accordingly, the analytical relations derived below should not be interpreted as unique predictive laws, but rather as illustrative energetic realizations associated with the specific closure family represented by Eq. (19). In this sense, Fig. 3a is intended to visualize how different choices of the phenomenological exponent α modify the mapping between energy excess and rupture velocity within the same energetic framework.

4. Analytical Results and Scaling Laws

An alternative perspective is obtained by considering the threshold condition $G(v) = G_c$, which defines the onset of rupture propagation in classical fracture mechanics. Substituting this condition into Eq. (10) yields

$$F(v) = \frac{G_c}{G_0}. \tag{20}$$

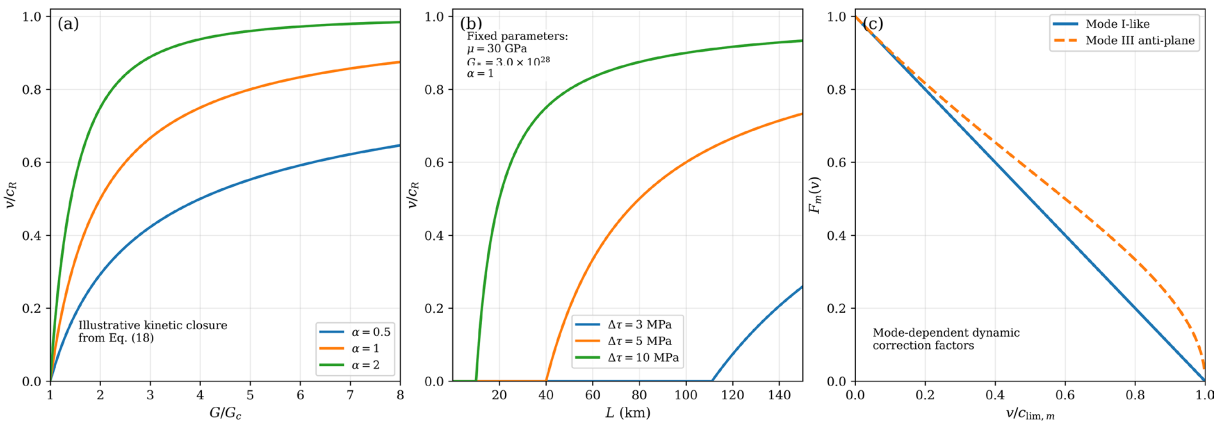

Figure 3. Analytical scaling relations and energetic structure of dynamic rupture propagation. (a) Illustrative family of admissible phenomenological kinetic closures relating normalized rupture velocity v/c_R to the energy ratio G/G_c for different values of the exponent α. The panel visualizes how different closure choices modify the energetic mapping between rupture velocity and energy excess within the same admissible framework. The point $G/G_c = 1$ corresponds to the energetic onset threshold for admissible rupture propagation, for which the kinetic closure yields vanishing rupture velocity. (b) Illustrative scaling of normalized rupture velocity with rupture dimension L for representative stress-drop levels, using fixed values of shear modulus μ, exponent α, and an effective scaling constant G_*. Very low stress-drop conditions may remain near threshold over much of the plotted range. (c) Illustrative comparison of mode-dependent dynamic correction factors in sub-Rayleigh rupture regimes. The Mode-I-like asymptotic branch exhibits a progressive reduction of the available energy flux as rupture velocity approaches the Rayleigh wave speed c_R, whereas the Mode III anti-plane shear formulation approaches zero asymptotically at the shear wave speed c_s following the analytical relation introduced in Section 2. The panel highlights that the energetic correction factor is not universal but depends on the fracture mode and rupture regime.

Eq. (20), however, should not be interpreted as an independent predictive relation for rupture velocity. Rather, it expresses a limiting consistency condition associated with the threshold state $G = G_c$. In the present framework, sustained dynamic propagation requires $G > G_c$ and therefore must be described through an additional kinetic relation, such as Eq. (19).

In this framework, the condition $G = G_c$ corresponds to vanishing rupture velocity in accordance with Eq. (13) and therefore cannot describe sustained dynamic propagation. This clarifies that Eq. (20) should be interpreted as a boundary condition rather than as a predictive relation for rupture speed.

