The determination of earthquake mechanism, using both longitudinal and transverse waves

V. I. KEYLIS - BOROK

Introduction

The practical aspect of a method of determining earthquake mechanism (the "fault plane solution") is dealt with here. This method has much in common with the well-known methods of Byerly (especially, taking into account the improvements suggested by A. R. Ritsema) and of Japanese seismologists: the principal difference lies in the fact that transverse waves are widely used here, which renders the solution unambiguous (*). The worked out theoretical principles permit as well to use simply the amplitude ratio of different waves, but in practice it is less reliable.

In § 1 the theoretical properties of the waves caused by the dipole with moment are described. Such model of a seismic source is the most often encountered. The method is applicable to any other models as well, but their theoretical features are considered very briefly.

We do not deal with the effect of the interfaces and the inhomogeneity of the medium on the form and intensity of displacements; the methods of eliminating of these factors are described in detail in an other article.

In § 2 the general pattern of the interpretation is given.

In §§ 3-6 the successive steps of the interpretation of observations are presented.

(*) The experience in interpretation of a great number ($\sim$ 300) of sources (and even the correlation of S-phases at different stations) acquired since 1947, testifies that the study of transverse waves is as reliable as PP, PKP, pP etc. (provided the seismograms and not the questionnaires are used).

The Wolf stereographic projection used for interpretation is described in the Supplement.

§ 1. – Initial formulas

Earthquake foci are generally equivalent to the dipole with moment (fig. 1). The axis x corresponds to the motion direction, $y = 0$ – to the fault plane.

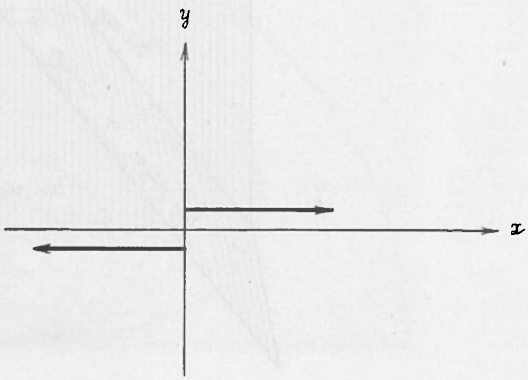


Fig. 1 – The dipole with moment (the symmetrical fault).

In this paragraph the main properties of elastic waves caused by the dipole with moment (and some other sources) in a homogeneous medium will be considered.

1. *The formulas for displacements in a homogeneous medium at great distance "r" from the source.*

At great distances r the displacement in a longitudinal wave is directed along the ray and is completely determined by one (any) component. We shall consider the value of the total displacement vector u_a; u_a is regarded positive when it is directed *from* the source.

In a transverse wave the displacement is determined by two components (the third one can be found from the condition that the displacement vector is perpendicular to the ray). We shall consider two independent components, u_b^P in the plane of incidence (SV wave) and u_b^H in the direction perpendicular to the plane (SH wave) — see fig. 2. u_b^H is regarded positive when it is directed *clockwise*, assuming the focus to be in the centre (or to the right if one looks at the observational point from the focus). u_b^P is positive when its horizontal component is directed *from* the focus. For instance, fig. 2 shows $u_b^H > 0$ and $u_b^P > 0$.

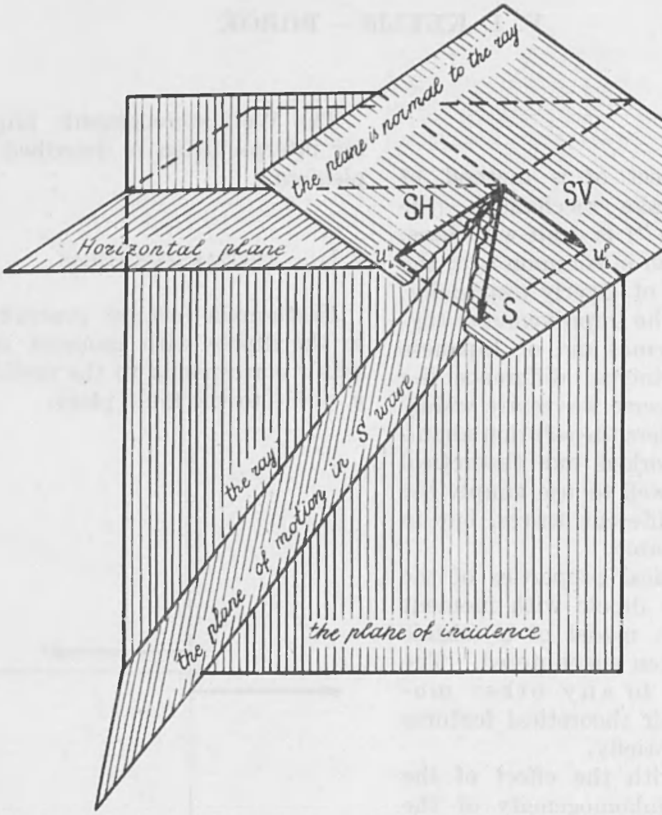


Fig. 2 — The resolution of S-wave into SV and SH.

It would be convenient to introduce two systems of coordinates (x, y, z) and (x, y, z) (the beginning of both systems coinciding with the focus) so that to obtain more compact formulas for u_a, u_b^H, u_b^P. The axes x and y relate to the direction of the dipole axes as it is shown in fig. 1. The axis x is directed to the East, y to the North, z upward.

At great distances r the formulas for independent components are as follows [V. I. Keylis-Borok et al., 1957]:

$$4\pi\rho u_a \sim \frac{xy}{a^3 r^3} K'(t - r/a) \qquad [1]$$

$$4\,\pi \varrho u_b^H \simeq -\frac{y g^H}{b^3 r^3} K' \ (t - r/b) \qquad [2]$$

$$4\,\pi \varrho u_b^P \simeq -\frac{y g^P}{b^3 r^3} K' \ (t - r/b) \qquad [3]$$

Here,

ϱ — density; a, b — longitudinal and transverse wave velocities; $K\,(t)$ intensity-time function.

$$g^H = \frac{\bar{y}\alpha_x - \bar{x}\beta_x}{\sin i} \qquad [4]$$

$$g^P = \frac{x \cos^2 i - \bar{z}\gamma_x}{\sin i \ \cos i} \qquad [5]$$

i — angle of incidence $\left(\sin\ i = \frac{\sqrt{x^2 + y^2}}{r}\right)$

2. *The nodal lines and the distribution of signs of u_a, u_b^H, u_b^P in each point.*

The *nodal surfaces* where all or some of the displacements u_a, u_b^H, u_b^P come out zero can be easily obtained from formulas [1]-[3].

All the nodal surfaces go through the focus. However, in practice the diplacements vanish only on that part of the surfaces where the values of r are sufficiently great (i. e., where approximate formulas [1]-[3] are valid). Formulas [1]-[3] give the following nodal lines for the dipole with moment:

a) plane $y = 0$, where $u_a = u_b^H = u_b^P = 0$.

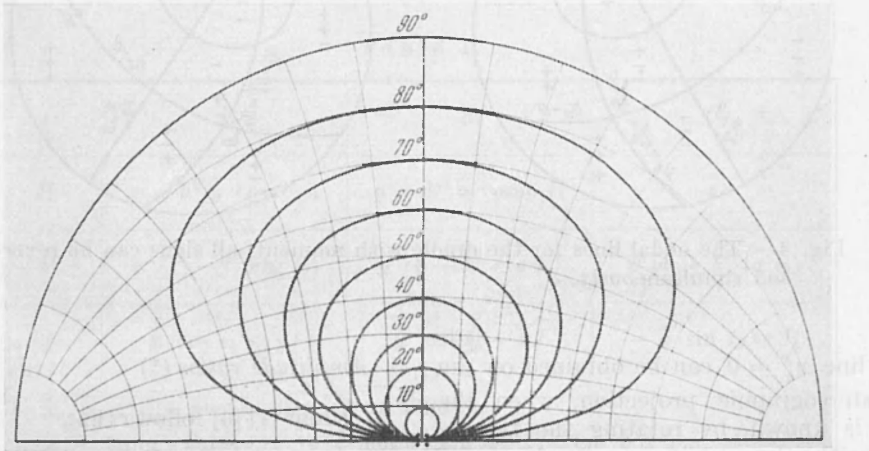


Fig. 3 - The nodal lines for SV-wave on a Wolf stereographic projection.

α_x; β_x; γ_x — angles made by the axis x andthe axes $\bar{x}$, $\bar{y}$, $\bar{z}$ respectively (so that $x = \bar{x}\alpha_x + \bar{y}\beta_x + \bar{z}\gamma_x \ldots$ [6]). The formulas for other sources may be derived from [1], [2], [3] if to replace the factor "y" by the following symbols: a or b for simple force; x for the dipole without moment; y for the superposition of the dipoles with moment and without moment (see fig. 5*d*); y^2/a or y^2/b — for the double dipole with moment . . . and so on; for various sources K is represented in various units, which can be neglected in the present article.

b) plane $x = 0$, where $u_a = 0$,

c) plane $g^H = 0$ or $\bar{x}_c \cdot \bar{y} - \bar{y}_c \cdot \bar{x} = 0$, where $u_b^H = 0$.

Here $\bar{x}_c$, $\bar{y}_c$ are the coordinates of any point on the axis x. This plane is vertical and includes the axis x.

d) cone $g^P = 0$, or $x(\bar{x}_c\bar{z} - \bar{z}_c\bar{x}) + y(\bar{y}_c\bar{z} - \bar{z}_c\bar{y}) = 0$, where $u_b^P = 0$.

This one is an elliptical cone; its opposite rulings are the axes $\bar{z}$, and x. The cone axis lies in the plane xz and bisects the angle made by x and z.

The projection of the cone on a horizontal plane is a circle the diameter of which connects the epicentre with the axis x.

