

Love wave dispersion in anisotropic visco-elastic medium

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ABSTRACT

The paper presents a study on Love wave propagation in a anisotropic visco-elastic layer overlying a rigid half space. The characteristic frequency equation is obtained and the variation of the wave number with frequency under the combined effect of visco-elasticity and anisotropy is analysed in detail. The results show that the effect of visco-elasticity on the wave is similar to that of anisotropy as long as the coefficient of anisotropy is less than unity.

RIASSUNTO

La nota presenta uno studio sulla propagazione delle onde di Love in uno strato anisotropo visco-elastico che giace su uno spazio semi rigido. E' stata ricavata l'equazione caratteristica della frequenza, ed è stata

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analizzata dettagliamente la variazione del numero d'onda con la frequenza, sotto l'effetto combinato della visco-elasticità e dell'anisotropia. I risultati mostrano che l'effetto della visco-elasticità sul numero d'onda è simile a quello dell'anisotropia, finchè il coefficiente di anisotropia è inferiore all'unità.

1. INTRODUCTION

Study of the propagation behaviour of surface waves in a visco-elastic and anisotropic models is of great interest for the accurate inversion of the observed surface wave data. The results of numerical model studies of Schwab and Knopoff (1972), for an anelastic medium, and those of Crampin and Taylor (1971), for anisotropic medium, shown that both these factors can have a considerable influence on the propagation behaviour of surface waves. In order to understand their propagation behaviour in more realistic conditions it will be interesting to study their propagation in visco-elastic anisotropic medium.

In the present paper we examine the dispersion behaviour of Love wave in a simple two layer model, consisting of a visco-elastic (Voigt type) anisotropic layer overlying a rigid half space. The results are then compared with those of Negi and Upadhyay (1968) who have studied the effect of anisotropy on the Love wave dispersion, for a similar model.

2. MATHEMATICAL FORMULATION

The geometry of the problem is illustrated in Fig. 1. Rectangular co-ordinate system is considered with the Z-axis directed vertically downward and the X-Y plane coinciding with the free surface. Considering Love wave propagation in the positive X-direction, neglecting body forces and assuming small defor-

mations, the equation of motion of Love wave in a visco-elastic anisotropic layer can be written as:

$$\frac{\partial}{\partial t^2} \frac{\partial^2 v}{\partial x^2} = \left(N + N' \frac{\partial}{\partial t} \right) \frac{\partial^2 v}{\partial x^2} + \left(L + L' \frac{\partial}{\partial t} \right) \frac{\partial^2 v}{\partial x^2} \quad [1]$$

where,

N is the modulus of rigidity and N' is the coefficient of visco-elasticity in the X -direction.

and

L and L' are the corresponding quantities in the Z -direction.

In representing a harmonic wave travelling in the X -direction we choose a solution of the following form:

$$\phi = V(Z) \exp [i(\omega t - KX)] \quad [2]$$

where, $V(Z)$ is areal function of Z and K is a complex quantity equal to $(k + ik)$, whose real part represents the wave number and the imaginary part represents the damping coefficient. Putting this value of $V(Z)$ in equation [1] we get:

$$(L + iL') \frac{d^2 V}{dZ^2} + [\rho \omega^2 - K^2 (N + iN' \omega)] V = 0 \quad [3]$$

The above equation reduces to:

$$\frac{d^2 V}{dZ^2} + S^2 V = 0 \quad [4]$$

and its solution can be written as:

$$V = A \exp (iSZ) + B \exp (-iSZ) \quad [5]$$

where,

A and B are constants and S is complex quantity given by the relation:

$$S^2 = \frac{\rho\omega^2 - K^2(N + iN'\omega)}{L + iL'\omega} \quad [6]$$

Boundry conditions of zero stress at the free surface and of zero displacement at the interface require that:

$$\frac{dV}{dZ} \Big|_0 = 0 \quad \text{at } Z = 0 \quad [7]$$

and

$$V = 0 \quad \text{at } Z = H \quad [8]$$

From equations [5, 7] and [8] we get:

$$A = B \quad [9]$$

and

$$A \exp(iSH) + B \exp(-iSH) = 0 \quad [10]$$

equations [9] and [10] yield,

$$\exp(2iSH) = -1$$

or

$$S^2 H^2 = (n + 1/2)^2 \lambda^2 \quad [11]$$

putting the value of S^2 in equation [10] and separating into real and imaginary parts, we get:

$$\begin{aligned} & RH^2 [w_n^2 (1/\beta_o^2 - aQ) - a - bw_n Q (p-1)] / C + \\ & - iRH^2 [w_n^2 \left(\frac{Qw_n}{\beta_o^2} + bQ^2 \right) + b + aw_n Q (p-1)] = \\ & = C \left(n + \frac{1}{2} \right)^2 \lambda^2 \end{aligned} \quad [12]$$

where,

$$\frac{L}{L'} = Q \quad \text{and} \quad \frac{N}{N'} = pQ \quad [13]$$

By definition (Yanakawa and Sato, 1964), $1/Q$ represents the specific dissipation factor for shear wave.

$$R = N/L \quad , \quad \sqrt{\frac{1}{N/\rho}} \quad , \quad C' = (1 + Q^2 w_n^2)$$

and

$$a = k_o^2 - k_1^2 \quad , \quad b = 2 k_o k_1$$

equating real and imaginary parts and putting $g = 1$, we get:

$$RH^2 [(w_n/\beta_o)^2 - aC'] = (n + 1/2)\lambda^2 C' \quad [14]$$

$$\frac{w_n^3 Q}{\beta_o^2} + bC' = 0 \quad [15]$$

equations [14] and [15] yield:

$$a = \frac{w_n^2}{C' \beta_o^2} - \frac{(n + 1/2)^2 \lambda^2}{RH^2} \quad [16]$$

$$b = \frac{w_n^3 Q}{C' \beta_o^2} \quad [17]$$

wave number k_o an damping coefficient k_1 for various modes are given by the relations:

$$\begin{aligned} k_o(n) &= \sqrt{\frac{1}{2} \left(\sqrt{a^2 + b^2} + a \right)} \\ k_1(n) &= \sqrt{\frac{1}{2} \left(\sqrt{a^2 + b^2} - a \right)} \end{aligned} \quad [18]$$

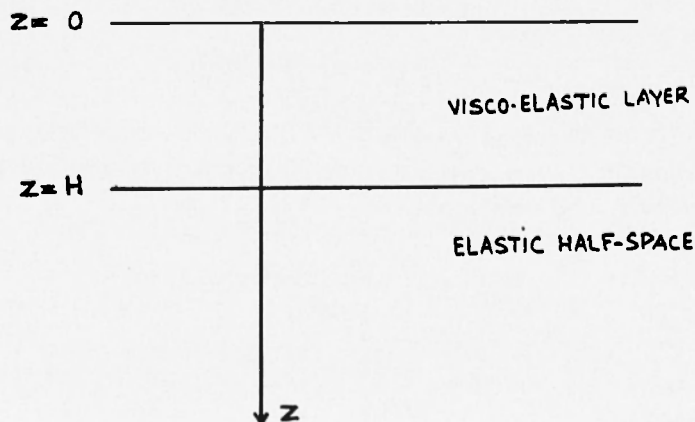
Equation [18], considered along with [16] and [17] shows $k_o(n)$ would be real for all values of w and thus the propagation should take place at all frequencies without any cut off. However, when $Q \ll 1$ (as is the case for real earth), the contribution of ' b ' at low frequencies will be negligible and therefore the dispersion behaviour of Love waves will be same as for elastic earth.

3. WAVE NUMBER VARIATION AND THE EFFECT OF VISCO-ELASTICITY ON THE PHASE GROUP VELOCITIES

Equations [18] along with [16] and [17], has been used to analyse the wave number and, phase and group velocities for following two cases, assuming $Q = 50$. The value of Q of this order for upper crust have been reported by Mitchell (1973).