The transition from the threshold regime ($G \approx G_c$) to dynamic propagation ($G > G_c$) is captured by the kinetic relation Eq. (18), which maps potential energy excess into rupture velocity. In this sense, Eqs. (18) and (19) describe complementary aspects of the problem: the former governs dynamic evolution, while the latter defines the admissibility threshold. Accordingly, Eq. (19) is useful for identifying the threshold regime $G \approx G_c$, whereas the dynamic branch shown in Fig. 3a is governed by the phenomenological closure Eq. (18). The two relations therefore play different roles and should not be conflated. The implications of the kinetic closure can be visualized in an energetic phase diagram in the $(G/G_c, v/c_R)$ plane (Fig. 4). In this representation, the threshold condition $G/G_c = 1$ defines the onset of admissible rupture propagation, whereas the subsequent evolution of rupture velocity depends on the adopted kinetic closure.

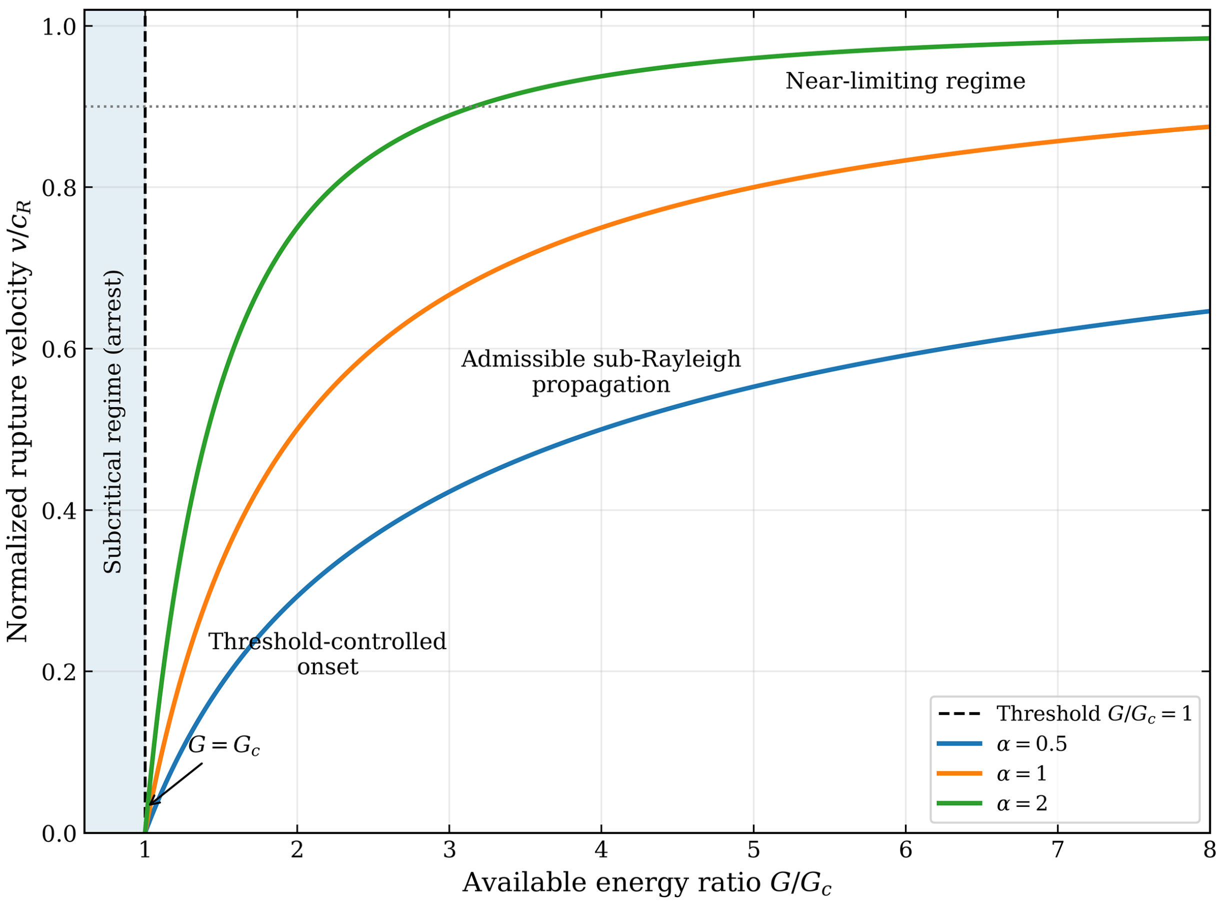

Figure 4. Analytical scaling relation Energetic phase diagram for sub-Rayleigh rupture propagation. The diagram summarizes the structure of the proposed framework in the $(G/G_c, v/c_R)$ plane. The vertical dashed line marks the energetic admissibility threshold $G/G_c = 1$, separating the subcritical regime, in which sustained rupture propagation is not possible, from the dynamically admissible regime. The solid curves represent illustrative kinetic branches derived from Eq. (19) for different values of the exponent α, showing how different phenomenological closures map energy excess into rupture velocity within the sub-Rayleigh domain. As $G/G_c \to 1$, rupture velocity tends to zero, whereas increasing energy excess drives rupture toward a near-limiting regime in which v/c_R approaches unity asymptotically. The figure emphasizes the distinction between the energetic threshold condition and the closure-dependent dynamic branch.

For shear fractures relevant to earthquakes, the quasi-static energy release rate scales as (e.g., Kanamori and Brodsky, 2004)

$$G_0 \sim \frac{(\Delta\tau)^2}{\mu} L, \tag{21}$$

where $\Delta\tau$ is the stress drop, L is a characteristic rupture dimension, and μ is the shear modulus. For the purpose of deriving the analytical scaling relation below, the parameters G_c, μ, and the closure exponent α are treated as scale-independent effective quantities.

Substituting Eq. (21) into Eq. (19) yields

$$v \approx c_R \left[1 - \left(\frac{G_c}{\mu(\Delta\tau)^2 L}\right)^{\alpha}\right], \tag{22}$$

indicating that rupture velocity increases with stress drop and rupture size (Fig. 3b), under the simplifying assumption that the effective fracture energy G_c and other material parameters remain constant. In natural fault systems, however, G_c may evolve with stress drop, rupture dimensions, accumulated slip, and dissipative processes associated with off-fault damage and distributed inelastic deformation. Under such conditions, the scaling trends predicted by