The cone projection on a Wolf stereographic projection is an oval passing through the projection of the axis z (the centre) and the projection of the axis x; a family of such ovals for various inclinations of the axis x is given in fig 3; the line of symmetry in fig. 3 coincides with $u_b^H = 0$.

is also of great importance. All possible combinations of signs (top to bottom: u_a, u_b^H, u_b^P) are shown in fig. 4. All the signs certainly can be *simultaneously* reversed. The nodal lines are unambiguously related to the main parameters of dislocation in a focus: $y = 0$ determines the fault plane; axis x (a point on this plane) — the motion direction.

Fig. 5 shows the nodal lines and the correlation of signs for some other sources. For details see [Keylis-Borok et al., 1957].

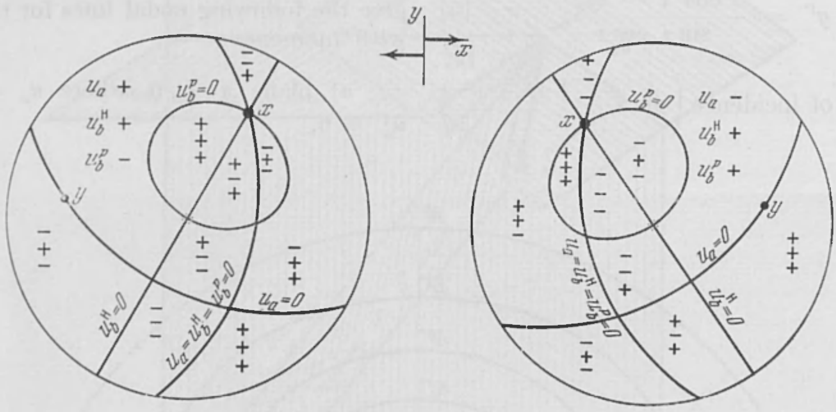


Fig. 4 – The nodal lines for the dipole with moment; all signs can be reversed simultaneously.

The line $u_b^P = 0$ can be obtained on the Wolf stereographic projection, when the axis x is known, by rotating the tracing paper with the axis x on it about the centre of fig. 3 so that the axis x should coincide with the line of symmetry; then the oval in fig. 3 passing through the axis x will represent the line $u_b^P = 0$.

Interpretation requires the knowledge of the theoretical position of the nodal lines on a Wolf stereographic projection. They are given in fig. 4. $x = 0$ and $y = 0$ are the projections of two perpendicular planes.

$u_b^H = 0$ is a straight line passing through the centre and the axis x (the pole of the line $x = 0$). $u_b^P = 0$ is one of the ovals from fig. 3.

The distribution of the signs of u_a, u_b^H, u_b^P in different regions between the nodal lines

3. *Amplitude ratios* (*)

From [1]-[6] follows that

$$k \frac{u_a}{u_b^P} = \frac{\bar{x}}{g^P} = \frac{\bar{x}\alpha_x + \bar{y}\beta_x + \bar{z}\gamma_x}{\bar{y}\alpha_x - \bar{x}\beta_x} \sin i \qquad [7]$$

$$k \frac{u_a}{u_b^P} = \frac{\bar{x}}{g^P} = \qquad [8]$$

$$= \frac{\bar{x}\alpha_x + \bar{y}\beta_x + \bar{z}\gamma_x}{(\bar{x}\alpha_x + \bar{y}\beta_x)\cos^2 i - \bar{z}\gamma_x \sin^2 i} \sin i \cos i$$

$$\frac{u_b^P}{u_b^H} = \frac{g^P}{g^H} = \frac{(\bar{x}\alpha_x + \bar{y}\beta_x)\cos^2 i - \bar{z}\gamma_x \sin^2 i}{(\bar{y}\alpha_x - \bar{x}\beta_x)\cos i} \qquad [9]$$

(*) This point may be omitted if as usually we use only the displacement signs.

Formulas [1]-[3] give $k = \frac{a^3}{b^3}$: however, in practice it may be *considerably less* due to the fact, that the source is not a point, and the medium is not ideally elastic.

Let

$$k \frac{u_a}{u_b^H} = h^H \quad : k \cdot \frac{u_a}{u_b^P} = h^P \; ; \; f = \frac{u_b^P}{u_b^H} \; .$$

Then [7]-[9] can be easily written down as follows:

$$A_H \alpha_x + B_H \beta_x + C_H \gamma_x = 0 \qquad [7a]$$

$$A_P \alpha_x + B_P \beta_x + C_P \gamma_x = 0 \qquad [8a]$$

$$A_f \alpha_x + B_f \beta_x + C_f \gamma_x = 0 \qquad [9a]$$

The formulas for A, B, C are given in table 1. These formulas are also true for the sources (*b*), (*c*), (*a*), (*f*) (fig. 5).

if the medium were homogeneous. Then applying the elastic wave theory for a homogeneous medium we determine the source equivalent to the focus.

The interpretation includes the following steps:

1. The determination of the initial observations (ground displacement components).

2. The reduction of observations to a homogeneous medium.

a) the eliminating of the effect of interfaces and of the deflection of a ray from a straight line.

b) the plotting of the initial observations on a Wolf stereographic projection (see Supplement).

Table 1

i	A_i	B_i	C_i
H	$\bar{x} - h^H \bar{y}$ cosec i_h	$\bar{y} + h^H \bar{x}$ cosec i_h	$\bar{z}$
P	$\bar{x}$ $(1 - h^P$ ctg $i_h)$	$\bar{y}$ $(1 - h^P$ ctg $i_h)$	$\bar{z}$ $(1 + h^P$ tg $i_h)$
f	$\bar{x}$ cos $i_h - \bar{y} f$	$\bar{y}$ cos $i_h + \bar{x} f$	$- \bar{z}$ sin i_h tg i_h

$\bar{x}$, $\bar{y}$, $\bar{z}$ can be substituted as follows:

$\bar{x} = \sin a \cdot \sin i_h$; $\bar{y} = \cos a \cdot \sin i_h$; $\bar{z} = \cos i_h$

§ 2. – General pattern of the interpretation of observations.

The initial data for interpretation are the signs of the arrivals of longitudinal and transverse seismic waves. The worked out theory permits to use amplitude ratios as well (without any additional computations, with the help of special nomograph — figs. 13), however in fact it is often considerably less reliable.

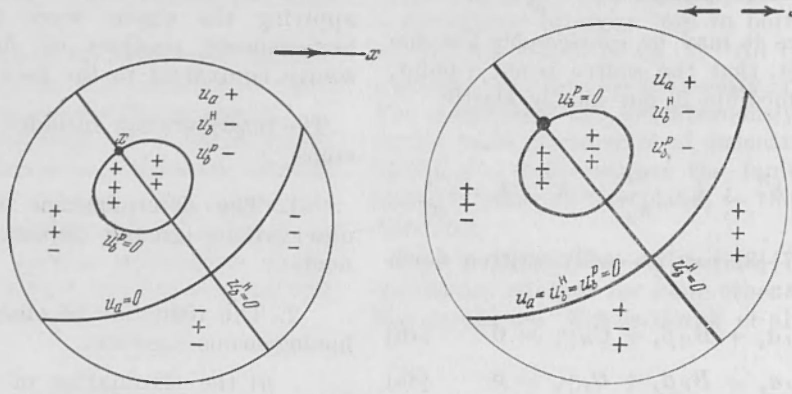
The interpretation of observations first implies the determination of the initial data

3. The determination of the dynamic parameters of a focus (fault plane solution).

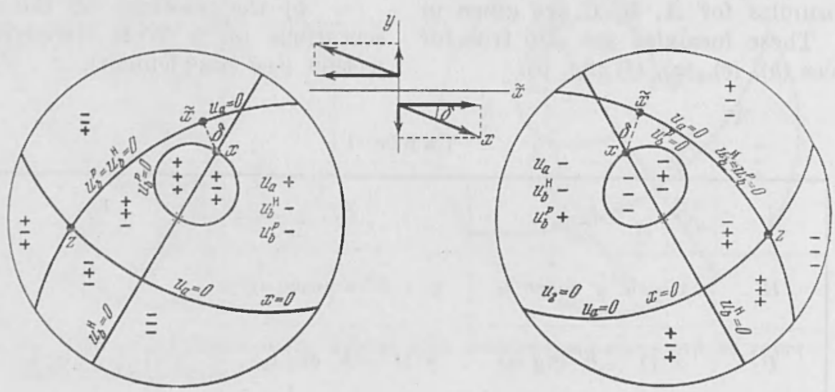
a) The drawing of nodal lines basing upon the signs of displacements and their correlation in each point, the amplitude ratio being taken into account whenever possible.

b) The estimation of the accuracy of interpretation.