3.1 *Isotropic visco-elastic case ($R = 1$)*

From the results given in Table 1, it can be readily observed that for a visco-elastic surface layer, the value of $k_o H$ is smaller than its corresponding value for a perfectly elastic layer. The effect of visco-elasticity, is therefore to decrease the wave



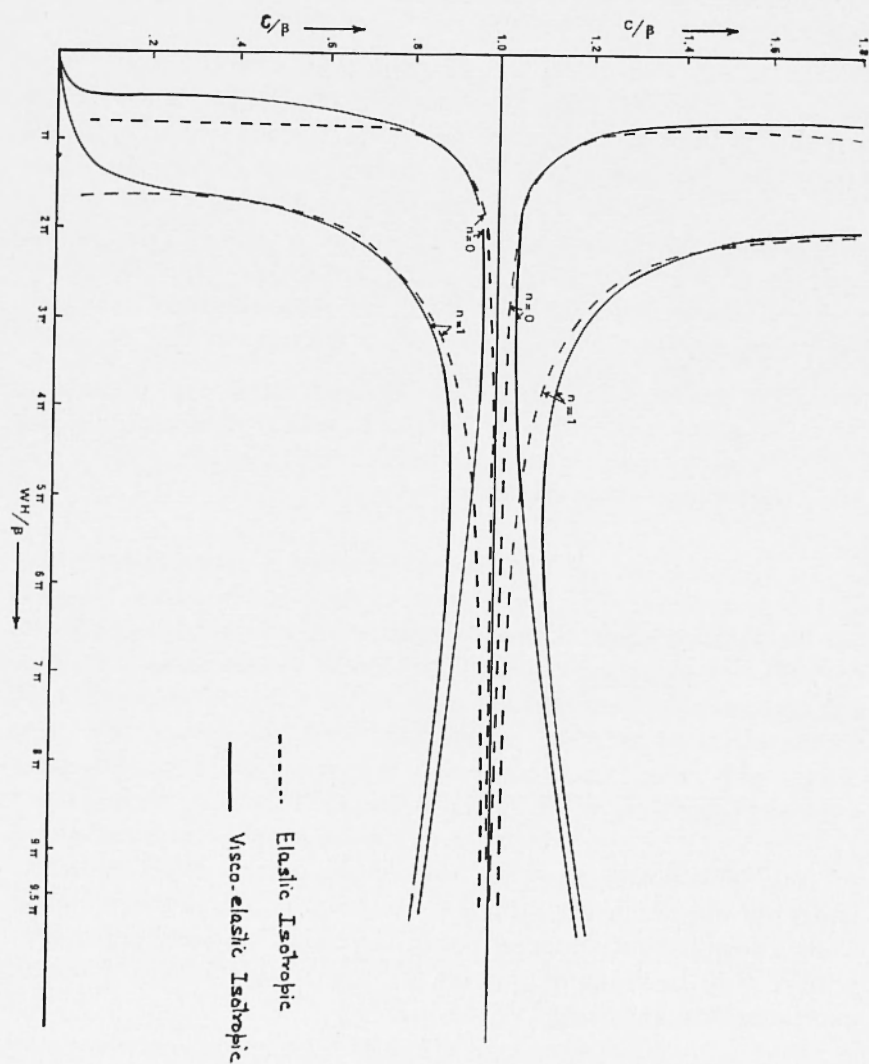
number. This result is qualitatively similar to that brought about by anisotropy ($R < 1$), (Negi and Upadhyay, 1968 except that in the case of visco-elasticity, the effect goes on increasing with frequency.

To study the effect of visco-elasticity on the phase and group velocities, equation [14] has been analysed and the results are shown in Fig. 2. Unlike the perfect elastic case in which the phase and group velocities converge to a common limit as $w_n \rightarrow 0 \cdot C$ (Hudson, 1962); in the present case the phase and group velocities start diverging after a certain value of w_n , because of the increasing effect of visco-elasticity. However, at low frequencies when the effect of visco-elasticity is small, the phase and group velocities follow the same trend as found in the case of a perfectly elastic case.