Eq. (22) may become weakened, modified, or partially suppressed. Therefore, the scaling expressed by Eq. (22) should be interpreted as a conditional trend valid for fixed energetic and material parameters, rather than as a general predictive law. These scaling relations should be interpreted as closure-dependent analytical mappings between energetic ratios and rupture kinematics, rather than as universal predictive laws. Their quantitative form depends on the adopted phenomenological kinetic relation and on the specific choice of the closure parameters. Although the qualitative dependence of rupture velocity on stress drop and rupture size is robust within energetically admissible formulations, different kinetic closures may lead to different quantitative scaling trends and asymptotic behaviors. The present formulation therefore aims primarily to provide a transparent energetic framework illustrating how rupture kinematics may emerge from energetic constraints once a closure relation is specified.

It should be noted that the approach does not explicitly resolve the physical processes governing dissipation and post-front sliding along the evolving rupture interface, such as frictional weakening, off-fault damage, or inelastic deformation. As a result, while the scaling relations describe how rupture velocity depends on energetic ratios, they do not predict the detailed acceleration history nor explain why rupture velocities in natural earthquakes typically remain below the theoretical upper bound. In particular, if fracture energy increases with stress drop or rupture size, the apparent scaling between rupture velocity, $\Delta\tau$, and L may be weakened or altered.

Linearizing the kinetic relation Eq. (13) leads to

$$\frac{dv}{dt} \approx \frac{dV}{dG}\,\frac{dG}{dt}, \tag{23}$$

and dynamic rupture stability requires

$$\frac{dG}{da} > \frac{dG_c}{da}. \tag{24}$$

Using the linearized expressions for rupture response and stability introduced in Eqs. (23) and (24), a first-order approximation of the dynamic correction factor can be written in the near-critical regime. For the illustrative Mode-I-like sub-Rayleigh branch adopted in the simplified scaling of Fig. 3c, the dynamic correction factor may be approximated near the limiting velocity as

$$F_I(v) \sim 1 - \frac{v}{c_R}, \tag{25}$$

implying

$$G(v) \to 0 \text{ as } v \to c_R. \tag{26}$$

Eq. (26) reflects the assumed Mode-I-like form of the dynamic correction factor $F_I(v)$, for which the available dynamic energy release rate vanishes as v approaches c_R. This behavior should not be interpreted as a universal property of earthquake ruptures. As shown in Fig. 3c, Mode III anti-plane shear ruptures follow a different dynamic correction factor and approach their limiting velocity at c_s, whereas Mode II ruptures may exhibit still more complex behavior, including supershear propagation. Accordingly, Eq. (25) is used here only as an illustrative sub-Rayleigh asymptotic branch, not as a general dynamic correction law.