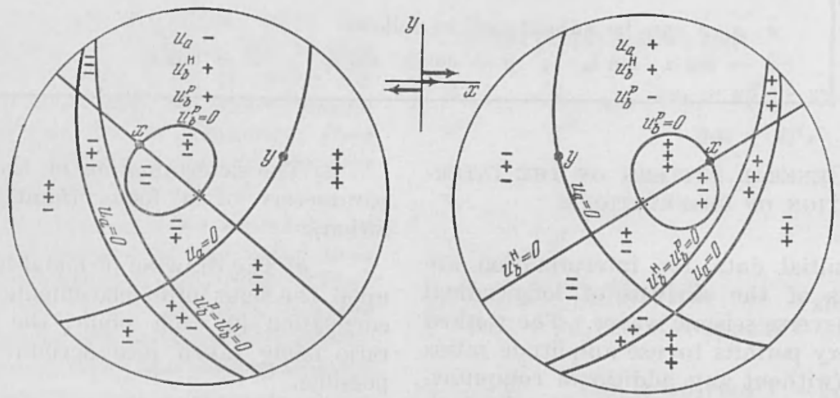
Now we shall describe all the successive steps.



a) the simple force (very asymmetrical fault) *b*) the dipole without moment

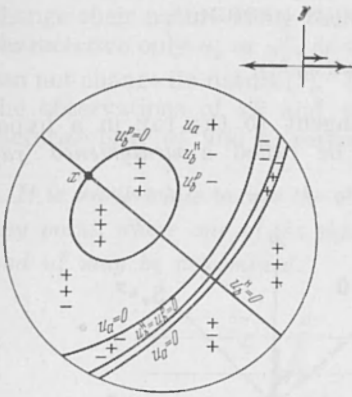


c) the superposition of dipoles with and without moment



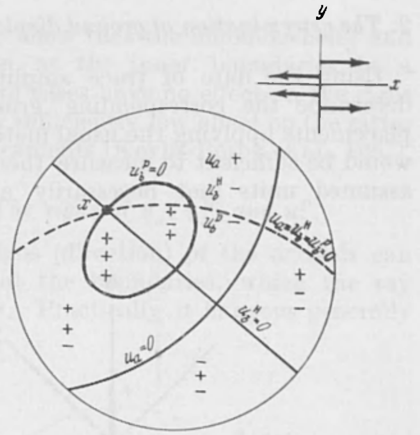
d) the superposition of *a*) and dipole with moment

Fig. 5 – The nodal lines for various sources. All signs can be reversed simultaneously.

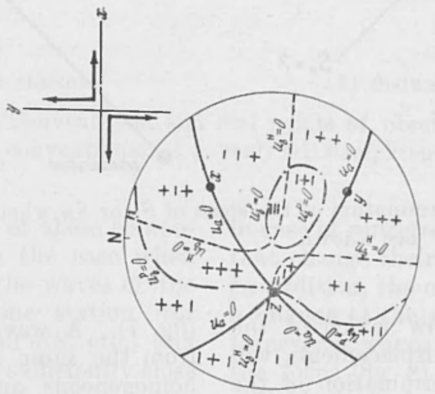


e) the superposition of a) and b)

Fig. 5 (*continuation*) The nodal lines for various sources. All signs can be reversed simultaneously.



f) the double dipole with moment



g) two dipoles with moment.

§ 3. – THE DETERMINATION OF INITIAL OBSERVATIONS.

1. *Measurements on seismograms.*

The signs and amplitudes of the first arrivals in all body and diffracted (head) seismic waves (the paths of which are known) can serve as initial data (in any combination).

Let P_z and P_v be vertical and horizontal components of the ground displacement (for the case of an incident longitudinal wave); S_H, S_v — horizontal components on the ground displacement in SH and SV waves (for the case of an incident transverse wave); S_z is the vertical component of the ground displacement (for an incident transverse wave); S_z and S_v are the components of the SV wave.

For the disturbance of the earth surface caused by the arrival of a longitudinal wave (P, SP, etc.) only one component is independent-either vertical or horizontal.

For the displacement caused by a transverse wave two independent components can be measured: S_H and S_v or S_z.

If the data are not too numerous, it happens to be useful to determine the signs and to measure the amplitudes for all the interpreted phases on all available records. Not all the measurements can give independent initial data; but the rest can be used for the control and recognition of anomalies in azimuths and angles of emergence (these anomalies can indicate the necessity of accounting for the corresponding inhomogeneity of the medium).

2. *The determination of ground displacements.*

Using the date of trace amplitudes we determine the corresponding ground displacements applying the usual methods. It would be sufficient to measure them in any assumed units (not necessarily absolute).

§ 4. – Reduction of observations to a homogeneous medium.

1. *General statements.*

A semi-tangent to the ray in a hypocentre will be called *a straightened ray*

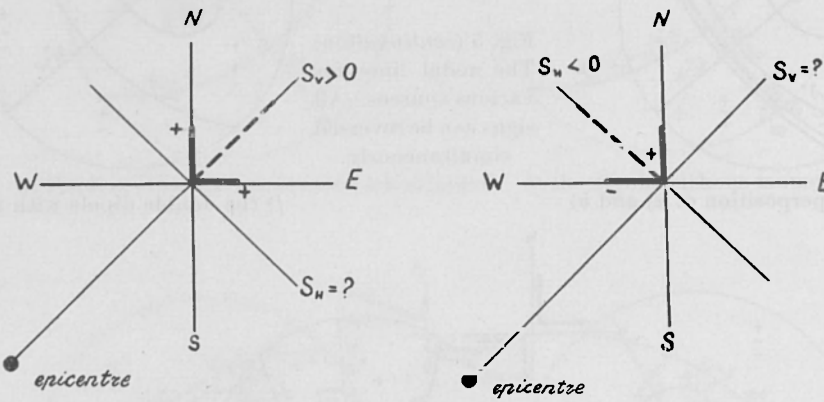


Fig. 6 – The determination of the signs of S_v or S_H when only the signs of NS and EW are known.

It should be noted now which of the observations of ground displacements will be necessary for the determination of the signs of u_a, u_b^P, u_b^H.

For the determination of the signs of u_a it is sufficient to known only the sign of P_z or P_v.

The determination of the signs of u_b^H and u_b^P require the knowledge of the direction of the total horizontal ground displacement (it should be resolved into S_v and S_H) and thus, — the values of the NS and EW components. When only the signs of these components being known, the sign of one of the components S_H or S_v can be determined (fig. 6).

Finally, the sign of S_z unambiguously determines the sign of u_b^P, provided the angle of incidence to the earth surface is less than critical.

As a result of this part of interpretation a table should be made for all the found signs and amplitudes of P_z, P_v, S_H, S_v, S_z in various waves.

(fig. 7). A wave that would be observed from the same focus if the medium were homogeneous and ideally elastic will be called a *primary wave.*

Using the observed ground displacements (they can be P_z, P_v, S_H, S_v, S_z) we shall try to find *such signs (and maybe ratios) of* u_a, u_b^P, u^H *in the primary wave that would be observed on the straightened rays at great distance from the source.* The data sought for are constant along a straightened ray beginning with sufficiently great distance; this permits to ascribe them to *conventional points* of observations — the stereographic projection of the straightened rays.

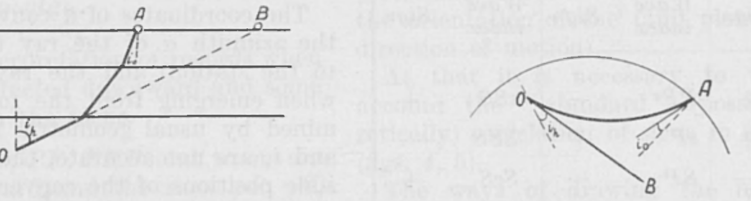
The same idea lies in the method of Byerly except that he takes another projection and does not consider transverse waves.

2. *Use of observations.*

If the nature of the wave does not change along its path then u_a is determined by P_z or P_v; u_b^H — by S_H, and u_b^P — by S_v or S_z.

The observations of the waves that change their nature along their paths may characterize only u_a or u_b^P, as the SH wave can not change its nature (*). (For example the observations of sP and pS may characterize only u_b^P and u_a respectively).

It is worth-while to use the observations in any point, where one of the signs of u_a, u_b^H, *and* u_b^P *may be determined.*



a) near stations — *b*) distant stations

Fig. 7 - Conventional and real points of observation. (B — conventional, A — real) OB straightened rays.

The ratio of the values of these components will be used only in the case when they are determined for the waves of the same path recorded at one station (for example, P and S, PP and SS, etc.) and consequently correspond to sufficiently close conventional points (theoretically it is possible to use the displacement ratio in different points; however, it is more complicated and, as experience shows, considerably less reliable).

3. *Determination of signs and displacement ratio in the primary wave.*

One should eliminate the effect of the inhomogeneity of the medium so that to determine the displacements in a primary wave.

In practice one can eliminate the effect of only known boundaries, and the curvature of the ray. However, the experience in interpretation and the theoretical calculations show that the inhomogeneity and refraction at the inner boundaries in a number of cases have no effect on the signs and have sufficiently low effect on the ratios of displacements [Keylis-Borok et al., 1957].

a) *The signs of* u_a, u_b^H, *and* u_b^P.

The signs (direction) of the arrivals can change at the boundaries, which the ray traverses. Practically it happens generally in case of reflection or in case of such waves that change their nature along their paths. In addition, the curvature of the ray causes a change of the sign of the SV arrival in transverse waves emerging *downward* from the focus (fig. 8).

Table 2 can be utilized to determine the signs of u_a, u_b^P taking into consideration the signs of the observed displacements. The table is compiled taking into account the surface of Earth, the curvature of the rays and the two boundaries of the crust of primary importance.

The effect of any other boundary will be introduced into this if we take into account the signs of the corresponding coefficients given in the theory of plane waves.

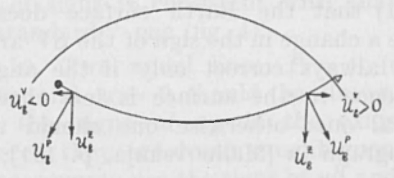


Fig. 8 - The changing of the sign of SV due to the curvature of the ray.

(*) An exception is the case of non-parallel boundaries when it is necessary to resolve a transverse wave into SH and SV for each plane of incidence anew; in this case the surface displacements S_v and S_H will be linear functions of u_b^H and u_b^P.

Table 2

The occurrence of like and unlike signs in the primary wave displacements and the earth surface displacements.
" + " indicates that the signs are the same, " — " that the signs are different. For transverse waves the sign corresponds to the horizontal S_v component of ground displacements. In a primary wave u_a or u_b^P can be determined when the first letter in the wave index is P or S respectively. u_b^H has the same sign as S_H.