3.2 Anisotropic visco-elastic case

a) For $R > 1$ — Comparing the results given in column 5 and 6 of table 2, with those in column 2 of the same table, it can be observed that, whereas the effect of anisotropy is to increase the k_o value, visco-elasticity tends to decrease it. Two effects therefore counteract each other. Since the effect of visco-elasticity goes on increasing with frequency, the two effects will annul each other at a frequency w_e (say). For frequencies higher than w_e , the visco-elastic effect will be predominant and there will be a net decrease in $k_o H$ under the combined effect of two. This is qualitatively equivalent to the effect of an anisotropic elastic layer with $R < 1$, (Negi and Upadhyay, 1968). Thus under the combined effect of visco-elasticity and anisotropy with $R > 1$, the medium appears as being simply anisotropic with coefficient of anisotropy $R < 1$.

b) For $R < 1$ — Since the effect of both visco-elasticity and anisotropy with coefficient $R < 1$, is to decrease the $k_o H$ value the combined effect of two is to enhance the effect of each other. The results computed for $R = 0.69$ are shown in column 3 and 4 of Table 2.



4. EQUIVALENCE OF PHASE VELOCITY

At a frequency w_e where the effect of visco-elasticity and anisotropy with coefficient $R > 1$, annul each other, the waves will propagate with same phase velocity as they will do in a isotropic elastic medium.

Equating k_e value for a wave propagating with frequency w_e in a visco-elastic anisotropic medium, eq. [18], to its corresponding value in the isotropic elastic model (Hudson, 1962 eq. [23]) we get:

$$\frac{1}{2} \left(\sqrt{a^2 + b^2} + a \right) = \left(\frac{w_e H}{\beta_0} \right)^2 - \left(n + \frac{1}{2} \right)^2 \lambda^2 \quad [19]$$

Substituting the values of a and b in the above equation we get:

$$4R^2 [C^2 (M - P)^2 - (M - P) MC^3] + 4(M - P) C^3 PR - M^2 J^2 = 0 \quad [20]$$

where,

$$M = (w_e H / \beta_0)^2, \quad P = (n + 1/2) \quad \text{and} \quad J = w_e Q$$

therefore we get

$$R = \frac{P \pm \sqrt{p^2 + M^2 J^2 [(1/C^4) - M/(M - P) C^3]}}{2 \left[\frac{(M - P) - M}{C} \right]} \quad [21]$$

From the above equation one can deduce the value of the coefficient of anisotropy which will annul the known visco-elastic effect at a given frequency.

5. CONCLUSION

From the above discussion of the results it can be concluded that the effect of the visco-elasticity on the wave number variation with frequency is similar to that produced by anisotropy with coefficient <1 . When the coefficient of anisotropy is >1 , the effect of visco-elasticity is reflected as a decrease in the effect of anisotropy. For a given frequency there will be a combination of the coefficients of visco-elasticity and anisotropy which will cancel the effect of each other and the propagation will take place as in isotropic elastic medium.

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TABLE 1

Comparison of the wave number variation with frequency for elastic $KH(E)$ and visco-elasticity $KH(V E)$ cases for first three modes.

$\frac{\omega_n H}{\beta_o}$							
	0.5	0.8	1.2	2.0	3.0	5.0	8.0
Fundamental Mode	$kH(E)$	—	0.6245	1.0908	1.9365	2.9580	4.9749
	$kH(VE)$		0.6232	1.0866	1.9174	2.8955	4.7054
1st Mode	$kH(E)$	—	—	—	1.3229	2.5981	4.7697
	$kH(VE)$				1.3065	2.5342	4.4967
2nd Mode	$kH(E)$	—	—	—	—	1.6583	4.337
	$kH(VE)$					1.6287	3.9862

TABLE 2

Comparison of the wave number variation with frequency for elastic isotropic $KH(E)$, $R = 1.0$, anisotropic $KH(EA)$, $R = 0.6844$ and visco-elastic anisotropic $KH(VEA)$, $L/L' = 50$ cases for fundamental mode.

$\frac{\omega_n H}{\beta_0}$	$R = 1.0$	$R = 0.68$		$R = 1.44$	
	$KH(E)$	$KH(EA)$	$KH(VEA)$	$KH(EA)$	$KH(VEA)$
0.8	0.6245	0.5219	0.5208	0.6829	0.6816
1.2	1.0908	1.0355	1.0313	1.1253	1.1211
1.6	1.5198	1.4806	1.4707	1.5447	1.5349
2.0	1.9364	1.9058	1.8867	1.9561	1.9371
2.5	2.4494	2.4253	2.3885	2.4650	2.4284
3.0	2.9580	2.9380	2.8754	2.9709	2.9085

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