Within this simplified energetic framework, it is useful to introduce an effective scaling relation for radiated energy in order to describe how the excess energy available beyond fracture resistance may be partitioned during dynamic rupture. The radiated seismic energy scales as

$$E_r \sim \int [G(v) - G_c]\, da, \tag{27}$$

indicating that increased seismic radiation is associated with conditions in which the energy release rate exceeds the effective fracture resistance. This formulation should be interpreted as a simplified energetic partitioning, where the term $G - G_c$ represents the energy available for radiation and other dynamic processes. In general, however, seismic radiation may also occur near the propagation threshold, and the partitioning between fracture energy and radiated energy depends on additional physical mechanisms not explicitly resolved in this framework, including rupture acceleration, near-tip kinetic energy, frictional heating, off-fault inelastic deformation, and other dissipative processes. Eq. (27) should therefore be interpreted as an effective energetic scaling, rather than as a uniquely derived partition law.

5. Discussion

The framework developed in this study is intended to provide a unified physical-mathematical synthesis of dynamic fracture propagation grounded in energetic principles, explicitly building upon classical results in fracture mechanics and seismology (Griffith, 1920; Rice, 1968; Freund, 1990), rather than to introduce a new rupture theory. By explicitly coupling bulk elastodynamics, fracture energetics, and rupture kinetics, the formulation clarifies how rupture velocity, energy partitioning, and seismic radiation emerge as interdependent quantities rather than independent inputs. In particular, the framework does not aim to predict the actual rupture velocity observed in specific earthquakes, which depends on poorly constrained dissipation mechanisms and fault-zone properties. Instead, it provides a physically consistent envelope within which rupture dynamics must operate.

Figure 4 provides a compact graphical summary of this distinction, highlighting that energetic constraints define the admissible domain of rupture propagation, whereas the specific dynamic trajectory within this domain remains controlled by the phenomenological kinetic closure. Within this representation, the condition $G = G_c$ defines the energetic threshold for admissible rupture onset, whereas finite rupture velocity emerges only once excess energy above the fracture threshold is converted into dynamic propagation through the kinetic closure relation.

It is important to emphasize that the present framework does not propose new fracture mechanisms or alternative constitutive laws for earthquake rupture. Rather, its novelty lies in the explicit analytical organization of energetic constraints, elastodynamic corrections, and phenomenological kinetic closures into a unified structure that makes their logical interdependence transparent. In many existing formulations, these ingredients appear implicitly within numerical implementations or are embedded within specific frictional or constitutive assumptions. The present approach instead isolates the energetic backbone common to a broad class of rupture models and expresses it in a compact analytical form. This provides a physically interpretable reference framework for understanding how rupture velocity, energy partitioning, admissibility conditions, and dynamic corrections interact within dynamically propagating fractures.

A key result is the explicit role of the energy release rate G as the central mediator between elastic loading and rupture dynamics. While classical fracture mechanics emphasizes the Griffith criterion $G = G_c$ as a threshold condition (Griffith, 1920; Rice, 1968), numerous studies have shown that earthquake ruptures operate in a strongly dynamic regime where wave radiation plays a fundamental role (Husseini et al., 1975; Andrews, 1976; Madariaga, 1976). The present formulation makes this coupling explicit by decomposing the energy budget into fracture energy and radiated energy (Fig. 1), providing a transparent interpretation of rupture acceleration and radiation efficiency (Kanamori and Anderson, 1975; Abercrombie, 1995). It should be emphasized, however, that while the energetic decomposition itself follows directly from elastodynamic fracture mechanics, quantitative predictions of rupture speed and acceleration require the specification of a kinetic closure relating energy excess to rupture advance.