Wave index	*Sign*	*Wave index*	*Sign*	*Wave index*	*Sign*
$\bar{P}$	+	PP^x	—	sSS	+
P	+	sP	+	SSS	—
pP	—	SP	—	ScS	+
PP	—	$\bar{S}$	+	sS^x	—
pPP	+	S (*)	+	SS^x	+
PPP	+	S (**)	—	pS	—
PcP	+	sS	—	PS	—
pP^x	—	SS	+		

(*) The ray moves upward from the focus.
(**) The ray moves downward from the focus.

Besides table 2 is compiled on two assumptions:

1) that the Earth surface does not cause a change in the sign of the SV arrival. It is always correct only if the angle of iucidence to the surface is less than the critical one; otherwise one should use a nomograph in [Malinovskaja, p. 151];

2) the sign of PP, pP changes (and sS, SS does not) after reflection at the surface. It is correct practically for all epicentral distances if a_0 (P-wave velocity at the surface) is not more than 5.5-6 km/sec, which generally occurs. The case of greater a_0 is considered by J. H. Hodgson and R. E. Ingram (*Bull. Seism. Soc. Amer.* 46, N. 3, 1956).

b) *Determination of amplitudes.*

For the elimination of the effect of the boundaries traversed by a ray it is necessary to divide the ground displacements by plane wave reflection and refraction coefficients. These coefficients are given in [Keylis-Borok et al., suppl. IV].

It must be borne in mind that for the angles of incidence greater than the critical angle u_b^P is the least reliable.

4. *Plotting of conventional points on a Wolf stereographic projection.*

The coordinates of a conventional point: the azimuth a of the ray (from epicentre to the station) and the ray inclination i_h when emerging from the focus are determined by usual geometric methods. If a and i_h are not accurate, the region of possible positions of the conventional point is outlined. The observations of longitudinal and transverse waves with identical rays (for instance, P and S, PP and SS, pP and sS and so on) at one station may be referred (in first approximation) to one conventional point. Strictly saying, it is not absolutely accurate. However, the discrepancy of the conventional points usually may be neglected; but the procedure of the interpretation based on signs will not change if for greater accuracy we divide the conventional points mentioned above.

a) Near stations.

a can be directly measured on a map (if there are no azimuthal anomalies pointing out to the presence of inclined interfaces). i_h is determined in accordance with the seismogeological conditions of the region.

For the case of non-horizontal interfaces a and i_h may be determined by making certain constructions on a Wolf stereographic projection. These constructions suggested by E. N. Bessonova are described in [Keylis-Borok et al.] and are significant when sharp inclined interfaces are observed near the focus.

b) Distant stations.

At distant earthquakes the inhomogeneities of the earth crust have a less distorting effect on the rays.

a can be determined on a Wolf stereographic projection; it should be kept in mind that the difference of a from the azimuth from the station to the epicentre is not 180° due to non-parallel meridians.

The plots for determining i_h and i_o (angle of incidence to the surface) for various waves and various h drawn on the basis (for distant stations) of the Jeffreys-Bullen travel-time curve are given in figs. 16*a-f*. For the determination of i_h one can also use the tables of "extented distances" compiled by J. H. Hodgson; these distances are equal to ctgi_h.

c) Joint interpretation of records when some rays are directed downward and some — upward from the focus.

The rays *P*, *S*, *PP*, *PKP* and so on are directed downward and the rays *pP*, *sS*, etc. — upward from the focus. The rays $\bar{P}$, $\bar{S}$, from near earthquakes are directed upward, and the rays of head waves — downward from the focus. However, all the conventional points (for all waves) should be plotted on one semi-sphere (better on the upper one).

If the straightened ray does not traverse this semi-sphere (the wave propagating downward from the focus), the conventional point should be taken on the extension of the ray in the opposite direction (i_h remains the same, to a the value of 180° is added).

The signs and ratios of u_a, u_b^H, u_h^P in such a conventional point will be the same as on the straightened ray for the dipole with moment and sources, *c*, *b*, *g* (fig. 5), etc.; for the sources *a*, *f*, etc. the signs considered should be reversed.

As a result of this part of interpretation all the conventional points of observations are plotted on a Wolf stereographic projection, if the sign of even one of the displacement components for these points being found; determined signs of u_a, u_b^H, $u_b^{..}$ are put near them (top to bottom); the unknown signs are designated by a wavy line $\sim$.

For the conventional points where all the amplitudes of u_a, u_b^H, u_h^P are known it is necessary also to determine the ratios u_a/u_b^H, u_b^P/u_b^H.

§ 5. – Determination of the dynamic parameters of a focus (fault plane solution).

1. *Drawing of nodal lines.*

The next and the principal problem is to draw one of the theoretical systems of nodal lines (figs. 4, 5) on a Wolf stereographic projection in accordance with the observed distribution of the signs of diplacements (according to § 1 the nodal lines determine the orientation of the fault plane and the direction of motion).

At that it is necessary to take into account the "standard" (possible theoretically) correlation of signs in each point (figs. 4, 5).

The ways of drawing the nodal lines can be explained best of all by examples (here, as for any kind of interpretation, it is difficult to make the general rules exhausting). In figs. 9-12 four examples are given. They are based on the experience of practical interpretation, but greatly simplified.

Fig. 9. Two nodal lines are necessary for dividing the signs of u_a, however, they are not drawn unambiguously (fig. 9*a* and fig. 9*b*). It can be easily seen that taking into account the signs of $u_b^{..}$ and u_b^I we can draw the nodal lines only in one way, as is shown in fig. 9*b*. The obtained combination of signs is consistent with that of the "standard" one (fig. 4).

Fig. 10. Two nodal lines: $P = 0$ (i. e. $u_a = 0$) and $y = 0$ should be drawn to divide the signs of u_a; in the example considered they are drawn unambiguously. It is necessary to use the signs of $u_b^{..}$ and u_b^I so that to find out which of these lines corresponds to the fault plane $y = 0$; the signs can be dibided in two ways shown in fig. 10*a* and 10*b*. The comparison of both variants with the "standard" one (fig. 4) leads to the conclusion that such a combination of signs as obtained in fig. 10*a* cannot exist in reality.

Thus, the only possible variant of interpretation is that shown in fig. 9*b*.

Fig. 11. The signs of u_a $u_b^{..}$, and u_b^P, can be divided in two ways as it is shown in

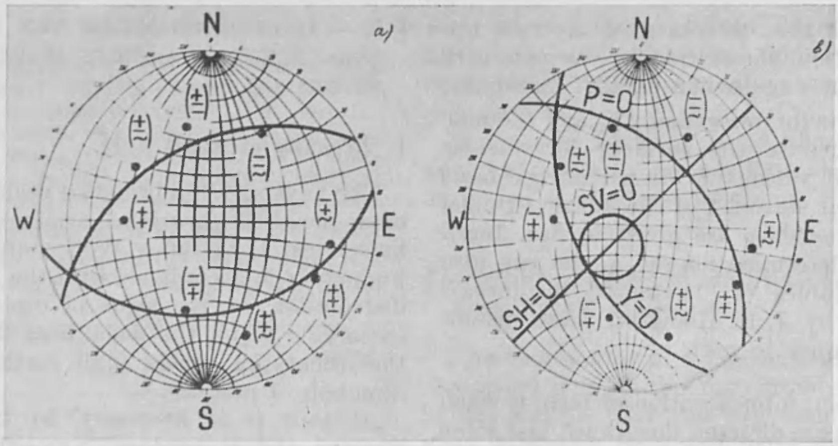


Fig. 9

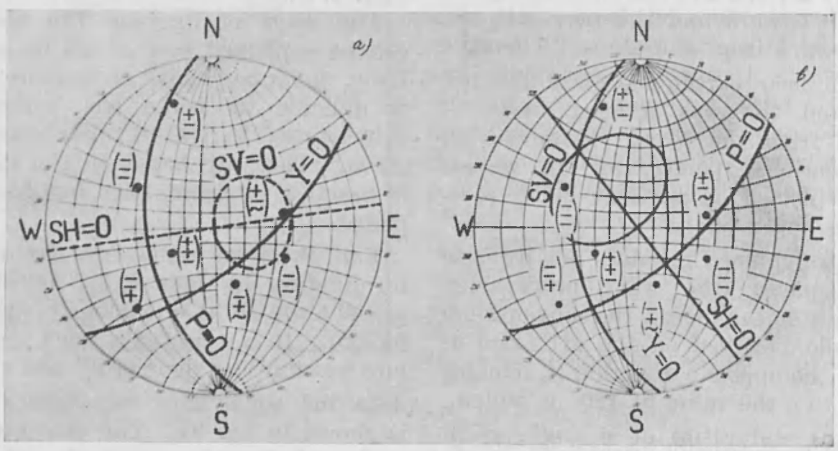
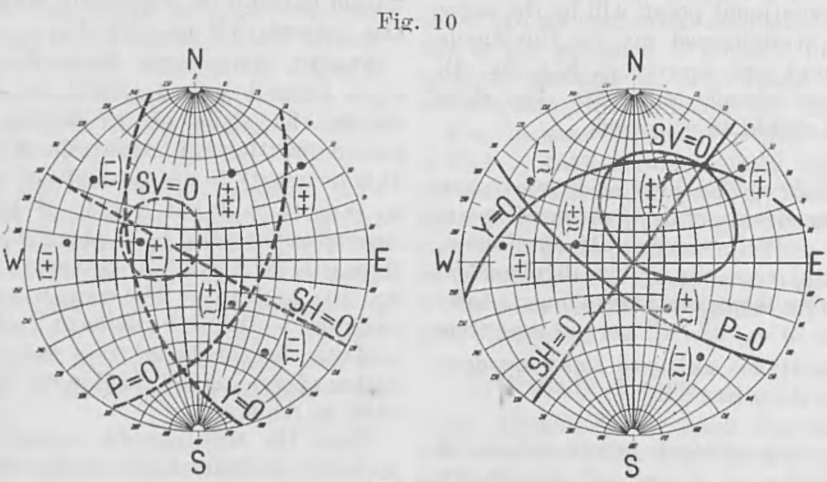


Fig. 10



a) Fig. 11 b)

Fig. 9-11 – Examples of the drawing of nodal lines. On $Y = 0$ $u_a = u_b^P = u_b^H = 0$.

fig. 11*a* and 11*b*. However, after comparison with the " standard " distribution the variant of 11*a* should be neglected.

Fig. 12. The interpretation with the help of signs is ambiguous: the system of nodal lines can be drawn in two ways (fig. 12*a* and 12*b*), both variants are consistent with fig. 4. However, it is possible to establish which of the two indicated variants corresponds to the reality if the projection of the axis x is determined using the ratio of amplitudes.

Here for the upper right point we had: $h^H = -2.5$; $h^P = 7.0$; $f = -0.35$; and for the lower (second to the right) $h^H = 0.25$; $h^P = 0.5$; $f = +0.5$. In this case the variant in fig. 12*a* should be neglected (see the next po nt).

It would be sufficient to determine only the region, where the projection of the axis x lies.

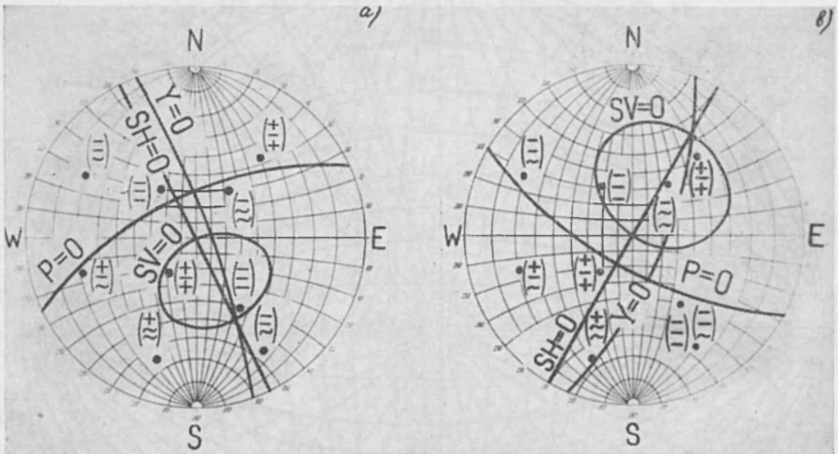


Fig. 12 – Examples of the drawing of nodal lines. On $Y = 0$ $u_a = u_b^P = u_b^H = 0$.

2. *The determination of the axis x (direction of motion) using displacement ratios.*

The displacement ratios are considerably greater effect by the inhomogeneities and non ideally elastic properties of the earth than the signs; at the same time the effect of these factors can only partly be allowed for in reducing the observations to a homogeneous medium.

Therefore, only the approximate direction of the axis x — the region of its projection on a Wolf stereographic projection — can be determined, and it is far from being every case. However, it is not connected with additional computation and often may be useful.

Equations [7a]-[9a] § 2 can be used for determining the axis x. Each equation determines the plane, containing this axis (only two of these equations are independent). If u_a, u_b^H, u_b^P are known at some point the axis x can be found as the intersections of two such planes. The graphical way (on a Wolf stereographic projection) of determining the axis x is the most convenient one. The projections of the planes mentioned above on a Wolf stereographic projection are arcs of a great circle (they will be called " x-arcs " further on); the intersection of them determines the projection of the axis x. In practice the determination of the axis x on a Wolf stereographic projection is purely graphical, using the nomographs given in [Malinovskaja] and in fig. 13 (*).

(*) For a more complete set of nomographs apply to the Institute of the Earth's Physics.

On the nomographs there are " x-arcs " for various values of h^H, h^P, f. These values are plotted near the " x-arcs ".

Different nomographs correspond to different values of i_h — the inclination of a straightened ray. i_h coincide with the centre of the nomograph, and the mark of the azimuth of the conventional point of observation with the upper end of the vertical diameter of the nomograph (then the conventional point on the tracing paper will coincide with the

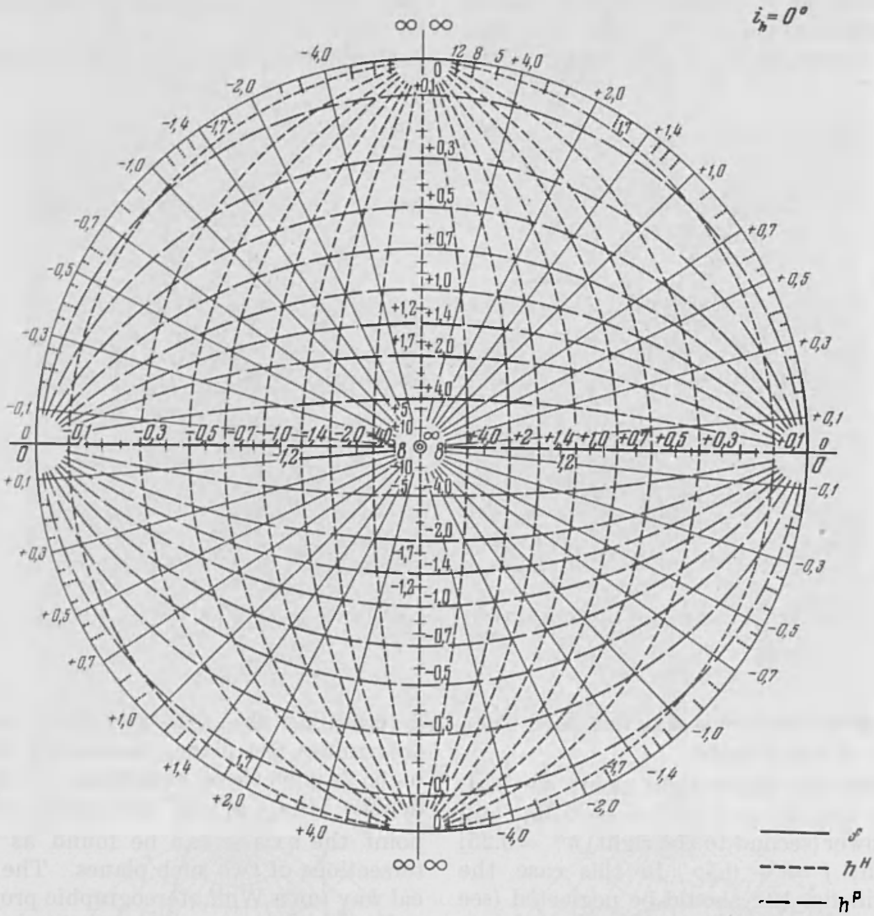


Fig. 13*a* – The nomographs for determining motion direction using amplitude ratio.

In order to find an " x-arc " it is necessary: 1) to choose a nomograph with the values of $i = i_h$ (if the nomograph with the required value of i_h is absent the whole construction is made for two neighbouring values of i_h and then interpolation is carried ot); 2) to superimpose tracing paper on the nomograph; 3) to make the centre of the projection on the tracing paper coincide with the double circle on the nomograph); 4) to copy the " x-arcs " corresponding to the given values of h^H, h^P, f on the tracing paper. The intersection of the " x-arcs " gives the projection of the axis x. In practice we do not draw the whole arcs and find *immediately* their point of intersection.

The " x-arcs " constructed for three displacement ratios at one station intersect in one point as one of these values is not independent.

Besides, it is necessary to note the following properties of "x-arcs": the "x-arcs" constructed for f and h^H pass through the point of observation and the pole of this point respectively; and the "x-arcs" constructed for h^P have the azimuth perpendicular to the azimuth of the station.

These properties should be taken into account when trying to determine the axis of x using "x-arcs" for the amplitude ratios of the same components at various stations. For instance, i_h being great, then all the "x-arcs" for h^H come closer to the projection centre, no matter what is the actual direction of the axis x (since the poles of all the conventional points gather around the centre).

The value of k should be found empirically by: 1) selecting such earthquakes for which the axis x is determined only from signs and 2) determining k so that the displacement ratio should give a close result. These same values of k may be used for studying the focii of a given region.

It proceeds from experience that k should be taken less than $(a/b)^3$. For the Tango earthquake 1927, $1 \leqslant k \leqslant 3$; it was established in the process of interpretation of

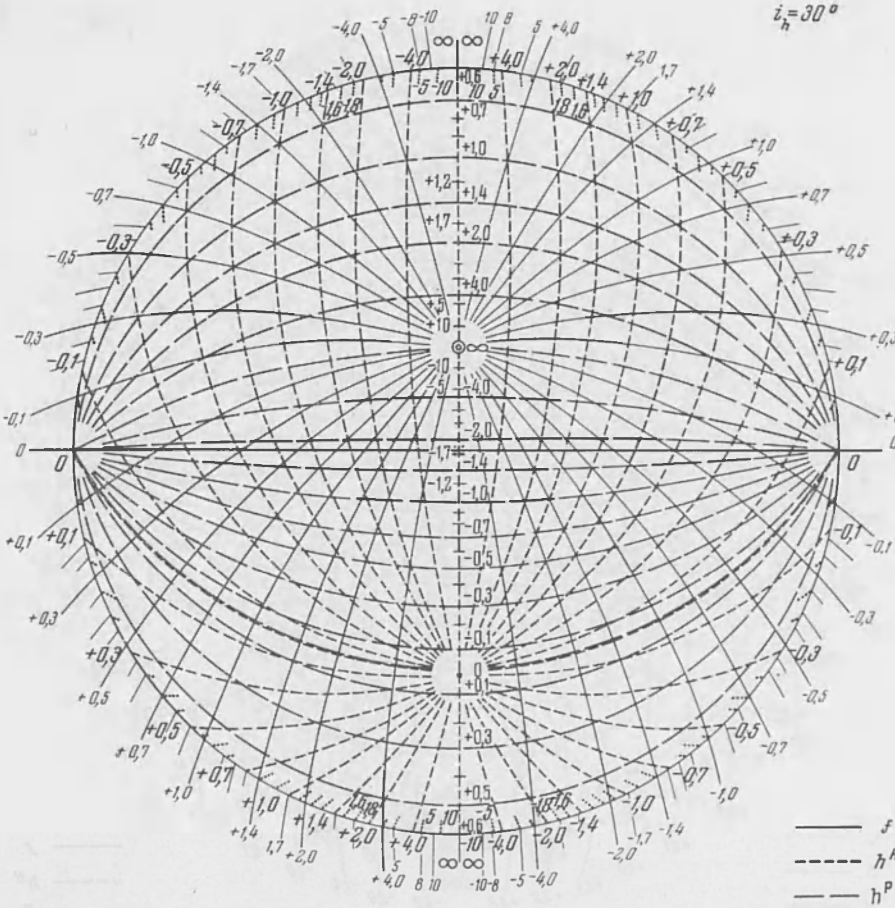


Fig. 13*b* – The nomographs for determining motion direction using amplitude ratio.

a valuable set of seismograms collected by Dr. Hodgson, E. A. (Dominion Observatory, Canada), and kindly sent to me by Dr. Hodgson, J. H.