An important implication of the present formulation concerns the mode dependence of dynamic energetic corrections. In classical dynamic fracture mechanics, the relationship between rupture velocity and energy release rate depends fundamentally on the fracture mode and on the admissible elastodynamic regime. Tensile Mode I cracks, anti-plane shear Mode III ruptures, and in-plane shear Mode II ruptures are characterized by different dynamic correction factors and, in some cases, different limiting propagation velocities. The present framework therefore should not be interpreted as imposing a universal Rayleigh-wave limitation on earthquake rupture dynamics. Instead, the sub-Rayleigh energetic branch developed here represents one admissible realization within a broader class of mode-dependent and fault-complexity-dependent rupture behaviors (e.g., Zaccagnino and Doglioni, 2022). In particular, the explicit inclusion of the Mode III anti-plane shear correction factor highlights that even within sub-Rayleigh regimes the asymptotic energetic structure differs from the simplified Mode-I-like approximation adopted in the analytical scaling relations. Fully dynamic Mode II rupture, including supershear propagation, requires more general energetic corrections and remains beyond the scope of the present simplified formulation.

The mathematical structure summarized in Fig. 2 provides a conceptual overview of the interdependence between governing equations, energetic constraints, and rupture dynamics. Similar conclusions have been reached in numerical and boundary integral studies of dynamic rupture, which demonstrated the necessity of energetic closure relations to achieve physically consistent solutions (Cochard and Madariaga, 1994; Geubelle and Rice, 1995; Dunham et al., 2011). In this sense, the present framework provides a compact analytical counterpart to more computationally intensive approaches. Its primary value lies in clarifying the energetic structure common to these approaches, rather than in replacing detailed numerical or constitutive rupture models.

The analytical scaling laws derived in Section 4 offer additional physical insight. In particular, the dependence of rupture velocity on stress drop and rupture size (Fig. 3b) reflects how increasing energy availability shifts the system toward higher velocity regimes within the admissible domain, consistent with both theoretical predictions and observational inferences for large earthquakes (Kostrov, 1964; Kanamori and Anderson, 1975; Venkataraman and Kanamori, 2004). Moreover, the saturation of rupture velocity near the Rayleigh wave speed (Fig. 3c) emerges as a direct consequence of elastodynamic energy constraints, in agreement with classical results from dynamic fracture theory (Freund, 1990; Broberg, 1999). Figure 3c further illustrates that the asymptotic decay of the dynamic correction factor depends on the fracture mode, with Mode III exhibiting a different energetic structure from the Mode-I-like sub-Rayleigh approximation adopted in the simplified analytical scaling.

The energetic scaling discussed above is broadly consistent with observational constraints on earthquake rupture dynamics and seismic radiation efficiency reported in the seismological literature. Large crustal earthquakes commonly exhibit rupture velocities corresponding to a substantial fraction of the Rayleigh or shear wave speed, while remaining below the theoretical limiting velocity in most sub-Rayleigh events (e.g., Madariaga, 1976; Heaton, 1990; Venkataraman and Kanamori, 2004). Similarly, observed variations in apparent stress and radiation efficiency are generally interpreted as reflecting differences in the partitioning between fracture energy, frictional dissipation, off-fault damage, and radiated seismic energy (Kanamori and Anderson, 1975; Abercrombie, 1995; Rice, 2006). Within this perspective, seismic radiation efficiency and apparent stress can be interpreted as observational manifestations of how the excess energetic budget $G - G_c$ is partitioned between radiated seismic waves and competing dissipative processes during rupture propagation. Within the present framework, such variability can be interpreted as corresponding to different energetic trajectories and kinetic closures within the admissible rupture domain. For typical crustal earthquake parameters, stress drops of several MPa and rupture dimensions ranging from kilometers to hundreds of kilometers naturally place the system within the intermediate-to-high G/G_c regime illustrated schematically in Figs. 3 and 4, consistent with dynamically propagating sub-Rayleigh rupture.

These results can be interpreted within the unified energetic framework summarized in Fig. 4. While Fig. 3 illustrates how rupture velocity varies as a function of specific physical parameters such as stress drop and rupture length, Fig. 4 provides a complementary representation in which these effects are recast in terms of the dimensionless energy ratio G/G_c. In this representation, variations in stress drop and rupture size effectively correspond to trajectories in the $(G/G_c, v/c_R)$ phase space, moving the system across different dynamical regimes.