Any excess in the accepted value of k can be easily noticed from the displacement of the axis x to the corresponding conventional points.

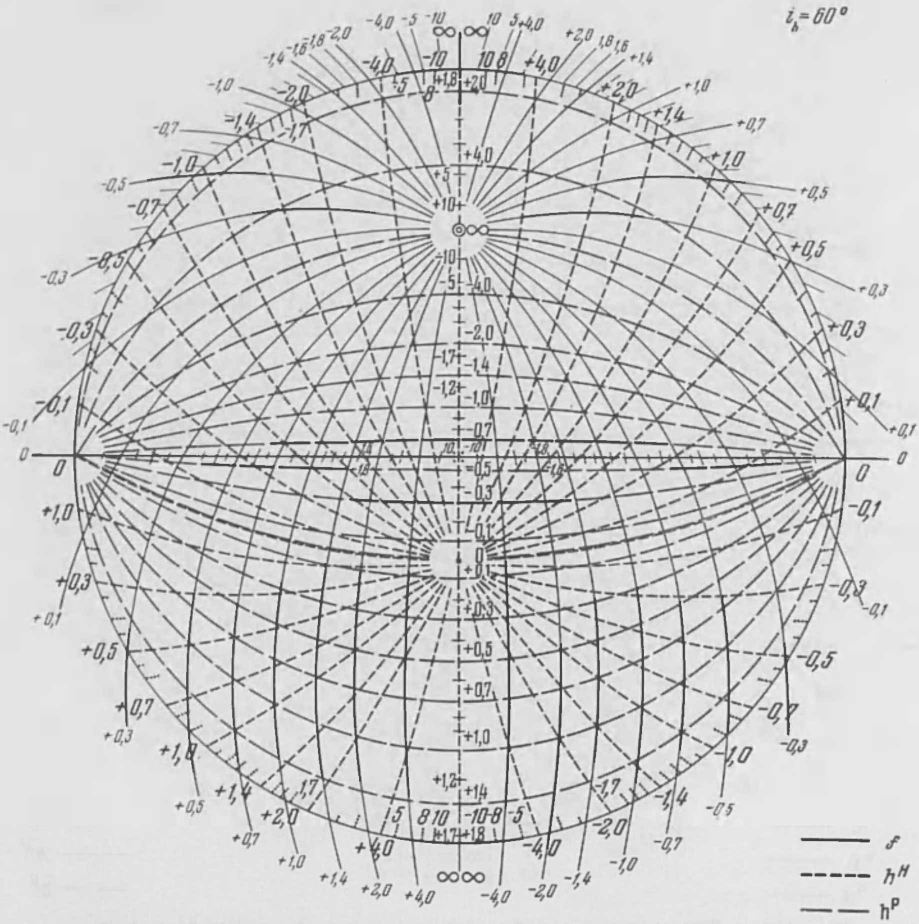


Fig. 13c – The nomographs for determining motion direction using amplitude ratio.

§ 6. – Estimation of accuracy

The accuracy of the interpretation of each earthquake is estimated directly in the process of drawing the nodal lines: each system of the initial data determines the nodal lines in such a way that they can be transferred in certain limits which indicate the possible errors.

It should be borne in mind that these limits essentially depend on the errors of the coordinates of the conventional points themselves (i. e. of the ray directions in the hypocentre).

The main source of errors (especially for near earthquakes recorded by high frequency instruments) is the unknown structure of medium, foremost, the interfaces near the focus.

The complete estimation of the accuracy requires that the interpretation should be conducted for the extreme possible positions of the conventional points.

For the interpretation on a large scale it is necessary to investigate throughly the possible errors of at least one or hypocentres of each group.

The author is greatly obliged to Prof. P. Caloi for his valuable remarks and help in publication of the present article.

Institute of the Earth's Physics,
Ac. Sci. USSR, Moscow.

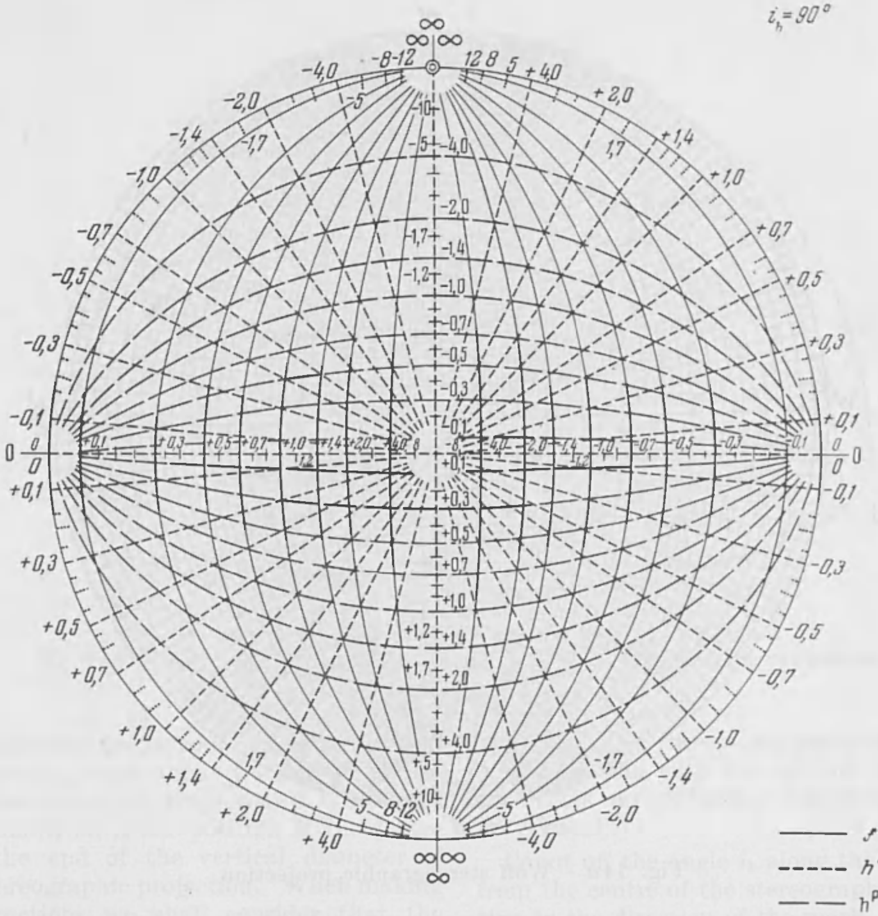


Fig. 13*d* – The nomographs for determining motion direction uring amplitude ratio.

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SUPPLEMENT. PRINCIPAL CONSTRUCTIONS ON A WOLF STEREOGRAPHIC PROJECTION.

Fig. 14*a* represents a Wolf stereographic projection which is a stereographic projection of a semi-sphere on a plane. The scheme of the semi-sphere being projected is shown in fig. 14*b*. The centre of the projection with four points around it corresponds to the vertical axis 00. The circumference bounding the projection corresponds to the circle upon which the semi-sphere is supported to stand.

Let the ends of the diameter NS (fig. 14*b*) be the poles of the sphere; draw two systems of lines on the sphere: meridians (sections of the sphere by planes forming different angles with the axis 00) and parallels (section of the sphere by planes perpendicular to NS). These lines should not taken for geographical meridians and parallels since the centre of the sphere coincides with the hypocentre.

The angle β is counted off along the horizontal diameter from the centre to a given meridian. The angle γ for each parallel is counted off along anyone of the meridians (each unit is equal to 2° in fig. 14*a*).

The straight line and the plane passing through the centre of the sphere are represented on the Wolf stereographic projection respectively by a point and by such an arc which after being rotated coincides with one of the meridians. The pole of the plane means the projection of its normal.

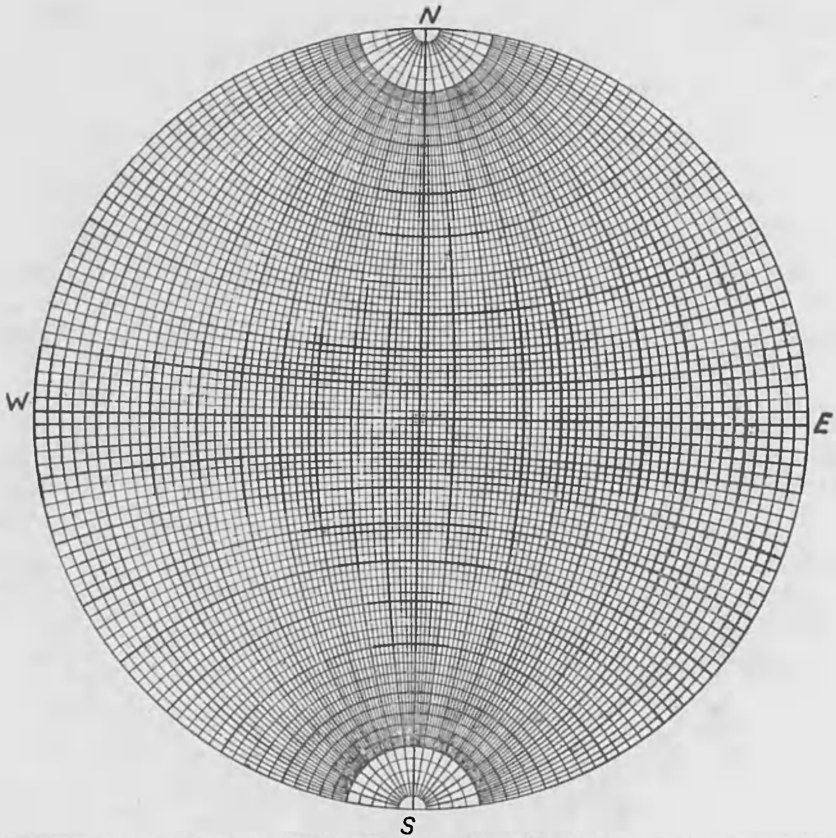


Fig. 14*a* – Wolf stereographic projection.

The pole of the line is the projection of its normal lying in one vertical plane with tihs line.

All constructions on a Wolf projection can be most conveniently fulfilled on a tracing paper being superimposed on the

α of the horizontal projection) is given. Find the projection of this line (i. e. the point having the coordinates α, i_h).

On the outer circle (the bound of the stereographic projection) count off the angle α clockwise and make a mark.

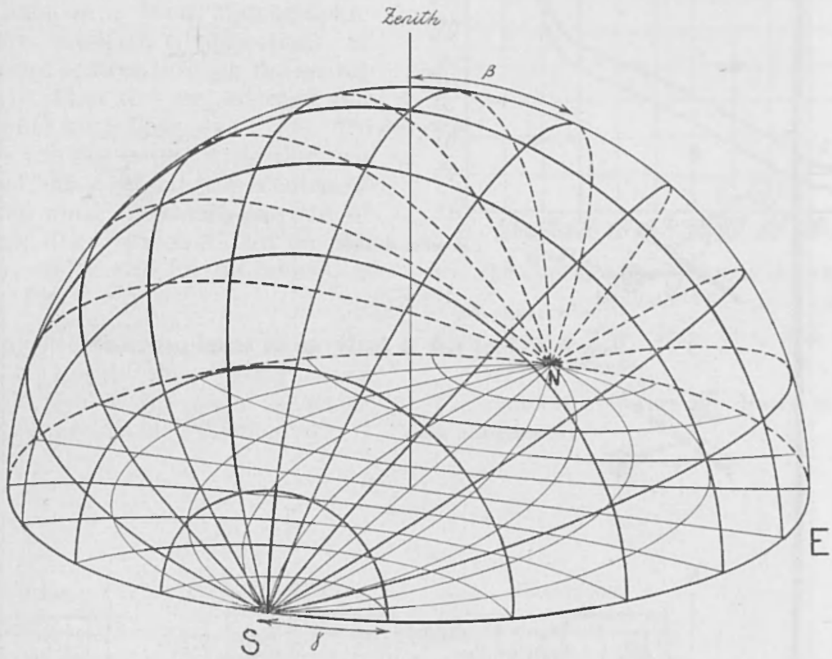


Fig. 14 *b* – Meridians and parallels traced on a Wolf stereographic projection.

stereographic projection. Before rotating the tracing paper about the centre of the Wolf stereographic projection it is necessary to mark on it the position of the centre and the end of the vertical diameter of the stereographic projection. When making constructions we shall consider that the direction upward ($\bar{z}$ axis) is projected in the centre of the stereographic projection, the axis $\bar{N}$ being directed to the North ($\bar{y}$) the axis $\bar{E}$ to the East ($\bar{x}$).

Consider now the principal constructions on a Wolf projection.

The planes and the straight lines considered below pass through the centre of the sphere.

1. The direction of a straight line (the inclination i_h to the vertical, the azimuth

By rotating the tracing paper make this mark coincide with one of the diameters (it makes no difference either horizontal or vertical).

Count off the angle i_h along the diameter from the centre of the stereographic projection to the direction of the mark.

Now the point being found has the coordinates α, i_h.

2. The azimuth A of the dipping and the dip e of a plane are given. Find the projection of the plane.

Make a mark for the azimuth A on the bound of the stereographic projection. Rotating the tracing paper make this mark coincide with the horizontal diameter. Count off the dip along the same horizontal diameter from the stereographic projection

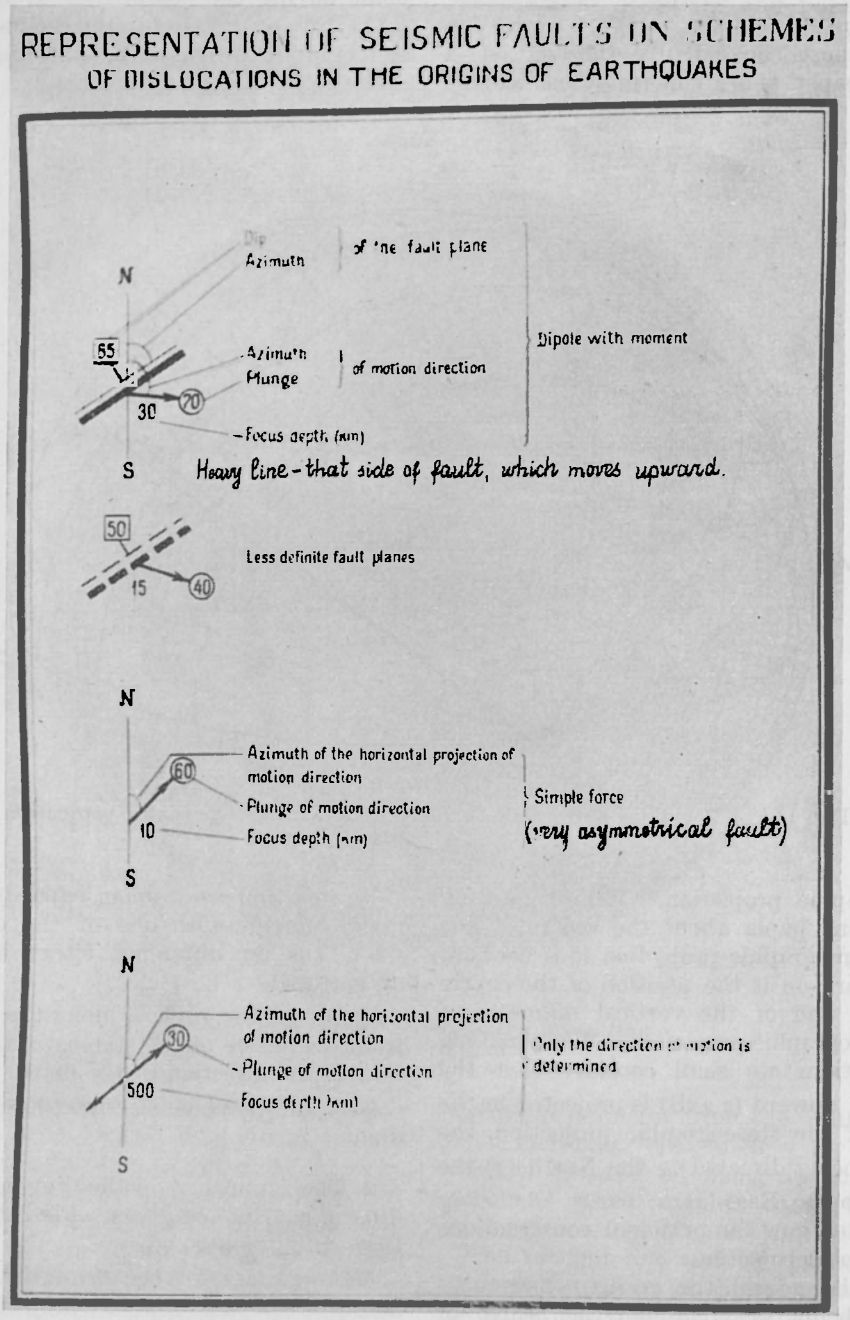


Fig. 15 - The representation of the found fault plane solutions on maps. The heavy line corresponds to that side of fault which moves upward.

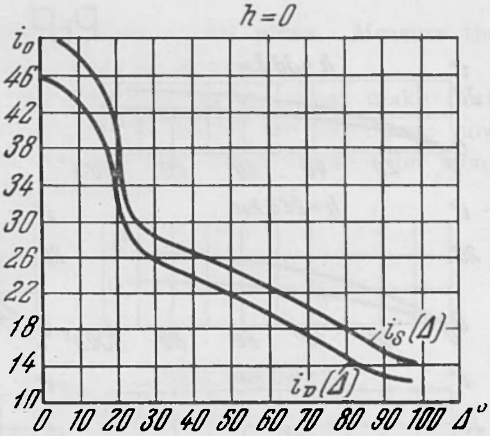
bound which is opposite to the mark of the azimuth.

Now the meridian passing trhough the found point is the plane being sought for (the coordinates of this point, as is evident, are $180 + A$, $90 - e$).

3. Two points on a Wolf stereographic projection are given (i. e. projections of the straight lines passing through the center of the sphere). Find the projection of the plane containing both lines.

Rotate the tracing paper with the two points plotted on it about the centre of the projection until they fall on one of the meridians; draw this meridian on the tracing paper, and it will be the projection being sought for.

4. The projection of a plane is given. Find its pole. Rotate the tracing paper until the projection of the plane coincides with one of the meridians. Then count off



h = the depth of the source is shown near each curve.

Figs. 16 *a-f* – The values of i_h and i_0 for various waves.