This combined perspective clarifies that the scaling relations are not independent empirical laws, but rather specific projections of a more general energetic structure. The threshold condition $G/G_c = 1$ defines the onset of admissible propagation, whereas the subsequent evolution toward higher rupture velocities reflects the increase of available energy and its partitioning between fracture and radiation (Kanamori and Anderson, 1975; Abercrombie, 1995; Venkataraman and Kanamori, 2004). The approach to the Rayleigh wave speed therefore appears as a geometric property of the admissible phase space rather than a consequence of a particular constitutive assumption, consistent with classical dynamic fracture theory (Freund, 1990; Broberg, 1999), while quantitative details may depend on the adopted dissipation and kinetic closure. In this respect, the specific kinetic relation adopted in this study (Eq. 19) should be viewed as a representative example rather than a unique description of rupture dynamics. Its role is to make explicit how different choices of closure translate into different rupture behaviors within the same energetic framework. In this context, well-established features of dynamic rupture, such as the limiting role of the Rayleigh wave speed and the vanishing of the energy release rate as this limit is approached, should not be regarded as new results of the present work. Instead, they emerge naturally within the proposed formulation as consequences of the energetic structure, reinforcing its internal consistency rather than constituting independent predictions.

A key limitation of the present framework is that it does not resolve the physical processes governing how rupture accelerates toward the limiting velocity. In particular, the absence of an explicit dissipation model implies that the approach cannot describe the detailed pathway to the asymptotic regime, nor can it explain why rupture velocities inferred from seismic observations are typically lower than the Rayleigh wave speed. These discrepancies are widely attributed to frictional complexity, off-fault damage, heterogeneous stress conditions, and evolving inelastic deformation processes that may locally modify the effective elastic response itself. Such effects are not explicitly represented within the present small-strain energetic formulation, which assumes an approximate separation between elastic energy release and dissipative processes, but could in principle be incorporated through more general extensions of the framework. If the effective fracture energy increases systematically with rupture dimensions, stress drop, or cumulative slip due to enhanced off-fault damage and distributed inelastic dissipation, the analytical scaling relations derived under constant-G_c assumptions may become substantially modified or partially suppressed. Frictional energy loss outside the principal slip zone and distributed microcracking around the rupture front may substantially modify the effective energetic budget of dynamic rupture (Andrews, 2005; Yamashita, 2000).

An important conceptual distinction concerns the respective roles of near-tip fracture energetics and frictional sliding during earthquake rupture. In the present framework, rupture propagation is governed at leading order by the energetic balance at the rupture tip and by the associated elastodynamic energy flux, consistent with classical dynamic fracture mechanics. Frictional constitutive laws primarily describe the mechanical behavior of the already-formed sliding interface behind the rupture front, controlling stress drop evolution, heat production, and post-front slip history rather than the fundamental energetic admissibility of rupture advance itself. In this sense, friction should be regarded as an emergent property of the evolving fault interface rather than as the primary mechanism governing crack-tip propagation. Moreover, weakly nonlinear theories of dynamic fracture have shown that the near-tip region may depart from the predictions of linear elastic fracture mechanics due to large strain effects and nonlinear elastic corrections (e.g., Bouchbinder et al., 2014). These nonlinear near-tip processes are not explicitly resolved within the present coarse-grained energetic formulation, which instead focuses on the large-scale energetic structure of dynamically propagating rupture. Consistent with recent studies emphasizing the spatial scale dependence of fault physical parameters (e.g., Zaccagnino et al., 2025), the present framework therefore distinguishes between energetic processes associated with crack-tip propagation and those governing frictional sliding and dissipation along the mature fault interface.

An additional limitation is that the present formulation does not account for supershear rupture regimes, which are known to occur in Mode II faulting under favorable conditions. This limitation is particularly relevant in light of recent results showing that transonic and supershear crack propagation may arise even for tensile cracks when geometric nonlinearities are included (Pundir et al., 2024). In such cases, rupture velocities can exceed the Rayleigh wave speed and approach or surpass the shear wave speed, requiring modified energetic formulations. The current framework is therefore restricted to sub-Rayleigh rupture dynamics.

An important implication of this energetic perspective concerns the interpretation of radiated seismic energy. Eq. (26) shows that radiation efficiency is controlled by the ratio between the energy release rate and the effective fracture resistance, rather than by rupture velocity alone. This result provides a natural bridge between fracture mechanics and seismological observations of variable apparent stress and radiation efficiency (Abercrombie, 1995; Venkataraman and Kanamori, 2004), without requiring ad hoc assumptions on frictional behavior. Nevertheless, the mapping between observed radiation efficiency and underlying rupture processes remains sensitive to how fracture energy and other dissipative mechanisms are parameterized.

It is important to emphasize that this representation is an idealized energetic partition. In dynamic rupture processes, the separation between fracture energy and radiated energy is not strictly binary, and part of the energy flux may be continuously redistributed between near-tip dissipation and wave radiation. Therefore, the condition $G > G_c$ should not be interpreted as a strict threshold for the onset of radiation, but rather as an indicator of the energetic regime in which radiation becomes increasingly efficient.