The symbols of waves are drawn in each figure.

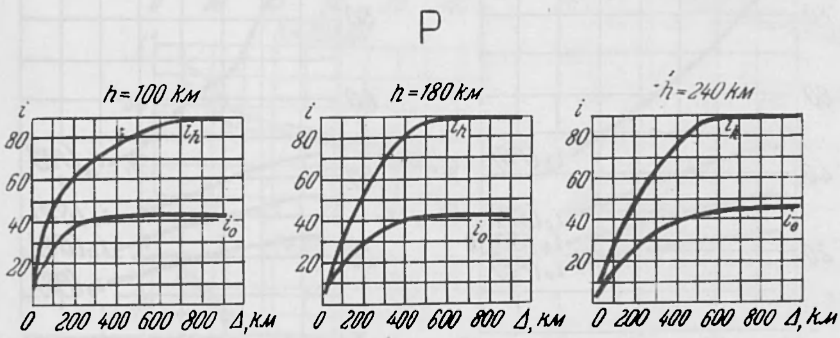


Fig. 16 *b*

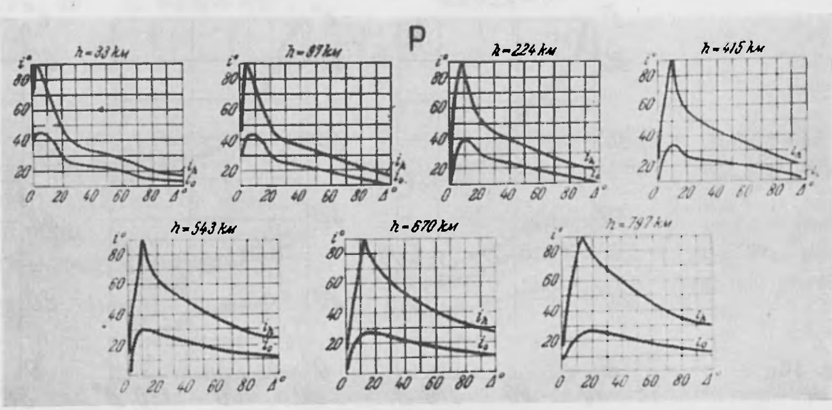


Fig. 16 *c*

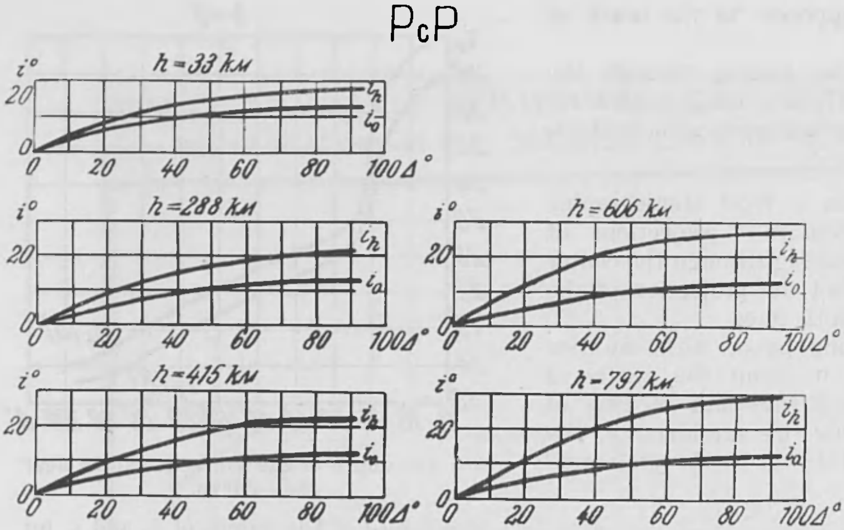


Fig. 16d

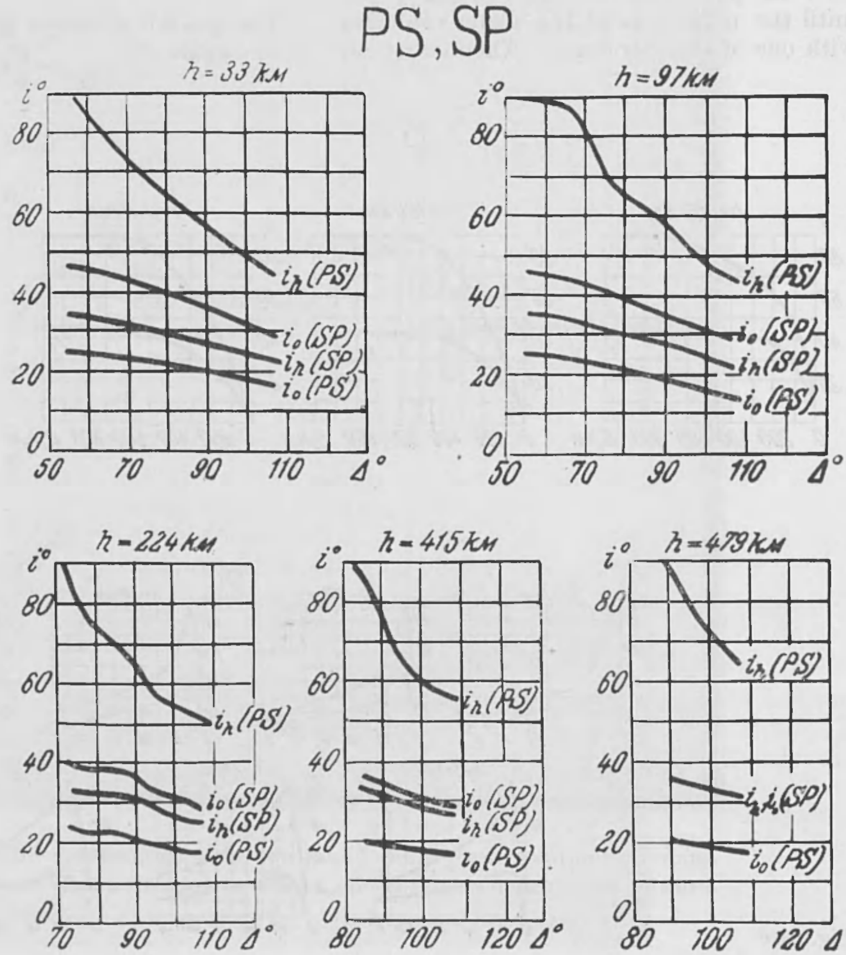


Fig. 16e

90° along the horizontal diameter from the plane to the direction of the centre. Now the found point is the pole.

5. The pole of a plane is given. Find the projection of the plane (the equator of the given pole).

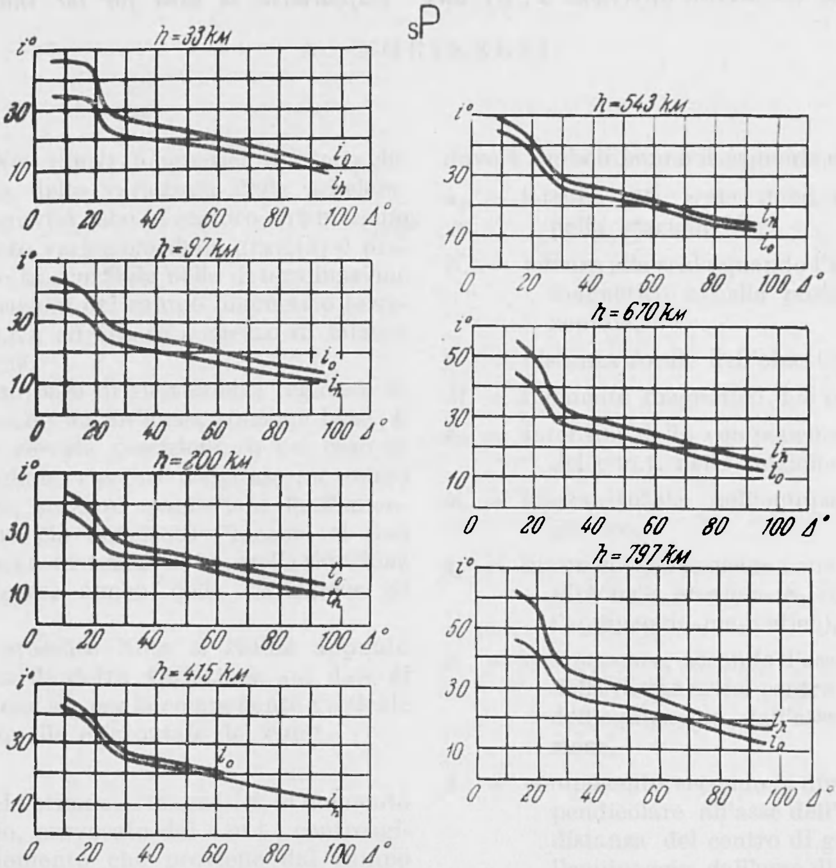


Fig. 16

Make the pole fall on the horizontale diameter and count off 90° along it to the direction of the centre. Draw the meridian through the found point; then this meridian is the equator bring sought for.

6. Two points — the projections of straight lines — are given. Measure the angle made by them.

Rotating the tracing paper make both points fall on one of the meridians; now the angle sought for is counted off along this meridian.

7. The projections of a plane and a straight line are given. Find the angle made by them.

Make the projection of the plane coincide with one of the meridians; now the angle sought for is counted off along the parallel passing through the given point.

ABSTRACT

The interpretational aspect of determining fault plane solution for earthquake sources with the use of both longtitudinal and transverse waves of various types is described. The first arrival direction of P, SV, SH and the ratios of their amplitudes can be employed. The use of the arrival directions P, SV and SH (and especially their combination at each point) sharply lessens the quantity of observations required and makes results unambiguous.

The properties of various sources (the dipole with moment in most detail) are considered.

Wolf stereographic projection discribed in Supplement is used for the interpretation.