Finally, the generality of the formulation helps clarify how energetic constraints can be extended to more complex rupture scenarios, while also delineating the limits of the present framework. Velocity-dependent fracture energy, spatial heterogeneity, and coupling with off-fault damage or inelastic deformation can be incorporated within the same energetic framework, consistent with previous studies emphasizing the role of damage and frictional complexity in earthquake rupture (Ben-Zion and Sammis, 2003; Ampuero and Rubin, 2008; Rice, 2006). As such, the present model provides a flexible theoretical basis for interpreting both numerical simulations and observational constraints on dynamic earthquake rupture. More general formulations including nonlocal constitutive behavior, internal material length scales, and evolving damage-dependent elastic properties would require extensions beyond the present local energetic framework.

Open challenges remain in quantifying fracture energy in heterogeneous fault systems, in capturing the coupling between near-tip and off-fault dissipation, and in assessing how scale-dependent processes modify effective energetic budgets – issues that require integration of energetic frameworks with laboratory experiments, numerical simulations, and observational data.

6. Conclusions

In this study, we recast the energetic backbone of dynamic fracture and earthquake rupture propagation into a compact analytical form suitable for interpretation and benchmarking. By combining elastodynamic energy release with an energetic admissibility condition and an explicit kinetic closure, the approach makes transparent which aspects of rupture speed and saturation follow from radiation-modified energy release and which depend on the adopted kinetic relation. The resulting scaling relations provide illustrative closure-dependent connections between rupture speed, stress drop, rupture size, and fracture energy within the adopted energetic formulation, offering an interpretable baseline against which more detailed constitutive or numerical rupture models can be compared.

The analysis highlights the central role of the energy release rate G as the key quantity linking elastic loading to rupture propagation. Rather than treating rupture velocity or radiation efficiency as prescribed parameters, the proposed framework shows how both quantities are constrained by the balance between available elastic energy, fracture resistance, and elastodynamic wave radiation, within the limits of the adopted energetic description. This energetic perspective naturally explains the existence of rupture velocity limits and the observed variability in seismic radiation efficiency.

A key result is the derivation of analytical scaling relations linking rupture velocity to stress drop and rupture size. These relations demonstrate that, within the adopted constant-G_c energetic approximation, faster rupture propagation is favored by larger stress drops and larger rupture dimensions. Importantly, these results are obtained without specifying a particular friction law in explicit constitutive form, although their quantitative realization remains dependent on the adopted kinetic closure.

The mathematical formulation presented here treats dynamic fracture as a moving boundary-value problem, in which rupture propagation is governed by an energetic admissibility condition and a kinetic relation linking energy excess to rupture speed. This structure provides a compact analytical counterpart to more complex numerical and boundary integral models and clarifies the logical connections between governing equations, boundary conditions, and rupture dynamics.

Overall, the proposed framework offers a transparent and physically grounded basis for interpreting earthquake rupture processes across a wide range of scales. Its generality allows for straightforward extensions to include spatial heterogeneity, velocity-dependent dissipation, and coupling with off-fault damage or inelastic deformation. As such, it provides a useful reference model for future theoretical developments and for the interpretation of numerical simulations and observational constraints on dynamic earthquake rupture. The proposed energetic framework therefore also provides an idealized reference structure for interpreting how excess rupture energy may be partitioned between seismic radiation and competing dissipative processes in natural earthquakes.

Although the present formulation is not intended as a predictive rupture inversion model, its energetic structure remains broadly compatible with observed ranges of earthquake rupture velocities and seismic radiation efficiencies reported in natural earthquakes.

Its primary contribution is therefore conceptual, structural, and interpretative rather than constitutive. The framework does not seek to replace detailed numerical rupture simulations or physically complete constitutive descriptions, but instead to provide a transparent analytical reference structure that clarifies the energetic relationships linking rupture admissibility, dynamic corrections, kinetic closure assumptions, and rupture kinematics. In this sense, the proposed formulation is intended as a physically interpretable benchmark framework that may facilitate the comparison, interpretation, and conceptual organization of more complex dynamic rupture models.

Data availability statement. This study is theoretical in nature and does not use external datasets. All information necessary to evaluate the results is contained within the manuscript

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